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1. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

- (a) more than 2 daisies. (3)  
(b) either 5 or 6 daisies. (2)

The botanist decides to count the number of daisies,  $x$ , in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \quad \sum x^2 = 1386$$

- (c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places. (3)  
(d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model. (1)  
(e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square. (2)

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### **Question 1 continued**

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**Question 1 continued**

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**Q1**

**(Total 11 marks)**



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2. The continuous random variable  $X$  is uniformly distributed over the interval  $[-2, 7]$ .

(a) Write down fully the probability density function  $f(x)$  of  $X$ .

(2)

(b) Sketch the probability density function  $f(x)$  of  $X$ .

(2)

Find

(c)  $E(X^2)$ ,

(3)

(d)  $P(-0.2 < X < 0.6)$ .

(2)



**Question 2 continued**

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**Q2**

**(Total 9 marks)**



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**Turn over**

3. A single observation  $x$  is to be taken from a Binomial distribution  $B(20, p)$ .

This observation is used to test  $H_0 : p = 0.3$  against  $H_1 : p \neq 0.3$

- (a) Using a 5% level of significance, find the critical region for this test.  
The probability of rejecting either tail should be as close as possible to 2.5%.

(b) State the actual significance level of this test.

The actual value of  $x$  obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer. (2)

(c) State a conclusion that can be drawn based on this value giving a reason for your

answer.



**Question 3 continued**

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**Q3**

**(Total 7 marks)**



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4. The length of a telephone call made to a company is denoted by the continuous random variable  $T$ . It is modelled by the probability density function

$$f(t) = \begin{cases} kt & 0 \leq t \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the value of  $k$  is  $\frac{1}{50}$ . (3)
- (b) Find  $P(T > 6)$ . (2)
- (c) Calculate an exact value for  $E(T)$  and for  $\text{Var}(T)$ . (5)
- (d) Write down the mode of the distribution of  $T$ . (1)

It is suggested that the probability density function,  $f(t)$ , is not a good model for  $T$ .

- (e) Sketch the graph of a more suitable probability density function for  $T$ . (1)
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### **Question 4 continued**

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**Question 4 continued**

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**Q4**

**(Total 12 marks)**



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**Turn over**

5. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component. (2)

(b) Find the probability that there are at least 2 defective components in the box. (3)

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components. (4)

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**Question 5 continued**

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**Q5**

(Total 9 marks)



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6. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result. (7)

(b) State an assumption that has been made about the visits to the server. (1)

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In a random two minute period on a Saturday the web server is visited 20 times.

- (c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday. (6)



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## **Question 6 continued**

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**Question 6 continued**

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**Q6**

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7. A random variable  $X$  has probability density function given by

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$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the cumulative distribution function  $F(x)$  can be written in the form  $ax^2 + bx + c$ , for  $1 \leq x \leq 4$  where  $a$ ,  $b$  and  $c$  are constants. (3)

(b) Define fully the cumulative distribution function  $F(x)$ . (2)

(c) Show that the upper quartile of  $X$  is 2.5 and find the lower quartile. (6)

Given that the median of  $X$  is 1.88

- (d) describe the skewness of the distribution. Give a reason for your answer.



**Question 7 continued**

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### **Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**Q7**

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

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