

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
ADVANCED SUBSIDIARY GCE
4776/01
MATHEMATICS (MEI)
Numerical Methods

MONDAY 24 MAY 2010: Afternoon
DURATION: 1 hour 30 minutes

SUITABLE FOR VISUALLY IMPAIRED CANDIDATES

Candidates answer on the Answer Booklet

OCR SUPPLIED MATERIALS:

8 page Answer Booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer ALL the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

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SECTION A (36 marks)

- 1 (i) Show that the equation**

$$\frac{1}{x} = 3 - x^2 \quad (*)$$

has a root, α , between $x = 1$ and $x = 2$.

Show that the iteration

$$x_{r+1} = \frac{1}{3 - x_r^2},$$

with $x_0 = 1.5$, converges, but not to α . [5]

- (ii) By rearranging (*), find another iteration that does converge to α . You should demonstrate the convergence by carrying out several steps of the iteration. [3]**

- 2 A function $f(x)$ has the values shown in the following table.**

x	2.8	3	3.2
$f(x)$	0.9508	0.9854	0.9996

- (i) Taking the values of $f(x)$ to be exact, use the forward difference method and the central difference method to find two estimates of $f'(3)$. State which of these you would expect to be more accurate. [5]**
- (ii) Now suppose that the values of $f(x)$ have been rounded to the four significant figures shown. Find, for each method used in part (i), the largest possible value it gives for the estimate of $f'(3)$. [2]**

- 3 (i) X is an approximation to the number x such that $X = x(1 + r)$. State what r represents.

Show that, provided r is small, $X^n \approx x^n(1 + nr)$. [4]

- (ii) The number $G = 0.577$ is an approximation to the number g . G is about 0.04% smaller than g . State, in similar terms, relationships between

(A) G^2 and g^2 ,

(B) \sqrt{G} and \sqrt{g} . [3]

- 4 The expression, $\sin x + \tan x$, where x is in radians, can be approximated by $2x$ for values of x close to zero.

- (i) Find the absolute and relative errors in this approximation when $x = 0.2$ and $x = 0.1$. [4]

- (ii) A better approximation is $\sin x + \tan x \approx 2\left(x + \frac{x^3}{k}\right)$, where k is an integer.

Use your results from part (i) to estimate k . [3]

- 5** A quadratic function, $f(x)$, is to be determined from the values shown in the following table.

x	1	3	6
$f(x)$	-10	-12	30

Explain why Newton's forward difference formula would not be useful in this case.

Use Lagrange's interpolation formula to find $f(x)$ in the form $ax^2 + bx + c$. [7]

SECTION B (36 marks)

- 6** The integral

$$I = \int_1^{1.8} \sqrt{x^3 + 1} \, dx$$

is to be estimated numerically. You are given that, correct to 6 decimal places, the mid-point rule estimate with $h = 0.8$ is 1.547 953 and that the trapezium rule estimate with $h = 0.8$ is 1.611 209.

- (i) Find the mid-point rule and trapezium rule estimates with $h = 0.4$ and $h = 0.2$.**

Hence find three Simpson's rule estimates of I . [7]

- (ii) Write down, with a reason, the value of I to the accuracy that appears to be justified. [2]**

- (iii) Taking your answer in part (ii) to be exact, show in a table the errors in the mid-point rule and trapezium rule estimates of I .

Explain what these errors show about

(A) the relative accuracy of the mid-point rule and the trapezium rule,

(B) the rates of convergence of the mid-point rule and the trapezium rule. [8]

- 7 (i) Show that the equation

$$x^5 - 8x + 5 = 0 \quad (*)$$

has a root in the interval $(0, 1)$.

Find this root, using the Newton-Raphson method, correct to 6 significant figures.

Show, by considering the differences between successive iterates, that the convergence of the Newton-Raphson iteration is faster than first order. [11]

- (ii) Equation (*) was given in part (i) above. You are now given that equation (*) has a root in the interval $(1.4, 1.5)$. Find this root, correct to 3 significant figures, using the secant method. Determine whether or not the secant method is faster than first order. [8]

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