edexcel

Mark Scheme (Results)

Summer 2012

GCE Mechanics M4
(6680) Paper 1

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

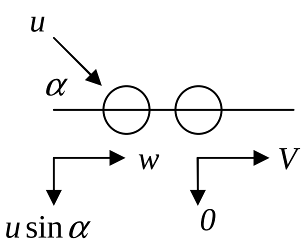
www.edexcel.com/contactus

earson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

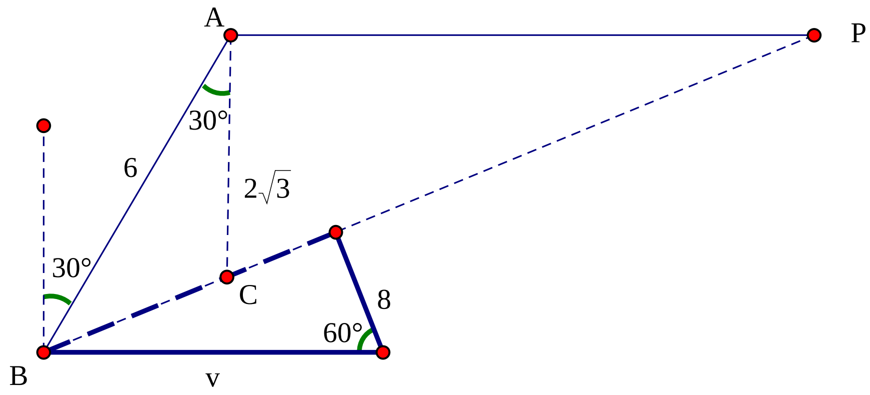
Summer 2012
Publications Code UA032684

June 2012
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks	Notes
1. (a)			
	$mu\cos\alpha = mw + 2mV$	M1 A1	CLM parallel to the line of centres. $\frac{4}{5}u = w + 2V$. Need all terms but condone sign errors.
	$eu\cos\alpha = -w \;+\; V$	M1 A1	Impact law. Must be the right way round. $\frac{3}{4}\times\frac{4}{5}u = V - w$
	$u\cos\alpha(e+1) = 3V \Rightarrow$ (i) $u = \frac{15V}{7}$	M1 A1	Eliminate w and solve for u in terms of V or v.v. 2.14V or better
	$\Rightarrow w = -\frac{2V}{7}$	A1	Solve for w in terms of V. -0.286V or better
	(ii) speed of S $=\sqrt{(\frac{-2V}{7})^2 + (u\sin\alpha)^2} = \frac{V\sqrt{85}}{7}$	M1 A1 (9)	Use of Pythagoras with their $u\sin\alpha$ and w. $\sqrt{\left(\frac{-2V}{7}\right)^2 + \left(\frac{15V}{7}\times\frac{3}{5}\right)^2}$ $\sqrt{\frac{85}{49}}V$, accept 1.32V or better

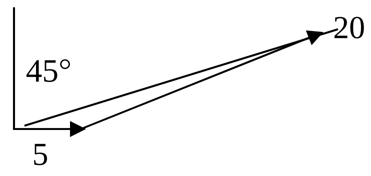
Question Number	Scheme	Marks	Notes
(b)	$\tan\theta = \dfrac{\frac{9V}{7}}{\frac{2V}{7}} = \frac{9}{2}$	M1	Direction of *S* after the collision. Condone $\frac{2}{9}$
		A1	$77.5°$ or $12.5°$ seen or implied
	defln angle = $180° - (\theta + \alpha)$	DM1	Combine their θ and α to find the required angle. e.g. $12.5° + \tan^{-1}\left(\frac{4}{3}\right)$
	$= 65.7°$ (3 sf)	A1	Accept 66°
		(4) **13**	

Question Number	Scheme	Marks	Notes
2.	With B as origin,		
	$\mathbf{r}_A = (6\sin 30\mathbf{i} + 6\cos 30\mathbf{j})$	M1	Express the original relative positions in component (vector) form – one term correct.
	$= (3)\mathbf{i} + (3\sqrt{3})\mathbf{j}$	A1	Both terms correct (substitution of trig values not required).
	$\mathbf{r}_B = vt\mathbf{i}$ or $\mathbf{v}_B = v\mathbf{i}$	B1	Position of B at time t (seen or implied)
	$(v-4)\mathbf{i} + (4\sqrt{3})\mathbf{j}$	M1	Express the relative velocity in component form – one term correct.
	or $\left(v - 8\sin 30\right)\mathrm{i} + \left(8\cos 30\right)\mathrm{j}$	A1	Both terms correct
	When B is $2\sqrt{3}$ km south of A,		
	$-3\sqrt{3} + 4\sqrt{3}t = -2\sqrt{3} \Rightarrow t = \frac{1}{4}$	M1 A1	Compare $\mathbf{j}$ displacement with $\pm 2\sqrt{3}$ and solve for t cao
	$vt - 3 - 4t = 0 \quad \Rightarrow v = 16$	M1 A1	Equate $\mathbf{i}$ displacement to zero and substitute their value of t. cao
	When B is due east of A,		
	$-3\sqrt{3} + 4\sqrt{3}t = 0 \Rightarrow t = \frac{3}{4}$ i.e. at 12.45 pm	M1 A1	Equate $\mathbf{j}$ displacement to zero and solve for t. Any equivalent form for the time.
	then distance AB = $16\text{x}\frac{3}{4} - 3 - 4\text{x}\frac{3}{4} = 6$ km.	M1 A1	Substitute their v & t in the $\mathbf{i}$ displacement and evaluate cao. Must be a scalar.
		13	
			See over page for geometrical alternative

or	Triangle *ABC*: cosine rule gives $BC^2 = 36 + 12 - 2 \times 6 \times 2\sqrt{3}\cos 30$	M1A1	The given information provides us with two triangles - velocities in bold. Fix *A* and *B* follows the path *BP*. *C* is the point when *B* is due South of *A*, and *P* when it is due East.
	Solve for *BC* and $\angle ABC$ $BC = 2\sqrt{3}$, → triangle is isosceles	M1A1	
	$\angle B$ in velocity triangle is 30°	B1	
	Trig in rt∠ triangle gives relative velocity $=8 \times \tan 60 = 8\sqrt{3}$	M1A1	
	∠APB = 30° (angles of a triangle) so triangle is isosceles and distance *AP* = 6km	M1A1	
	Using cosine rule or symmetry of isosceles triangle, distance *BP* = $6\sqrt{3}$	M1A1	
	Time taken = $\frac{6\sqrt{3}}{8\sqrt{3}} = \frac{3}{4}$ hr, time is now 12.45	M1A1	

3. (a)	$2mg - T - kv^2 = 2ma$	M1 A1	Equation of motion for particle of mass 2*m* aef
	$T - mg - kv^2 = ma$	M1 A1	Equation of motion for particle of mass m aef
	Adding, $mg - 2kv^2 = 3ma$		
	$\frac{2g}{3} - \frac{4kv^2}{3m} = 2v\frac{dv}{dx}$	DM1	Eliminate *T*, substitute for *a* and rearrange. Dependent on both previous M marks.
	$\frac{d(v^2)}{dx} + \frac{4kv^2}{3m} = \frac{2g}{3}$ *	A1	Reach **given answer** correctly
		(6)	
(b)	$IF = e^{\int \frac{4k}{3m} dx} = e^{\frac{4kx}{3m}}$	B1	
	$v^2 e^{\frac{4kx}{3m}} = \frac{2g}{3}\int e^{\frac{4kx}{3m}} dx = \frac{mg}{2k} e^{\frac{4kx}{3m}} (+C)$	M1 A1	Use integrating factor to obtain $\frac{d}{dx}\left(v^2 e^{\frac{4kx}{3m}}\right) = \frac{2g}{3} e^{\frac{4kx}{3m}}$ and integrate
	$v^2 = \frac{mg}{2k} + Ce^{\frac{-4kx}{3m}}$		
	$x = 0, v = 0 \Rightarrow C = -\frac{mg}{2k}$	M1	Use initial values to evaluate *C* or as limits in a definite integral and find an expression for v^2.
	$v^2 = \frac{mg}{2k}(1 - e^{\frac{-4kx}{3m}})$	A1	aef.
		(5)	
OR	Separate variables: $\int \frac{3m}{2mg - 4kv^2} dv^2 = \int 1 dx$	B1	CF $v^2 = Ae^{-\frac{4k}{3m}x}$
	$x = -\frac{3m}{4k}\ln\left\|2mg - 4kv^2\right\| (+C)$	M1A1	PI $v^2 = b \Rightarrow 0 + \frac{4k}{3m}b = \frac{2g}{3}$; GS $v^2 = Ae^{-\frac{4k}{3m}x} + \frac{mg}{2k}$
	$x = -\frac{3m}{4k}\ln\left\|\frac{2mg}{2mg - 4kv^2}\right\|$	M1	$x = 0,\ v = 0 \Rightarrow A = -\frac{mg}{2k}$
	$v^2 = \frac{mg}{2k}(1 - e^{\frac{-4kx}{3m}})$	A1	$v^2 = \frac{mg}{2k}(1 - e^{\frac{-4kx}{3m}})$

(c)	When $x = 0, T = \frac{4mg}{3}$	M1 A1	Substitute v = 0 in the initial equations and solve for T
	As $x \to \infty, T \to \frac{9mg}{6} = \frac{3mg}{2}$	M1	For large x, $v^2 \to \frac{mg}{2k}$. Substitute in the initial equations and solve for T
		A1	
	Hence, $\frac{4mg}{3} \le T < \frac{3mg}{2}$. *	A1	cwo – **answer is given**.
		(5)**16**	

4.(a)			
	$\frac{\sin\theta}{5}=\frac{\sin 45}{20}$	M1	Use a vector triangle to find θ. Condone the 5 ms^{-1} in the wrong direction.
		A1	Correct equation for θ
	$\theta = 10.182...$		
	Bearing is $45^\circ - \theta = 34.8 = 35^\circ$ (nearest degree)	M1	Use their angle correctly in their triangle to find the bearing.
		A1 (4)	Accept alternative forms e.g. N 35 E
OR	$SW \rightarrow (20\sin\theta)T = (5+20\cos\theta)T$	M1	45° rt angle triangle
	$3t^2+8t-5=0$, $t=\frac{-8+\sqrt{124}}{6}=0.5225...$	A1	t substitution leading to correct equation in t, use of $R\cos(\theta+\alpha)$ o.e.
	$\theta = 55.18...$ Bearing is $90-\theta = 34.8^\circ$	M1A1	
		(4)	
(b)	$v^2 = 5^2+20^2-2\text{x}5\text{x}20\cos 124.818...$ OR $v=\frac{20}{\sin 45}\times\sin 124.8$ OR $v = 5\cos 45 + 20\cos\theta$	M1	Complete method to find v
	$v = 23.22$	A1	Or better $\left(\frac{5\sqrt{2}+5\sqrt{62}}{2}\right)$
	$t=\frac{15}{23.22}=0.646$ h= 39 min (nearest min)	M1	$\frac{15}{\text{their } v}$
		A1	The Q specifies "nearest minute"
		(4)	
(c)	Due N, (since current affects both equally)	B1 (1)	cao
(d)	$t=\frac{4}{20}=0.2$ h = 12 min	B1	cso
		(1)**10**	

5. **(a)**		B1		GPE of rod e.g. $-Wa\cos 2\theta$
	$V = -Wa\cos 2\theta + \frac{1}{2}W\{3a-(L-6a\cos\theta-4a)\}$	M1		GPE of the particle e.g. $\frac{1}{2}W\{3a-(L-6a\cos\theta-4a)\}$ Condone 3a term missing.
		A1		Correct expression including the 3a (unless in the GPE for the rod) Accept aef e.g. $\sqrt{18a^2(1+\cos 2\theta)}$ for $6a\cos\theta$
	$= -Wa\cos 2\theta + 3Wa\cos\theta + (\frac{7Wa}{2} - \frac{WL}{2})$			
	$= Wa(3\cos\theta - \cos 2\theta) + \text{constant}$ *	A1		Obtain the **given answer** correctly
			(4)	
(b)	$\frac{dV}{d\theta} = Wa(-3\sin\theta + 2\sin 2\theta)$	M1		Differentiate the given V wrt θ
		A1		correct
	For equilibrium, $Wa(-3\sin\theta + 2\sin 2\theta) = 0$			
	$\sin\theta(4\cos\theta - 3) = 0$	DM1		Set their derivative = 0
		A1		First answer
	$\Rightarrow \theta = 0$ or $\theta = \cos^{-1}\left(\frac{3}{4}\right)$	A1		Second answer - ignore $\theta = -\cos^{-1}\left(\frac{3}{4}\right)$. 0.72 rads or better
	$\frac{d^2V}{d\theta^2} = Wa(-3\cos\theta + 4\cos 2\theta)$	M1		Obtain the second derivative of V and substitute one of their values for θ
	$\theta = 0: \frac{d^2V}{d\theta^2} = Wa > 0 \Rightarrow$ *stable*	A1		Correct working and conclusion for one value
	$\theta = \cos^{-1}\frac{3}{4}: \frac{d^2V}{d\theta^2} = -\frac{7Wa}{4} < 0 \Rightarrow$ *unstable*	A1		Correct working and reasoning for the second. ISW for work on $-\cos^{-1}\left(\frac{3}{4}\right)$
			(8)	
			12	

Question	Scheme	Marks		Notes
6.(a)	$T_1 = mg + T_2$	M1		No resultant force and use of Hooke's law
	$\frac{3mge}{a} = mg + \frac{mg(2a-e)}{a}$	A1		Correct equation in one unknown $\frac{3mg(AP-a)}{a} = mg + \frac{mg(3a-AP)}{a}$, $3AP - 3a = a + 3a - AP$
	$e = \frac{3a}{4} \Rightarrow AP = \frac{7a}{4}$ *	A1		Derive **given result** correctly.
				Condone verification for 3/3
			(3)	
(b)	$mg + T_2 - T_1 - mkv = m\ddot{x}$	M1		Equation of motion – requires all terms but condone sign errors.
		A1		o.e. Correct equation in T_1 & T_2.
	$mg + \frac{mg(\frac{5}{4}a - x)}{a} - \frac{3mg(\frac{3}{4}a + x)}{a} - mkv = m\ddot{x}$	DM1		Use Hooke's law with extensions of the form $ka \pm x$
		A1		o.e. Correct unsimplified
	$\ddot{x} + k\dot{x} + \frac{4g}{a}x = 0$ *	A1		**Given answer** derived correctly
			(5)	
(c)	For a damped oscillation, $k^2 < \frac{16g}{a}$	M1		AE will have complex roots
		A1		Correctly substituted inequality
	i.e. $k < 4\sqrt{\frac{g}{a}}$	A1		Only (Q gives k>0) $-4\sqrt{\frac{g}{a}} < k < 4\sqrt{\frac{g}{a}}$ is A0.
			(3)	
			11	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA032684 Summer 2012

For more information on Edexcel qualifications, please visit our website www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning