

Mark Scheme (Results)

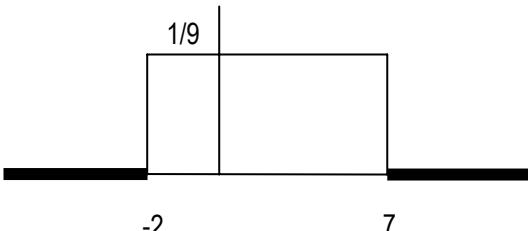
January 2009

GCE

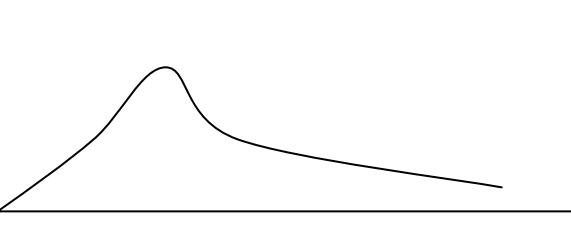
GCE Mathematics (6684/01)

January 2009
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1	The random variable X is the number of daisies in a square. Poisson(3)	B1
(a)	$1 - P(X \leq 2) = 1 - 0.4232 \quad 1 - e^{-3}(1 + 3 + \frac{3^2}{2!}) \\ = 0.5768$	M1 A1
(b)	$P(X \leq 6) - P(X \leq 4) = 0.9665 - 0.8153 \quad e^{-3}\left(\frac{3^5}{5!} + \frac{3^6}{6!}\right) \\ = 0.1512$	M1 A1
(c)	$\mu = 3.69$ $\text{Var}(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2 \\ = 3.73/3.72/3.71$	B1 M1 A1
(d)	For a Poisson model , Mean = Variance ; For these data $3.69 \approx 3.73$ \Rightarrow Poisson model	B1
(e)	$\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193$	M1
	allow their mean or var Awrt 0.193 or 0.194	A1 ft

Question Number	Scheme	Marks	
2 (a)	$f(x) = \begin{cases} \frac{1}{9} & -2 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$	B1 B1 (2)	
(b)		B1 B1 (2)	
(c)	$E(X) = 2.5$ $\text{Var}(X) = \frac{1}{12}(7+2)^2$ or <u>6.75</u> $E(X^2) = \text{Var}(X) + E(X)^2$ $= 6.75 + 2.5^2$ $= 13$	both M1 A1 alternative $\int_{-2}^7 x^2 f(x) dx = \left[\frac{x^3}{27} \right]_{-2}^7$ <p style="text-align: center;">$\int x^2 f(x) dx$ attempt to integrate and use limits of -2 and 7</p>	B1 M1 A1 (3)
(d)	$P(-0.2 < X < 0.6) = \frac{1}{9} \times 0.8$ $= \frac{4}{45}$ or 0.0889 Or equiv	M1 awrt 0.089 A1 (2)	

Question Number	Scheme	Marks
3 (a)	$X \sim B(20, 0.3)$ $P(X \leq 2) = 0.0355$ $P(X \geq 11) = 1 - 0.9829 = 0.0171$ Critical region is $(X \leq 2) \cup (X \geq 11)$	M1 A1 A1 (3)
(b)	Significance level = $0.0355 + 0.0171 = 0.0526$ or 5.26%	M1 A1 (2)
(c)	Insufficient evidence to reject H_0 Or sufficient evidence to accept H_0 /not significant $x = 3$ (or the value) is not in the critical region or $0.1071 > 0.025$ Do not allow inconsistent comments	B1 ft B1 ft (2)

Question Number	Scheme	Marks
4 (a)	$\int_0^{10} ktdt = 1$ $\left[\frac{kt^2}{2} \right]_0^{10} = 1$ $50k = 1$ $k = \frac{1}{50}$ <p style="text-align: right;">or Area of triangle = 1 or $10 \times 0.5 \times 10k = 1$ or linear equation in k</p> <p style="text-align: right;">cso</p>	M1 M1 A1 (3)
(b)	$\int_6^{10} ktdt = \left[\frac{kt^2}{2} \right]_6^{10}$ $= \frac{16}{25}$	M1 A1 (2)
(c)	$E(T) = \int_0^{10} kt^2 dt = \left[\frac{kt^3}{3} \right]_0^{10}$ $= 6\frac{2}{3}$ $\text{Var}(T) = \int_0^{10} kt^3 dt - \left(6\frac{2}{3} \right)^2 = \left[\frac{kt^4}{4} \right]_0^{10} - \left(6\frac{2}{3} \right)^2$ $= 50 - \left(6\frac{2}{3} \right)^2$ $= 5\frac{5}{9}$	M1 A1 M1; M1dep A1 (5)
(d)	10	B1 (1)
(e)		B1 (1)

Question Number	Scheme	Marks
5	<p>(a) X represents the number of defective components.</p> $P(X=1) = (0.99)^9 (0.01) \times 10 = 0.0914$ <p>(b) $P(X \geq 2) = 1 - P(X \leq 1)$ $= 1 - (p)^{10} - (a)$ $= 0.0043$</p> <p>(c) $X \sim Po(2.5)$</p> $P(1 \leq X \leq 4) = P(X \leq 4) - P(X=0)$ $= 0.8912 - 0.0821$ $= 0.809$ <p>Normal distribution used. B1 for mean only</p> <hr/> <p>Special case for parts a and b If they use 0.1 do not treat as misread as it makes it easier.</p> <p>(a) M1 A0 if they have 0.3874 (b) M1 A1ft A0 they will get 0.2639 (c) Could get B1 B0 M1 A0</p> <hr/> <p>For any other values of p which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0</p>	M1A1 (2) M1 A1/ A1 (3) B1B1 M1 A1 (4)

Question Number	Scheme	Marks
6 (a)(i)	$H_0 : \lambda = 7 \quad H_1 : \lambda > 7$ $X = \text{number of visits}, X \sim \text{Po}(7)$ $P(X \geq 10) = 1 - P(X \leq 9)$ $= 0.1695$ $1 - P(X \leq 10) = 0.0985$ $1 - P(X \leq 9) = 0.1695$ $\text{CR } X \geq 11$	B1 B1 M1 A1
	$0.1695 > 0.10, \quad \text{CR } X \geq 11$ Not significant or it is not in the critical region or do not reject H_0 The rate of visits on a Saturday is not greater/ is unchanged	M1 A1 no ft
(ii)	$X = 11$	B1
(b)	(The visits occur) randomly/ independently or singly or constant rate	B1 (7) (1)
(c)	$[H_0 : \lambda = 7 \quad H_1 : \lambda > 7 \quad (\text{or } H_0 : \lambda = 14 \quad H_1 : \lambda > 14)]$ $X \sim N(14, 14)$ $P(X \geq 20) = P\left(z \geq \frac{19.5 - 14}{\sqrt{14}}\right)$ $= P(z \geq 1.47)$ $= 0.0708 \quad \text{or } z = 1.2816$ $\pm 0.5, \text{ stand}$ $0.0708 < 0.10 \text{ therefore significant. The rate of visits is greater on a Saturday}$	B1;B1 M1 M1 A1dep both M A1dep 2 nd M (6)

Question Number	Scheme	Marks
7 (a)	$F(x_0) = \int_1^x -\frac{2}{9}x + \frac{8}{9} dx = \left[-\frac{1}{9}x^2 + \frac{8}{9}x \right]_1^x$ $= \left[-\frac{1}{9}x^2 + \frac{8}{9}x \right] - \left[-\frac{1}{9} + \frac{8}{9} \right]$ $= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$	M1A1 A1 (3)
(b)	$F(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$	B1B1J
(c)	$F(x) = 0.75 ; \quad \text{or } F(2.5) = -\frac{1}{9} \times 2.5^2 + \frac{8}{9} \times 2.5 - \frac{7}{9}$ $-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75$ $4x^2 - 32x + 55 = 0$ $-x^2 + 8x - 13.75 = 0$ $x = 2.5 \quad = 0.75$ <p style="text-align: right;">cso</p> <p>and $F(x) = 0.25$</p> $-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25$ $-x^2 + 8x - 7 = 2.25$ $-x^2 + 8x - 9.25 = 0$ $x = \frac{-8 \pm \sqrt{8^2 - 4 \times -1 \times -9.25}}{2 \times -1}$ $x = 1.40$ <p style="text-align: right;">quadratic 3 terms = 0</p>	M1; A1 M1 M1 dep M1 dep A1 (6)
(d)	$Q_3 - Q_2 > Q_2 - Q_1$ <p>Or mode = 1 and mode < median</p> <p>Or mean = 2 and median < mode</p> <p>Sketch of pdf here or be referred to if in a different part of the question</p> <p>Box plot with Q_1, Q_2, Q_3 values marked on</p> <p>Positive skew</p>	M1 A1 (2)