Mark Scheme (Results)
January 2010

GCE

Further Pure Mathematics FP3 (6676)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

Ask The Expert can be accessed online at the following link:

http://www.edexcel.com/Aboutus/contact-us/

January 2010

January 2010
FP3 Further Pure Mathematics 6676
Mark Scheme

Question Number	Scheme	Marks
Q1	Calculate $\left(\frac{dy}{dx}\right)_0 = 2\sin 1 = 1.683$	B1
	At $x = 0.1$, $y_1 = 1 + 0.1\ (2\sin 1) = 1.1683$ or awrt	M1 A1
	$x = 0.2$, $y_2 = 1.1683 + 0.1\ (0.1^2 + 2\sin 1.1683) = 1.3533$ awrt	M1 A1 **[5]**
	B1 may be implied 3dp lose last A1	

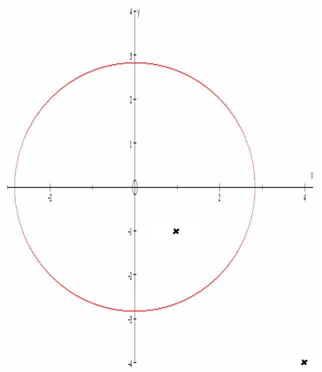
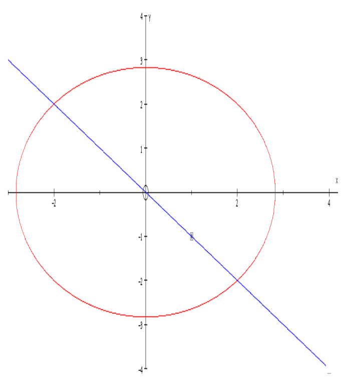
Question Number	Scheme	Marks
Q2 (a)	$\frac{dy}{dx} = 3x^2 \ln x + x^2$ $\frac{d^2y}{dx^2} = 6x \ln x + 5x$, and $\frac{d^3y}{dx^3} = 6 \ln x + 11$	M1 A1, M1A1ft, A1 (5)
(b)	Use of $x^3 \ln x = f(1) + (x-1)f'(1) + \frac{1}{2}(x-1)^2 f''(1) + \frac{1}{6}(x-1)^3 f'''(1)$ Evaluates $f(1)$, $f'(1)$, $f''(1)$ and $f'''(1)$	M1 M1
	So $x^3 \ln x = (x-1) + \frac{5}{2}(x-1)^2 + \frac{11}{6}(x-1)^3$	A1 (3) **[8]**
	(a) M1 is attempt at derivative involving product rule	

Question Number	Scheme	Marks
Q3 (a)	$\cos 5\theta = \text{Re}\left[(\cos\theta + i\sin\theta)^5\right]$ $= \cos^5\theta + 10\cos^3\theta\ \text{i}^2\sin^2\theta + 5\cos\theta\ \text{i}^4\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta\ \sin^2\theta + 5\cos\theta\ \sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta\ (1-\cos^2\theta) + 5\cos\theta\ (1-\cos^2\theta)^2$ $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (D)	M1A1 M1 M1 A1 (5)
(b)	$32x^5 - 40x^3 + 10x + 1 = 0 \Rightarrow 16x^5 - 20x^3 + 5x = -\frac{1}{2}$ so solve $\cos 5\theta = -\frac{1}{2}$ $5\theta = \frac{2\pi}{3}$, and $\frac{4\pi}{3}$ (ignore extra solutions) So $x = \cos\theta$, where $\theta =$ their $\frac{2\pi}{15}$ or $\frac{4\pi}{15}$ So x = 0.914 and 0.669	M1 A1, A1ft M1 A1, A1 (6) **[11]**
	In part (b) award M1 for +/- ½ A1 ft is for second solution consistent with first Accept answers which round to.. Ignore wrong or extra answers. Lose final A1 for 2dp	

Question Number	Scheme	Marks
Q4 (i)	$\begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}^n = \begin{pmatrix}1&0&0\\n&1&0\\n(n+2)&2n&1\end{pmatrix} = \begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}$ when $n = 1$ $\therefore$ true for $n = 1$	B1
	Assume true for $n = k$, then $\begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}^k = \begin{pmatrix}1&0&0\\k&1&0\\k(k+2)&2k&1\end{pmatrix}$	
	$\begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}^{k+1} = \begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}\begin{pmatrix}1&0&0\\k&1&0\\k(k+2)&2k&1\end{pmatrix}$ or $\begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}^{k+1} = \begin{pmatrix}1&0&0\\k&1&0\\k(k+2)&2k&1\end{pmatrix}\begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}$	M1
	i.e. $\begin{pmatrix}1&0&0\\1&1&0\\3&2&1\end{pmatrix}^{k+1} = \begin{pmatrix}1&0&0\\1+k&1&0\\\{3+2k+k(k+2)\}&2k+2&1\end{pmatrix} = \begin{pmatrix}1&0&0\\1+k&1&0\\\{3+4k+k^2\}&2k+2&1\end{pmatrix}$	M1
	$= \begin{pmatrix}1&0&0\\1+k&1&0\\(k+1)(k+3)&2k+2&1\end{pmatrix} = \begin{pmatrix}1&0&0\\n&1&0\\n(n+2)&2n&1\end{pmatrix}$ with $n = k+1$	A1
	($\therefore$ true for $n = k + 1$ if true for $n = k$) $\therefore$ true for $n \in \mathbf{Z}^+$ by induction.	A1 (5)
(ii)	Let $u_n = 2^{3n+1} + 5$, then $u_1 = 21$ which is divisible by 7 $\therefore$ true for $n = 1$	B1
	Assume true for $n = k$, then $u_k = 2^{3k+1} + 5$ is divisible by 7	
	Consider $u_{k+1} - u_k = (2^{3(k+1)+1} + 5) - (2^{3k+1} + 5) = 2^{3k+1}(2^3 - 1) = 2^{3k+1} \times 7$	M1, M1, A1
	As u_k and $u_{k+1} - u_k$ are both divisible by 7 $\therefore$ u_{k+1} is divisible by 7	
	($\therefore$ true for $n = k + 1$ if true for $n = k$) $\therefore$ true for $n \in \mathbf{Z}^+$ by induction	A1 cso (5) **[10]**
Alternatives for (ii)	Note: Accuracy marks only depend on first M1 Show that $u_0 = 7$ satisfies condition for $n = 0$, could earn first B1	
	Also $u_k = 2^{3k+1} + 5$ is divisible by 7 $\Rightarrow$ $2^{3k+1} + 5 = 7k \Rightarrow 2^{3k+1} = 7k - 5$	M1
	So $2^{3k+4} + 5 = 8(7k - 5) + 5 = 7(8k - 5)$ So divisible by 7	M1 A1
	$\therefore$ true for $n \in \mathbf{Z}^+$ by induction	A1 cso

Question Number	Scheme	Marks
Q5 (a)	$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 7 & 9 \\ -1 & 3 & 1 \end{vmatrix} = -20\mathbf{i} - 10\mathbf{j} + 10\mathbf{k} = -10(2\mathbf{i}+\mathbf{j}-\mathbf{k})$	M1 A1 (2)
(b)	The plane has equation **r.n** = **a.n ,** which is $-2x - y + z =$	M1
	i.e. $2x + y - z = 4$ o.a.e.	A1 (2)
(c)	The line l_1 passes through the point (1, 0, -2) and this lies in the plane	B1
	l_1 has direction **a** which is perpendicular to $\mathbf{a}\times\mathbf{b}$ so l_1 is parallel to the plane .(Thus l_1 lies in the plane.) Or $2(1+\lambda)+7\lambda-(9\lambda-2)=4$ for all values of λ, so line lies in plane	B1 (2)
(d)	**r.(2i + j - k) = (i + j + k) . (2i + j - k)**	M1
	i.e. $2x + y - z = 2$ o.a.e	A1 (2)
(e)	Either Distance from $2x + y - z = 4$ to origin is $\frac{4}{\sqrt{(2^2+(-1)^2+1^2)}} = \frac{4}{\sqrt{6}}$	M1
	Or Distance from $2x + y - z = 2$ to origin is $\frac{2}{\sqrt{(2^2+(-1)^2+1^2)}} = \frac{2}{\sqrt{6}}$	
	So distance between the planes is $\frac{4}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6}}$ $\left(=\frac{\sqrt{6}}{3}\right)$	M1, A1o.a.e (3)
		[11]

Question Number	Scheme	Marks
Q6 (a)	The eigenvalues satisfy the equation $\lvert \mathbf{M}-\lambda\mathbf{I}\rvert = 0$ so $(11-\lambda)(1-\lambda)-75=0$	M1 A1
	$\therefore \lambda^2-12\lambda-64=0$ so $\lambda = 16$ or -4.	M1 A1 (4)
(b)	$\lambda = 16: \begin{pmatrix} 11 & -5\sqrt{3} \\ -5\sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 16\begin{pmatrix} x \\ y \end{pmatrix}$ so an eigenvector is $k\begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$	M1 A1
	$\lambda = -4: \begin{pmatrix} 11 & -5\sqrt{3} \\ -5\sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -4\begin{pmatrix} x \\ y \end{pmatrix}$ so an eigenvector is $k'\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$	M1 A1 (4)
(c)	$\mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{2}k & \frac{1}{2}k' \\ \frac{-1}{2}k & \frac{\sqrt{3}}{2}k' \end{pmatrix}$, *where* $k = \pm 1$ *and* $k' = \pm 1$	M1, A1 (2)
(d)	$\mathbf{P}^{-1} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ o.e.	M1 A1ft
	$\mathbf{P}^{-1}\mathbf{MP} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 11 & -5\sqrt{3} \\ -5\sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & -4 \end{pmatrix}$	M1 A1ft (4) **[14]**

Question Number	Scheme	Marks
Q7 (a)	$(x-4)^2+(y+4)^2=4\{(x-1)^2+(y+1)^2\}$ $\therefore 3x^2+3y^2=24$ This is a circle with $r^2=8$ So $\lvert z\rvert=k$ and $k=2\sqrt{2}$	M1 A1 A1 B1 B1 (5)
(b)	 Circle centre O Point at (1, -1) Point at (4,-4)	B1 B1 B1 (3)
(c)	 Method of solution: e.g. diameter shown $4\sqrt{2}-r$ $4\sqrt{2}+r$	M1 A1ft A1ft (3)
(d)	Let $z=\sqrt{8}e^{i\theta}$, then $w=\sqrt{8}(e^{i\theta}+e^{-i\theta})$ i.e. $w=2\sqrt{8}(\cos\theta)$ So the locus is part of the real axis , i.e. Im (w) =0 And as $-1<\cos<1$, so the end points are $w=4\sqrt{2}$ and $w=-4\sqrt{2}$	.M1 A1 ft on *r* B1 M1 A1 (5) **[16]**
	Alternative method (d) Let $z=x+\mathrm{i}\,y$ and put $x^2+y^2=8$ to give $w=2x+0$ for M1 A1	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

January 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH