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# Mark Scheme (Results)

Summer 2012

International GCSE
Further Pure Mathematics
(4PM0) Paper 01

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# General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers may score full marks.

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

| Question Number | Scheme | Marks |
|---|---|---|
| 1. | $0 > (x-4)(2x-3+2x+1)$ *or* $0 > 4x^2 - 18x + 8$ | M1 A1 |
| | $0 > (x-4)(4x-2)$ | |
| | $\frac{1}{2} < x < 4$ | M1 A1 |
| | | (4) |
| | | **[4]** |
| 2. | (a) $5^2 = 7^2 + 8^2 - 2\times 7\times 8\cos A$ *or* $\cos A = \dfrac{8^2+7^2-5^2}{2\times 7\times 8}$ | M1 |
| | $25 = 49 + 64 - 112\cos A$ | |
| | $112\cos A = 88$ | |
| | $\cos A = \frac{88}{112} = 0.7857$ | A1 |
| | $\angle A = 38.2^{\circ}$ | A1 |
| | | (3) |
| | (b) $\frac{1}{2}\times 8\times 7\sin A = 28\sin 38.2^{\circ} = 17.3\ (\text{cm}^2)$ | M1 A1 |
| | | (2) |
| | | **[5]** |
| 3. | (a) $(1+x)^5 = 1 + \dfrac{5}{1}x + \dfrac{5\times 4}{1\times 2}x^2 + \dfrac{5\times 4\times 3}{1\times 2\times 3}x^3 + \dfrac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4}x^4 + x^5$ | M1 |
| | $= 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$ | A1 |
| | | (2) |
| | (b) $(1-2\sqrt{3})^5 = 1 + 5(-2\sqrt{3}) + 10(-2\sqrt{3})^2 + 10(-2\sqrt{3})^3 + 5(-2\sqrt{3})^4 + (-2\sqrt{3})^5$ | M1 |
| | $(= 1 - 10\sqrt{3} + 10(12) + 10(-8\times 3\sqrt{3}) + 5(16\times 9) - 32\times 9\sqrt{3})$ | |
| | $(= 1 - 10\sqrt{3} + 120 - 240\sqrt{3} + 720 - 288\sqrt{3})$ | |
| | $= 841 - 538\sqrt{3}$ | A1 A1 |
| | | (3) |
| | | **[5]** |

| Question Number | Scheme | Marks |
|---|---|---|
| 4. | $\alpha+\beta=\frac{7}{2},\ \alpha\beta=\frac{4}{2}$ oe | B1 |
| | Sum of roots = $\alpha+\frac{1}{\beta}+\beta+\frac{1}{\alpha}$ *or* $\left(\frac{\alpha\beta+1}{\beta}\right)+\left(\frac{\alpha\beta+1}{\alpha}\right)$ | |
| | $=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ $\qquad$ $=\frac{(\alpha+\beta)(\alpha\beta+1)}{\alpha\beta}$ | M1 |
| | $=\frac{7}{2}+\frac{\frac{7}{2}}{2}=\frac{21}{4}$ $\qquad$ $=\frac{\frac{7}{2}(2+1)}{2}=\frac{21}{4}$ | M1 A1 |
| | Product of roots = $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ *or* $\left(\frac{\alpha\beta+1}{\beta}\right)\left(\frac{\alpha\beta+1}{\alpha}\right)$ | |
| | $=\alpha\beta+\frac{1}{\alpha\beta}+2$ $\qquad$ $=\frac{(\alpha\beta+1)^2}{\alpha\beta}$ | M1 |
| | $= 2 + \frac{1}{2} + 2 = 4\frac{1}{2}$ $\qquad$ $=\frac{(2+1)^2}{2}=4\frac{1}{2}$ | A1 |
| | Equation is $x^2-\frac{21}{4}x+\frac{9}{2}=0$ | M1 |
| | $4x^2 - 21x + 18 = 0$ | A1 |
| | | (8) |
| | | **[8]** |

| Question Number | Scheme | Marks |
|---|---|---|

| | | |
|---|---|---|
| 5. | (a) $t_r = \log_a 2^r$ oe | B1 |
| | | (1) |
| | (b) $d = t_r - t_{r-1} = r\log_a 2 - (r-1)\log_a 2$ *or* terms are $(1+2+3+\cdots)\log_a 2$ *or* difference of any two consecutive terms | M1 |
| | $= \log_a 2$ | A1 |
| | | (2) |
| | (c) $S_n = \frac{n}{2}(2\log_a 2 + (n-1)\log_a 2)$ *or* $S_n = (1+2+3+\cdots+n)\log_a 2$ *or* $S_n = \dfrac{n}{2}(\log_a 2 + \log_a 2^n)$ | M1 |
| | $= \frac{n}{2}(n+1)\log_a 2$ * | A1 |
| | | (2) |
| | (d) $\log_a 6 + \log_a 12 + \ldots - (\log_a 2 + \log_a 4 + \cdots)$ | |
| | $= (\log_a 6 - \log_a 2) + (\log_a 12 - \log_a 4) + \cdots$ *or* $= (\log_a 2 + \log_a 3) + (\log_a 4 + \log_a 3) + \ldots - (\log_a 2 + \log_a 4 + \ldots)$ | M1 |
| | $= \log_a 3 + \log_a 3 + \cdots$ | M1 A1 |
| | $= n\log_a 3$ | A1 |
| | | (4) |
| | *Alternative* $T_n = \frac{n}{2}\left(2\log_a 6 + (n-1)\log_a 2\right)$ M1 | |
| | $T_n - S_n = \left(n\log_a 6 + \frac{n}{2}(n-1)\log_a 2\right) - \frac{n}{2}(n+1)\log_a 2$ | |
| | $= n\log_a 6 + \frac{n}{2}(n-1-n-1)\log_a 2$ M1 | |
| | $= n\log_a 6 - n\log_a 2 = n\log_a 3$ A1 A1 | |
| | | **[9]** |

| Question Number | Scheme | Marks |
|---|---|---|
| 6. | (a) Area of sector = ½ $r^2\theta$, Area of triangle $OPQ$ = ½ $r^2 \sin\theta$ | M1 M1 |
| | Area of segment = ½ $r^2\theta$ – ½ $r^2 \sin\theta$ = ½ $r^2 (\theta - \sin\theta)$ * | A1 |
| | | (3) |
| | (b) $\frac{dA}{d\theta} = \frac{1}{2}r^2(1-\cos\theta)$ | M1 A1 |
| | $\delta A \approx \frac{1}{2}r^2(1-\cos\theta)\delta\theta$ * | A1 cso |
| | | (3) |
| | (c) $0.05 = \frac{1}{2}\times 4^2(1-\cos\theta)\times 0.02$ | M1 |
| | $5 = 16(1-\cos\theta)$ | |
| | $\frac{5}{16} = 1-\cos\theta$ | |
| | $\cos\theta = \frac{11}{16}$ | M1 dep A1 |
| | $\theta = 0.8$ | A1 |
| | | (4) |
| | | **[10]** |
| 7. | (a) $\cos(2x+45) = \cos 2x\cos 45 - \sin 2x\sin 45$ | M1 |
| | $= \frac{\sqrt{2}}{2}\cos 2x - \frac{\sqrt{2}}{2}\sin 2x$ *or* $= \frac{1}{\sqrt{2}}\cos 2x - \frac{1}{\sqrt{2}}\sin 2x$ | A1 |
| | | (2) |
| | (b) $\frac{\sqrt{2}}{2}\cos 2x - \frac{\sqrt{2}}{2}\sin 2x = \frac{\sqrt{2}}{2}$ | M1 |
| | $\cos(2x+45) = \frac{\sqrt{2}}{2}$ | A1 |
| | $2x + 45 = 45, 315, 405, \ldots$ | M1 |
| | $2x = 0, 270, 360, \ldots$ | |
| | $x = 0^\circ, 135^\circ, 180^\circ, \ldots$ | A1 A1 |
| | | (5) |
| | (c) $\cos(2x+45) = \frac{\sqrt{2}}{2}(\cos 2x - \sin 2x)$ | |
| | $\Rightarrow (\cos 2x - \sin 2x) = \frac{2}{\sqrt{2}}\cos(2x+45)$ | |
| | maximum when $\cos(2x + 45) = 1$ | M1 |
| | $k = \frac{2}{\sqrt{2}} = \sqrt{2}$ | A1 |
| | | (2) |
| | (d) $\cos(2x + 45) = 1$ so $2x + 45 = 0, 360, \ldots$ | M1 |
| | $2x = -45, 315, \ldots$ | M1 |
| | Smallest positive value, $x = 157.5^\circ$ | A1 |
| | | (3) |
| | | **[12]** |

| Question Number | Scheme | Marks |
|---|---|---|

| | | |
|---|---|---|
| **8.** | (a) f(0) = 6 $\Rightarrow 0\times a+0\times b+0\times c+d=6$ | |
| | $\Rightarrow d = 6$ * | B1 |
| | | (1) |
| | (b) $a + b + c + d = -6$ and $-a + b - c + d = 12$ | M1 A1 |
| | $2b + 2d = 6$ | |
| | $2b = 6 - 12$ | M1 |
| | $b = -3$ | A1 |
| | | (4) |
| | (c) $a - 3 + c + 6 = -6$ $\Rightarrow a + c = -9$ (1) | M1 |
| | $27a + 9b + 3c + d = 0$ | M1 |
| | $27a - 27 + 3c + 6 = 0$ | |
| | $27a + 3c = 21$ $\Rightarrow 9a + c = 7$ (2) | A1 |
| | (2) – (1) $8a = 16$ | M1 |
| | $a = 2$, $c = -9 - 2 = -11$ | A1 A1 |
| | | (6) |
| | (d) $f(x) = (x - 3)(2x^2 + 3x - 2)$ | M1 |
| | $= (x - 3)(2x - 1)(x + 2)$ | M1 A1 |
| | | (3) |
| | | **[14]** |

| Question Number | Scheme | Marks |
|---|---|---|
| 9. | (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x$ | M1 |
| | at $P(4,4)$ $\frac{dy}{dx} = 2$ | A1 |
| | (i) tangent is $y - 4 = 2(x - 4)$ | M1 A1 |
| | $y = 2x - 4$ | |
| | (ii) normal is $y - 4 = -\frac{1}{2}(x - 4)$ | M1 A1 |
| | $y = -\frac{1}{2}x + 6$ | |
| | | (6) |
| | (b) Normal at $Q$ has gradient 2, so tangent has gradient $-\frac{1}{2}$ | |
| | $\frac{1}{2}x = -\frac{1}{2}$ | M1 |
| | $x = -1,\ y = \frac{1}{4},\ Q(-1, \frac{1}{4})$ | A1 |
| | | (2) |
| | (c) Normal at $Q$ | |
| | $y - \frac{1}{4} = 2(x+1)$ | M1 A1 |
| | $y = 2x + 2\frac{1}{4}$ | |
| | At $R$, $2x + 2\frac{1}{4} = -\frac{1}{2}x + 6$ | M1 |
| | $x = 1\frac{1}{2}$ | A1 |
| | | (4) |
| | (d) Tangent at $Q$ | |
| | $y - \frac{1}{4} = -\frac{1}{2}(x+1)$ *or* mid-point $QP$ is $\left(\frac{-1+4}{2}, \frac{\frac{1}{4}+4}{2}\right) = \left(\frac{3}{2}, \frac{17}{8}\right)$ | M1 A1 |
| | $y = -\frac{1}{2}x - \frac{1}{4}$ | |
| | at $S$, $2x - 4 = -\frac{1}{2}x - \frac{1}{4}$ *or* $RS$ is diagonal of rectangle $PQRS$ | M1 |
| | $x = 1\frac{1}{2}$ *or* so it passes through $\left(\frac{3}{2}, \frac{17}{8}\right)$ and $R\left(\frac{3}{2}, y\right)$ | A1 |
| | $RS$ is parallel to $y$-axis with reason to justify this. * | B1 cso |
| | e.g. $RS$ has equation $x = 1\frac{1}{2}$ | |
| | *or* $RS$ passes through two points with $x$-coordinate $1\frac{1}{2}$ | |
| | | (5) |
| | | **[17]** |

| Question Number | Scheme | Marks |
|---|---|---|
| 10. | (a) $M\,(1, 3)$ | B1 |
| | Gradient $AC = \frac{2}{-8}$ | |
| | $\Rightarrow$ gradient $l = -\left(\frac{-8}{2}\right) = 4$ | M1 |
| | $y - 3 = 4(x - 1) \Rightarrow y = 4x - 1$ | M1 A1 |
| | *Alternative* $AP^2 = (x+3)^2 + (y-4)^2$ and $CP^2 = (x-5)^2 + (y-2)^2$ where $P(x, y)$ lies on $l$. $x^2 + 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 - 4y + 4$ $y = 4x - 1$ | |
| | | (4) |
| | (b) $AC^2 = 8^2 + 2^2 = 68 \Rightarrow AC = \sqrt{68} = 2\sqrt{17}$ | M1 A1 |
| | | (2) |
| | (c) $\frac{1}{2}\sqrt{68} \times BM = 17\sqrt{2}$ | M1 |
| | $BM = \dfrac{34\sqrt{2}}{2\sqrt{17}} = \sqrt{34}$ | A1 |
| | | (2) |
| | (d) $AB^2 = AM^2 + BM^2 = \left(\sqrt{17}\right)^2 + \left(\sqrt{34}\right)^2 = 51$ | M1 |
| | $AB = \sqrt{51}$ | A1 |
| | | (2) |
| | (e) $B(x, y)$, $(y-4)^2 + (x+3)^2 = 51$ *or* $(x-1)^2 + (y-3)^2 = 34$ | M1 |
| | $y = 4x - 1$ so $(4x-5)^2 + (x+3)^2 = 51$ *or* $(x-1)^2 + (4x-4)^2 = 34$ | M1 |
| | $16x^2 - 40x + 25 + x^2 + 6x + 9 = 51$ *or* $(x-1)^2 + 16(x-1)^2 = 34$ | |
| | $17x^2 - 34x - 17 = 0$ *or* $17(x-1)^2 = 34$ | A1 |
| | $x^2 - 2x - 1 = 0$ | |
| | $x = \dfrac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$ *or* $x - 1 = \pm\sqrt{2} \Rightarrow x = 1 \pm \sqrt{2}$ | M1 A1 |
| | $(1+\sqrt{2},\ 3+4\sqrt{2})$ and $(1-\sqrt{2},\ 3-4\sqrt{2})$ oe | A1 |
| | | (6) |
| | | **[16]** |

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