

# Mark Scheme (Results) January 2009

GCE

GCE Mathematics (6667/01)

January 2009  
6667 Further Pure Mathematics FP1 (new)  
Mark Scheme

Question Number	Scheme	Marks
<b>1</b>	$x - 3$ is a factor $f(x) = (x-3)(2x^2 - 2x + 1)$ Attempt to solve quadratic i.e. $x = \frac{2 \pm \sqrt{4-8}}{4}$ $x = \frac{1 \pm i}{2}$	B1 M1 A1 M1 A1 <div style="text-align: right;">[5]</div>

Notes:

First and last terms in second bracket required for first M1

Use of correct quadratic formula for their equation for second M1

Question Number	Scheme	Marks
2	<p>(a) <math>6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1), -n</math></p> <p><math>= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6)</math> or <math>n(n+1)(2n+1) + (2n+1)n</math></p> <p><math>= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1) \quad *</math></p> <p>(b) <math>\sum_{r=1}^{20} (6r^2 + 4r - 1) - \sum_{r=1}^{10} (6r^2 + 4r - 1) = 20 \times 22 \times 41 - 10 \times 12 \times 21</math></p> <p style="text-align: center;"><math>= 15520</math></p>	<p>M1 A1, B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(5)</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;">[7]</p>

Notes:

(a) First M1 for first 2 terms, B1 for  $-n$   
 Second M1 for attempt to expand and gather terms.  
 Final A1 for correct solution only

(b) Require ( $r$  from 1 to 20) subtract ( $r$  from 1 to 10) and attempt to substitute for M1

Question Number	Scheme	Marks
3	<p>(a) <math>xy = 25 = 5^2</math> or <math>c = \pm 5</math></p> <p>(b) <math>A</math> has co-ords <math>(5, 5)</math> and <math>B</math> has co-ords <math>(25, 1)</math></p> <p>Mid point is at <math>(15, 3)</math></p>	<p>B1 (1)</p> <p>B1</p> <p>M1A1 (3)</p> <p>[4]</p>

Notes:

(a)  $xy = 25$  only B1,  $c^2 = 25$  only B1,  $c = 5$  only B1

(b) Both coordinates required for B1  
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4	<p>When <math>n = 1</math>, <math>LHS = \frac{1}{1 \times 2} = \frac{1}{2}</math>, <math>RHS = \frac{1}{1+1} = \frac{1}{2}</math>. So <math>LHS = RHS</math> and result true for <math>n = 1</math></p> <p>Assume true for <math>n = k</math>; <math>\sum_{r=1}^k \frac{1}{r(r+1)} = \frac{k}{k+1}</math> and so <math>\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}</math></p> $\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}$ <p>and so result is true for <math>n = k + 1</math> (and by induction true for <math>n \in \mathbf{Z}^+</math>)</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>B1</p> <p>[5]</p>

Notes:

Evaluate both sides for first B1

Final two terms on second line for first M1

Attempt to find common denominator for second M1.

Second M1 dependent upon first.

$\frac{k+1}{k+2}$  for A1

'Assume true for  $n = k$ ' and 'so result true for  $n = k + 1$ ' and correct solution for final B1

Question Number	Scheme	Marks
5	<p>(a) attempt evaluation of <math>f(1.1)</math> and <math>f(1.2)</math> (– looking for sign change)</p> <p><math>f(1.1) = 0.30875</math>, <math>f(1.2) = -0.28199</math> Change of sign in <math>f(x) \Rightarrow</math> root in the interval</p> <p>(b) <math>f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}</math></p> <p>(c) <math>f(1.1) = 0.30875..</math> <math>f'(1.1) = -6.37086..</math></p> $x_1 = 1.1 - \frac{0.30875..}{-6.37086..}$ <p><math>= 1.15(\text{to } 3 \text{ sig.figs.})</math></p>	<p>M1</p> <p>A1</p> <p>(2)</p> <p>M1 A1 A1</p> <p>(3)</p> <p>B1 B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>[9]</p>

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1

(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1

(c) awrt 0.309 B1 and awrt -6.37 B1 if answer incorrect

Evidence of Newton-Raphson for M1

Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question Number	Scheme	Marks
6	<p>At <math>n = 1</math>, <math>u_n = 5 \times 6^0 + 1 = 6</math> and so result true for <math>n = 1</math></p> <p>Assume true for <math>n = k</math>; <math>u_k = 5 \times 6^{k-1} + 1</math>, and so <math>u_{k+1} = 6(5 \times 6^{k-1} + 1) - 5</math></p> <p><math>\therefore u_{k+1} = 5 \times 6^k + 6 - 5 \quad \therefore u_{k+1} = 5 \times 6^k + 1</math></p> <p>and so result is true for <math>n = k + 1</math> and by induction true for <math>n \geq 1</math></p>	<p>B1</p> <p>M1, A1</p> <p>A1</p> <p>B1</p> <p>[5]</p>

Notes:

6 and so result true for  $n = 1$  award B1

Sub  $u_k$  into  $u_{k+1}$  or M1 and A1 for correct expression on right hand of line 2

Second A1 for  $\therefore u_{k+1} = 5 \times 6^k + 1$

‘Assume true for  $n = k$ ’ and ‘so result is true for  $n = k + 1$ ’ and correct solution for final B1

Question Number	Scheme	Marks
7 (a)	<p>The determinant is <math>a - 2</math></p> $\mathbf{X}^{-1} = \frac{1}{a-2} \begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>Attempt to solve <math>2 - \frac{1}{a-2} = 1</math>, or <math>a - \frac{a}{a-2} = 0</math>, or <math>-1 + \frac{1}{a-2} = 0</math>, or <math>-1 + \frac{2}{a-2} = 1</math></p> <p>To obtain <math>a = 3</math> only</p> <p>Alternatives for (b)            If they use <math>\mathbf{X}^2 + \mathbf{I} = \mathbf{X}</math> they need to identify <math>\mathbf{I}</math> for B1, then attempt to solve suitable equation for M1 and obtain <math>a = 3</math> for A1            If they use <math>\mathbf{X}^2 + \mathbf{X}^{-1} = \mathbf{O}</math>, they can score the B1 then marks for solving            If they use <math>\mathbf{X}^3 + \mathbf{I} = \mathbf{O}</math> they need to identify <math>\mathbf{I}</math> for B1, then attempt to solve suitable equation for M1 and obtain <math>a = 3</math> for A1</p>	M1 A1 (3)  B1  M1  A1 cso (3) [6]

Notes:

(a) Attempt  $ad-bc$  for first M1

$$\frac{1}{\det} \begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix} \text{ for second M1}$$

(b) Final A1 for correct solution only

Question Number	Scheme	Marks
8	<p>(a) <math>\frac{dy}{dx} = a^{\frac{1}{2}} x^{-\frac{1}{2}}</math> or <math>2y \frac{dy}{dx} = 4a</math></p> <p>The gradient of the tangent is <math>\frac{1}{q}</math></p> <p>The equation of the tangent is <math>y - 2aq = \frac{1}{q}(x - aq^2)</math></p> <p>So <math>yq = x + aq^2</math> *</p> <p>(b) <math>R</math> has coordinates <math>(0, aq)</math></p> <p>The line <math>l</math> has equation <math>y - aq = -qx</math></p> <p>(c) When <math>y = 0</math> <math>x = a</math> (so line <math>l</math> passes through <math>(a, 0)</math> the focus of the parabola.)</p> <p>(d) Line <math>l</math> meets the directrix when <math>x = -a</math>: Then <math>y = 2aq</math>. So coordinates are <math>(-a, 2aq)</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>B1</p> <p>M1A1</p> <p>(3)</p> <p>B1</p> <p>(1)</p> <p>M1:A1</p> <p>(2)</p> <p>[10]</p>

Notes:

(a)  $\frac{dy}{dx} = \frac{2a}{2aq}$  OK for M1

Use of  $y = mx + c$  to find  $c$  OK for second M1

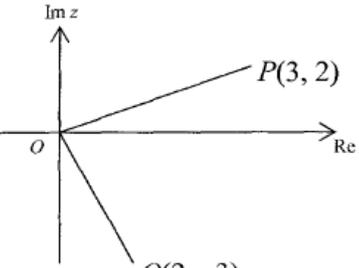
Correct solution only for final A1

(b)  $-1/(\text{their gradient in part a})$  in equation OK for M1

(c) They must attempt  $y = 0$  or  $x = a$  to show correct coordinates of  $R$  for B1

(d) Substitute  $x = -a$  for M1.

Both coordinates correct for A1.

Question Number	Scheme	Marks
9	<p>(a) <math display="block">z_2 = \frac{12-5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{36-24i-15i-10}{13}</math>  <math display="block">= 2-3i</math></p> <hr/> <p>(b)   <i>P: B1, Q: B1ft</i></p> <hr/> <p>(c) <math display="block">\text{grad. } OP \times \text{grad. } OQ = \frac{2}{3} \times -\frac{3}{2}</math>  <math display="block">= -1 \Rightarrow \angle POQ = \frac{\pi}{2} \quad (*)</math></p> <p>OR <math display="block">\angle POX = \tan^{-1} \frac{2}{3}, \angle QOX = \tan^{-1} \frac{3}{2}</math>  <math display="block">\text{Tan}(\angle POQ) = \frac{\frac{2}{3} + \frac{3}{2}}{1 - \frac{2}{3} \times \frac{3}{2}} \quad \text{M1}</math>  <math display="block">\Rightarrow \angle POQ = \frac{\pi}{2} \quad (*) \quad \text{A1}</math></p> <hr/> <p>(d) <math display="block">z = \frac{3+2}{2} + \frac{2+(-3)}{2}i</math>  <math display="block">= \frac{5}{2} - \frac{1}{2}i</math></p> <hr/> <p>(e) <math display="block">r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}</math>  <math display="block">= \frac{\sqrt{26}}{2} \text{ or exact equivalent}</math></p>	<p>M1 A1 (2)</p> <p>B1, B1ft (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2) [10]</p>

Notes:

(a)  $\times \frac{3-2i}{3-2i}$  for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

Question Number	Scheme	Marks
10	<p>(a) <b>A</b> represents an enlargement scale factor <math>3\sqrt{2}</math> (centre <math>O</math>)</p> <p><b>B</b> represents reflection in the line <math>y = x</math></p> <p><b>C</b> represents a rotation of <math>\frac{\pi}{4}</math>, i.e. <math>45^\circ</math> (anticlockwise) (about <math>O</math>)</p> <p>(b) <math>\begin{pmatrix} 3 &amp; -3 \\ 3 &amp; 3 \end{pmatrix}</math></p> <p>(c) <math>\begin{pmatrix} 3 &amp; -3 \\ 3 &amp; 3 \end{pmatrix} \begin{pmatrix} 0 &amp; 1 \\ 1 &amp; 0 \end{pmatrix} = \begin{pmatrix} -3 &amp; 3 \\ 3 &amp; 3 \end{pmatrix}</math></p> <p>(d) <math>\begin{pmatrix} -3 &amp; 3 \\ 3 &amp; 3 \end{pmatrix} \begin{pmatrix} 0 &amp; -15 &amp; 4 \\ 0 &amp; 15 &amp; 21 \end{pmatrix} = \begin{pmatrix} 0 &amp; 90 &amp; 51 \\ 0 &amp; 0 &amp; 75 \end{pmatrix}</math> so <math>(0, 0)</math>, <math>(90, 0)</math> and <math>(51, 75)</math></p> <p>(e) Area of <math>\Delta OR'S'</math> is <math>\frac{1}{2} \times 90 \times 75 = 3375</math></p> <p>Determinant of <b>E</b> is <math>-18</math> or use area scale factor of enlargement So area of <math>\Delta ORS</math> is <math>3375 \div 18 = 187.5</math></p>	<p>M1 A1</p> <p>B1 B1 (4)</p> <p>M1 A1 (2)</p> <p>B1 (1)</p> <p>M1A1A1A1 (4)</p> <p>B1</p> <p>M1A1 (3) [14]</p>

Notes:

(a) Enlargement for M1

$3\sqrt{2}$  for A1

(b) Answer incorrect, require **CD** for M1

(c) Answer given so require **DB** as shown for B1

(d) Coordinates as shown or written as  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$  for each A1

(e) 3375 B1

Divide by theirs for M1