

# GCE

## **Mathematics**

Advanced GCE

Unit 4730: Mechanics 3

### Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Any enquiries about publications should be addressed to:

OCR Publications PO Box 5050 Annesley NOTTINGHAM NG15 0DL

Telephone:0870 770 6622Facsimile:01223 552610E-mail:publications@ocr.org.uk

#### Annotations and abbreviations

Annotation in scoris	Meaning
√and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
٨	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning	
E1	Mark for explaining	
U1	Mark for correct units	
G1	Mark for a correct feature on a graph	
M1 dep*	Method mark dependent on a previous mark, indicated by *	
сао	Correct answer only	
oe	Or equivalent	
rot Rounded or truncated		
soi	Seen or implied	
www	Without wrong working	

#### Subject-specific Marking Instructions for GCE Mathematics (OCR) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

#### Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

#### Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

#### Mark Scheme

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

#### When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

#### When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

#### g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

	Question	Answer	Marks	Guidance
1	(i)	$[40d = 30 \times 2]$	M1	For taking moments about B for BC
		Distance is 1.5 m	A1	
			[2]	
	(ii)		B1	
		Horizontal component on <i>AB</i> at <i>B</i> is 40 N to the left	B1	
		For resolving forces on BC vertically, or	M1	$Y + 30 = 40$ , or $40 \ge 1/2 = Y \ge 2$
		taking moments about C		Accept directions on diagram, if not contradicted in text
		Vertical component on <i>AB</i> at <i>B</i> is 10 N down	A1	SR A1 if both magnitudes correct but directions wrong/not stated
			[4]	
	(iii)		M1	For taking moments about A for AB
		$(+/-)10 \times 2 + 60 \times 0.8d = (+/-)40 \times 1.5$	A1 FT	FT magnitudes of components at B; need to use ' $x = d\cos\theta$ '
		Distance is 0.833 m	A1	
			[3]	May see moments about A for $ABC$ (60 x $0.8d + 40$ x $3.5 = 30$ x $4 + 40^{\circ}$ x $1.5$ ) or
				moments about B for $AB$ – need to get equation with only 'd' unknown for M1
2	(i)	Since plane is smooth impulse is	B1	
		perpendicular to plane( so $\theta = 15$ )	[1]	
	(ii)	Use of $v^2 = (u^2) + 2 \times g \times 2.5$	M1	
		$v = 7 \text{ ms}^{-1}$	A1	
		after impact:		
		Speed parallel to plane is 7sin15°	B1	1.81(173)
		$u = 7\sin 15^\circ / \cos 60^\circ$	M1	Allow sin/cos errors
		u = 3.62	A1	
		$I = 0.45(7 \cos 15^\circ + u \sin 60^\circ)$	M1	Allow sin/cos errors or $I = 0.45(7 \cos 15^\circ + 7\sin 15^\circ \tan 60^\circ)$
		I = 4.45	A1	4.45477 May see $e = 0.464$
			[7]	
		Or For using a triangle with sides 3.15 (0.45	M1	Need 2 correct sides and 1 correct angle
		x 7), <i>I</i> and 0.45 x <i>u</i> (or 7, I/0.45 and <i>u</i> ) and	A 1	All correct
		correct angles 135°, 15° and 30°	A1	OR $I\cos 15^\circ = 3.15 + 0.45 \ u\cos 45^\circ$ M1
		Use of sin rule or cos rule (correct) $= 2.62$	M1	$I\sin 15^\circ = mu\cos 45^\circ$ B1
		u = 3.62	A1	Solve sim equations M1, dep attempt at two comps of <i>I</i>
		I = 4.45	A1	Answers A1A1

Q	Juestion	Answer	Marks	Guidance
3	(i)		M1	For using N's $2^{nd}$ law with $a = v dv/dx$ ; 3 terms
		$v  dv/dx = g - 0.0025v^2$	A1	
		$\int \frac{v dv}{g - 0.0025 v^2} = \int dx$	M1	For correctly separating variable and attempting to integrate
		$-200\ln(g - 0.0025v^2) = x (+A)$	A1	
		$A = -200 \ln g$	M1*	Attempt to find A from $Bln(C - Dv^2)$
		$[g - 0.0025v^2 = ge^{-0.005x}]$	*M1	For transposing equation to remove ln
		$v^2 = 400g(1 - e^{-0.005x})$	A1	
		$0 < e^{-0.005x} \le 1 \Rightarrow v^2$ cannot reach 400g	B1	dependent on getting other 7 marks.
		ie cannot reach 3920		Need ' $0 <$ ' oe
			[8]	
	(ii)	$v^2 = 400g(1 - e^{-0.5})$	M1	For substituting for x and evaluating v must have $v^2 = A + Be^{Cx}$ for (i), but not neces in this form
		Speed of P is $39.3 \text{ ms}^{-1}$	A1	
		-	[2]	
4	(i)		M1	For using the pce condone sin/cos and sign errors; need KE before and after and difference in PE
		$\frac{1}{2}mv^2 + mg(0.6)(1 - \cos\theta) = \frac{1}{2}m4^2$	A1	
		$v^2 = 4.24 + 11.76\cos\theta$	A1	AG
			M1	For using Newton's 2 <sup>nd</sup> law, condone sin/cos and sign erorrs; 3 terms needed
		$R - 0.45g\cos\theta = 0.45v^2/0.6$	A1	
		$R = 3.18 + 13.23 \cos \theta$	A1	
			[6]	
	(ii)		M1	For using $R = 0$
		$\cos\theta = -3.18/13.23$	A1 FT	$-0.24036$ or $-106/441$ or $\theta = 103.9^{\circ}$ ft from $R = A + B\cos\theta$ , where $A, B \neq 0$
		$[v^2 = 4.24 - 11.76 \times 3.18/13.23]$	M1	For substituting for $\cos \theta$
		Speed is $1.19 \text{ ms}^{-1}$	A1	CAO without wrong working
			[4]	

Q	uestion	Answer	Marks	Guidance
5	(i)	[0.8mgx/0.78 = mg(5/13)]	M1	For resolving forces and using $T = \lambda x / L$ at equilibrium position
		x = 0.375	A1	Accept 1.155 for $e + l$
		$PE = mg(0.78 + 0.375) \times 5/13$	B1 FT	FT value of x
		$EE = 0.8mg \times 0.375^2 \div (2 \times 0.78)$	B1 FT	FT value of x
		$[\frac{1}{2}mv^2 = m(4.353 0.7067)]$	M1	For using $\frac{1}{2}mv^2 = PE \log - EE gain$
		Maximum speed is $2.70 \text{ ms}^{-1}$	A1	
			[6]	
		OR at extension x		
		$PE = mg(x+0.78) \times \frac{5}{13}$	B1	
		$$ $0.8mgx^2$	B1	
		$EE = \frac{0.8mgx^2}{2 \times 0.78}$		
		$mg(x+0.78) \times \frac{5}{13} = \frac{1}{2}mv^2 + \frac{0.8mgx^2}{2 \times 0.78}$	M1	For using $\frac{1}{2}mv^2 = PE \log - EE$ gain
		$v^2 = -10.05x^2 + 7.53x + 5.88$		$v^{2} = -\frac{40 \times 9.8}{39}x^{2} + \frac{98}{13}x + \frac{9.8 \times 3.9 \times 2}{13}$
		$v^2 = -10.05(x^2 - 0.749x - 0.585)$		$v^{2} = -\frac{392}{39} \left( x^{2} - \frac{3}{4} x - \frac{3 \times 3.9 \times 2}{40} \right)$
		for attempting to complete square	M1	
		$v^2 = -10.05((x - 0.375)^2 - 0.726)$	A1	$v^{2} = -\frac{392}{39}((x - \frac{3}{8})^{2} - 0.725625)$
		Max speed is 2.70 ms <sup>-1</sup>	A1	
		*		Note, after getting equation for $v^2$ , can instead
				Differentiate $v^2$ wrt x M1
				Establish max at $x = 0.375$ A1
				Max speed 2.70 ms <sup>-1</sup> A1

Q	uestion	Answer	Marks	Guidance
	(ii)	$mg(0.78 + x) \times 5/13 = 0.8mgx^2 \div (2 \times 0.78)$	M1* A1	For using PE loss = EE gain or $mg(x) \times 5/13 = 0.8mg(x - 0.78)^2 \div (2 \times 0.78)$ if $PO = x$ or $mg(x+0.78+0.375) \times 5/13 = 0.8mg(x+0.375)^2 \div (2 \times 0.78)$ if $PO = x + 0.78 + 0.375$
		$[x^2 - 0.75x - 0.585 = 0 \text{ if } x \text{ is extension}]$ x = 1.2268 so Distance is 2.01 m	*M1 A1 [ <b>4</b> ]	For arranging in quadratic form and attempting to solve All nec terms required $[x^2 - 2.31x + 0.6084 = 0 \text{ if } PO = x]$ [20x <sup>2</sup> = 14.5125, if PO = x + 0.78 + 0.375] [x = 2.0068] [x = 0.8518]
		OR put $v = 0$ in $v^2$ equation from above Solve to get $x = 1.23$ (+0.78) = 2.01 m	M1A1ft M1A1	

	Question	Answer	Marks	Guidance
6	(i)		M1	For using $\frac{1}{2}m(u^2 - v^2) = 7.56$ and solving for <i>v</i> ; must use '5', allow sign error/ missing $\frac{1}{2}$ , missing <i>m</i> .
		$\frac{1}{2} \times 2(5^2 - v^2) = 7.56$ ( $v^2 = 17.44$ )	A1	
		Speed is $4.18 \text{ ms}^{-1}$	A1	Do not award if this is not candidate's final answer.
			[3]	
	(ii	$v_{Ay} = u_{Ay} = 5\sin\alpha = 4$	B1	
		$[v_{Ax}^2 + 4^2 = 17.44 \rightarrow v_{Ax}^2 = 1.44]$	M1	For using $v_{Ax}^{2} + v_{Ay}^{2} = 17.44$
		$v_{Ax} = \pm 1.2$ and $v_{Ax}$ must be less than 0.8		
		$\rightarrow$ Component has magnitude 1.2 ms <sup>-1</sup> and		
		direction to the left	A1	
			[3]	
	(iii		M1	For using the pcm parallel to loc must use $5\cos\alpha$ , 2, 0.8 and '1.2', 4 terms or
				equivalent, allow sign errors, condone one mass missing
		$2 \times 3 - m \times 2 = 2 \times (-1.2) + m \times 0.8$	A1 FT	FT incorrect $v_{AX}$
		m = 3	A1	CAO
			[3]	
	(iv	)  [e(3+2) = (1.2+0.8)]	M1	For using NEL with their '1.2' and $5\cos\alpha$ , 2 and 0.8; allow sign errors. Must be right
				way up
		e = 0.4	A1	
			[2]	

Q	Juestion	Answer	Marks	Guidance
7	(i)		M1	For using EPE = $\lambda x^2/2L$ for both strings for one position
		$E_{(AP=2.9)} = 120 \times 0.9^2/4 + 180 \times 0.1^2/6$		
		= (24.3 + 0.3) and		
		$E_{(AP=2.1)} = 120 \times 0.1^2/4 + 180 \times 0.9^2/6$		
		$=(0.3+24.3)$ $\rightarrow$ same for each position	A1	24.6 seen twice
		Conservation of energy $\rightarrow v = 0$ when AP		Need to point out that $v = 0$ when $AP = 2.1$ or $KE = 0$
		= $2.1$ , string taut here so taut throughout motion – oe,	B1	Dep on M1A1
		motion – oe,	[ <b>3</b> ]	Dep on MIAI
	(ii)	$T_A = 120(0.5 + x)/2, T_B = 180(0.5 - x)/3$	B1	soi
	(11)	$\begin{bmatrix} (30 - 60x) - (30 + 60x) = (+/-)0.8a \end{bmatrix}$	M1	For using Newton's 2 <sup>nd</sup> law; allow omission of 0.8
		a = -150x	Al	With no wrong working
			[3]	
	(iii)	SHM because $a = -k$ (where $k > 0$ )	M1	SHM because $a = -\omega^2 x$ or in words
		$[T = 2\pi / \sqrt{150}]$	M1	For using $T = 2 \pi / n$ ; must follow from (ii)
		Time interval is 0.257 s	A1 FT	FT $\pi$ ÷ candidate's <i>n</i> 0.256509
			[3]	
	(iv)	$[x = 0.4 \cos(\sqrt{150} \times 0.6) = 0.194]$	M1	For using $x = a\cos(0.6n)$ , where <i>n</i> follows from (ii) and <i>a</i> is numerical.
		[distance = 4a + (a - 0.194)]	M1	For using $T < 0.6 < 1.25$ T $\rightarrow$ distance = 4 $a + (a - x)$ ; may be implied by 1.6 <
				distance < 2.0
		Distance travelled is 1.81 m	A1	CAO, no wrong working
			[3]	
	(v)		M1	For using $\dot{x} = -an\sin(0.6n)$ , where <i>n</i> follows from (ii)
				Or using $v^2 = n^2(a^2 - x^2)$ , where <i>n</i> follows from (ii) and <i>x</i> follows from (iv)
				or using $\dot{x} = an \cos(0.6n)$ if $x = a\sin(0.6n)$ used in (iv), where <i>n</i> follows from (ii)
		Speed is $4.29 \text{ ms}^{-1}$ .	A1	Condone –4.29
			[2]	

OCR (Oxford Cambridge and RSA Examinations) 1 Hills Road Cambridge CB1 2EU

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#### **Education and Learning**

Telephone: 01223 553998 Facsimile: 01223 552627 Email: general.qualifications@ocr.org.uk

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