Write your name here

Surname	Other names

Edexcel
International GCSE

Centre Number | Candidate Number

Further Pure Mathematics

Paper 2

Tuesday 22 January 2013 – Afternoon
Time: 2 hours

Paper Reference
4PM0/02

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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PEARSON

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1

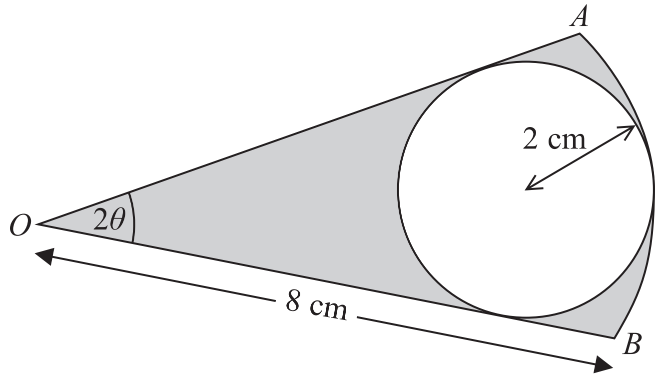

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector, *AOB* of a circle with centre *O* and radius 8 cm. A circle of radius 2 cm touches the lines *OA* and *OB* and the arc *AB*. Angle *AOB* is 2θ radians, $0 < \theta < \frac{\pi}{4}$.

(a) Find, to 4 significant figures, the value of θ

(3)

(b) Find, to 3 significant figures, the area of the region shaded in Figure 1.

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2 Using the identities $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan A = \frac{\sin A}{\cos A}$$

(a) show that $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(3)

(b) Hence show that

(i) $\tan 105° = \dfrac{1 + \sqrt{3}}{1 - \sqrt{3}}$ (ii) $\tan 15° = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}$

(4)

Question 2 continued

(Total for Question 2 is 7 marks)

3 (a) Expand $(1 + 3x^2)^{-\frac{1}{4}}$ in ascending powers of x up to and including the term in x^6, giving each coefficient as a fraction in its lowest terms.

(3)

(b) Find the range of values of x for which your expansion is valid.

(1)

$$f(x) = \frac{3 + kx^2}{(1 + 3x^2)^{\frac{1}{4}}} \qquad k \in \mathbb{R}^+$$

(c) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^6.

(3)

Given that the coefficient of x^4 in the series expansion of $f(x)$ is zero

(d) find the exact value of k.

(2)

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4 Differentiate with respect to x

(a) $3x \sin 5x$

(3)

(b) $\dfrac{e^{2x}}{4 - 3x^2}$

(3)

(Total for Question 4 is 6 marks)

5 $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(a) Use the above identity to show that $2\sin^2 A = 1 - \cos 2A$

(3)

(b) Hence find the value of k such that $\sin^2 2A = k(1 - \cos 4A)$

(1)

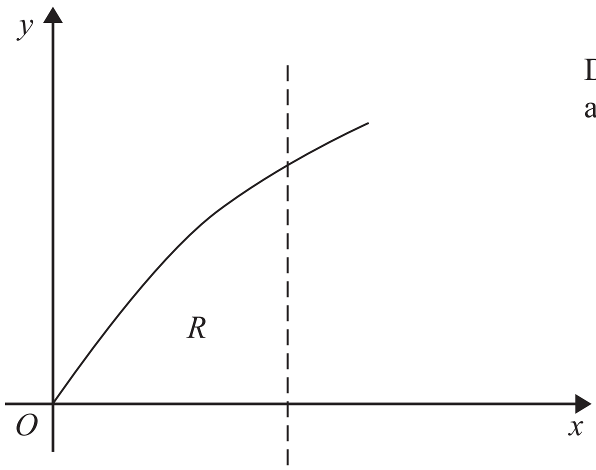

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Figure 2

Figure 2 shows part of the curve with equation $y = 3\sin 2x$. The region R, bounded by the curve, the positive x-axis and the line $x = \frac{\pi}{6}$, is rotated through 360° about the x-axis.

(c) Use calculus to find, to 3 significant figures, the volume of the solid generated.

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6 A solid paperweight in the shape of a cuboid has volume 15 cm^3. The paperweight has a rectangular base of length $5x$ cm and width x cm and a height of h cm. The total surface area of the paperweight is A cm^2.

(a) Show that $A = 10x^2 + \dfrac{36}{x}$

(3)

(b) Find, to 3 significant figures, the value of x for which A is a minimum, justifying that this value of x gives a minimum value of A.

(6)

(c) Find, to 3 significant figures, the minimum value of A.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7 The line l passes through the points with coordinates (1, 6) and (3, 2).

(a) Show that an equation of l is $y + 2x = 8$

(3)

The curve C has equation $xy = 8$

(b) Show that l is a tangent to C.

(3)

Given that l is the tangent to C at the point A,

(c) find the coordinates of A.

(2)

(d) Find an equation, with integer coefficients, of the normal to C at A.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8

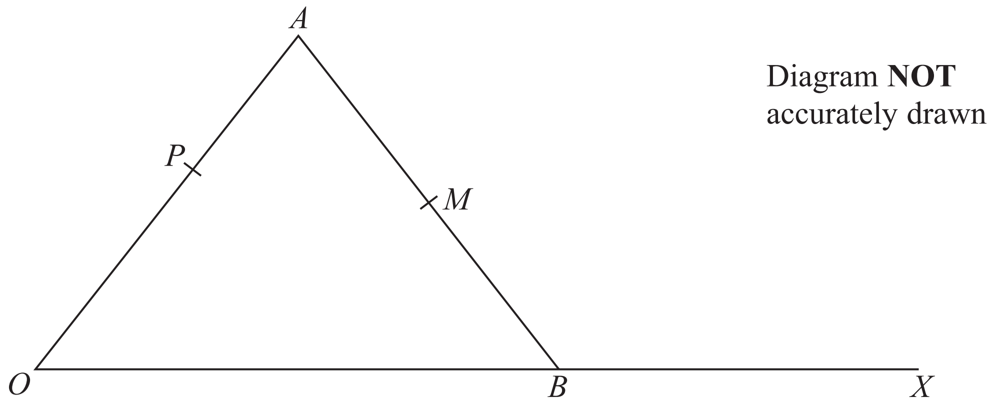

Figure 3

In Figure 3, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and M is the mid-point of AB.

The point P is on OA such that $OP:PA = 3:2$

The point X lies on OB produced.

(a) Find, as simplified expressions in terms of **a** and **b**,

(i) $\overrightarrow{AB}$ (ii) $\overrightarrow{OM}$ (iii) $\overrightarrow{PM}$

(6)

Given that P, M and X are collinear

(b) find, in terms of **b**, $\overrightarrow{OX}$

(4)

(c) Find the ratio (area ΔOAM) : (area ΔOAX).

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

9 The third and fifth terms of a geometric series S are 48 and 768 respectively. Find

(a) the two possible values of the common ratio of S,
(3)

(b) the first term of S.
(1)

Given that the sum of the first 5 terms of S is 615

(c) find the sum of the first 9 terms of S.
(4)

Another geometric series T has the same first term as S. The common ratio of T is $\frac{1}{r}$ where r is one of the values obtained in part (a). The nth term of T is t_n

Given that $t_2 > t_3$

(d) find the common ratio of T.
(1)

The sum of the first n terms of T is T_n

(e) Writing down all the numbers on your calculator display, find T_9
(2)

The sum to infinity of T is T_∞

Given that $T_\infty - T_n > 0.002$

(f) find the greatest value of n.
(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 16 marks)

10 Solve the equations

(a) $\log_x 1024 = 5$

(2)

(b) $\log_5 (6y + 11) = 3$

(3)

(c) $2\log_3 t + \log_t 9 = 5$

(6)

Question 10 continued

Question 10 continued

(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS