Mark Scheme (Results)

Summer 2013

GCE Statistics 4 (6686/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

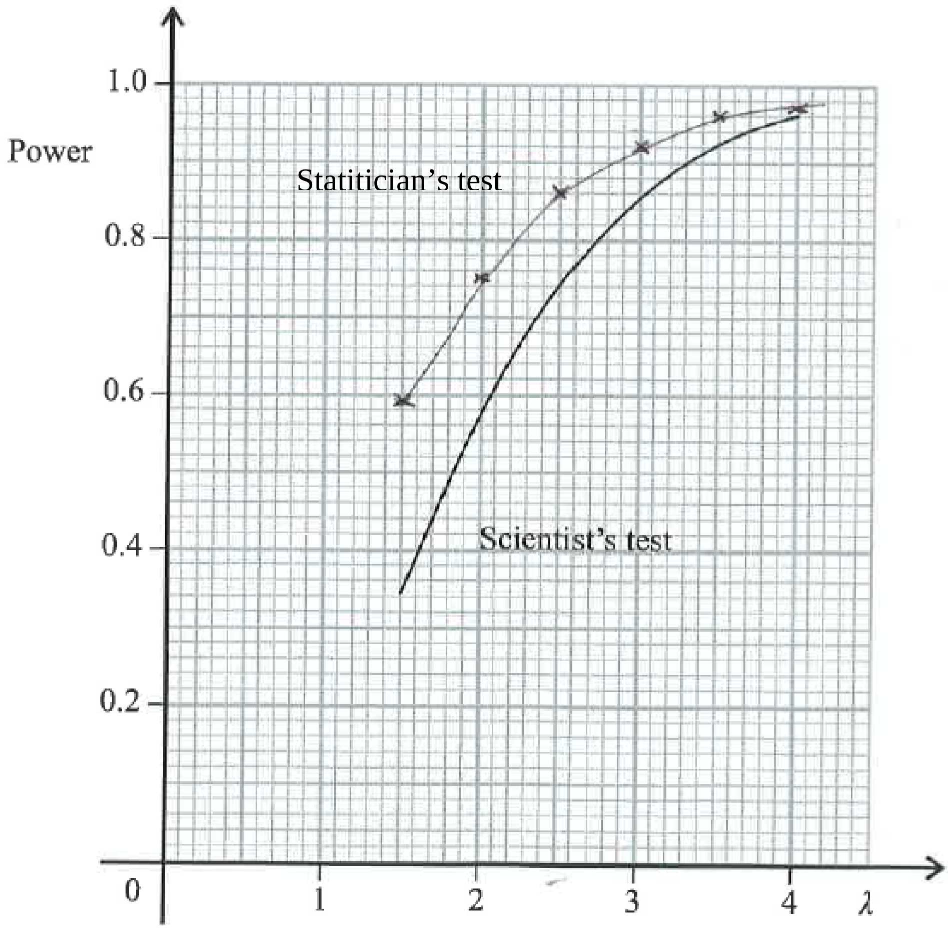
8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

Question Number	Scheme	Marks
1.	$H_0 : \sigma^2 = 2.4^2 \quad H_1 : \sigma^2 \neq 2.4^2$	B1
	$s^2 = \dfrac{1414.08 - 10\times\left(\dfrac{113.4}{10}\right)^2}{9} = 14.236$	M1 A1
	$\chi^2 = \dfrac{9s^2}{\sigma^2} = \dfrac{9\times 14.236}{2.4^2} = 22.24375$	M1 A1
	Critical Value $\chi_9^{\ 2}(0.025) = 19.023$	B1
	Significant result, there is evidence of a change in standard deviation <u>or</u> the data do not support George's belief	A1cso **(7)** **[7]**
	Notes	
	1st B1 Both hypotheses, must use σ. Allow $H_0 : \sigma = 2.4 \quad H_1 : \sigma \neq 2.4$ 1st M1 correct method used 1st A1 awrt 14.2 2nd M1 $\chi^2 = \dfrac{9\times\text{"}their\ s^2\text{"}}{2.4^2}$ 2nd A1 awrt 22.2 2nd B1 for critical value, this should be compatible with their alternative hypothesis (16.919 for one tail test) 3rd A1ft fully correct solution only	

Question Number	Scheme	Marks
2. (a)	d = Jan - June: -2, 1, -3, 2, -2, 3, 2, 2	M1
	$\bar{d} = 0.375, \quad \sum d^2 = 39 \Rightarrow s^2 = 5.4107...$ or s = 2.326…	M1, M1
	$t_7(0.025) = 2.365$	B1
	Confidence Interval: $0.375 \pm 2.365 \times \frac{2.326...}{\sqrt{8}}$	M1
	= **<u>(-1.57, 2.32)</u>** (o.e.)	A1,A1 **(7)**
(b)	$\mathrm{H}_0 : \mu_D = 0 \quad \mathrm{H}_1 : \mu_D \neq 0$	B1
	Comment that 0 is in the interval	M1
	Not sig, no evidence of a change in **mean time** to **assemble component**	A1ft **(3)** **[10]**
	Notes	
(a)	1st M1 for attempting differences 2nd M1 for attempting $\bar{d}$ 3rd M1 for attempting $s_d{}^2$, correct expression with their $\sum d^2$ and $\bar{d}$ or correct calculation (to 2 sf or better) 4th M1 for use of a correct CI formula, using a value for t and ft their values. 1st A1 for lower limit of -1.57 or -2.32 2nd A1 for corresponding upper limit	
S.C.	Allow A1A1 for (0, 2.32)	
(b)	B1 for both hypotheses using μ_D M1 for a comment about 0 being in (or out) of <u>their</u> interval A1 contextual conclusion – must include assemble components	
S.C.	If they have used difference in means test in part (a) to get the confidence interval then award the B1 for $\mathrm{H}_0 : \mu_x - \mu_y = 0 \quad \mathrm{H}_1 : \mu_x - \mu_y \neq 0$ or the correct hypotheses.	

Question Number	Scheme	Marks
3. (a)	$H_0 : \sigma^2_A = \sigma^2_B \quad H_1 : \sigma^2_A \neq \sigma^2_B$	B1
	$F = \frac{s_B{}^2}{s_A{}^2} = \frac{4.37^2}{4.24^2} = 1.0622...$	M1A1
	$F_{12,6}$ (0.01) = 7.72	B1
	Not sig, so no evidence of a difference in variances	A1ft **(5)**
(b)	$H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B$	B1
	$s_p{}^2 = \frac{6\times 4.24^2 + 12\times 4.37^2}{18}$ = 18.7238 or $s_p = 4.327...$	M1
	$t = \pm\frac{14.31 - 8.43}{s_p\sqrt{\frac{1}{7}+\frac{1}{13}}} = \pm\ 2.8985\ldots$ awrt 2.9	M1A1
	$t_{18}(0.01) = 2.552$	B1
	sig, there is evidence to support **archaeologist's** claim <u>or</u> there is evidence that bricks for site *B* have higher **mean compression** strength than those from site *A*.	A1ft **(6)**
(c)	The test in (b) requires $\sigma^2_A = \sigma^2_B$ and the test in part (a) shows that this is a reasonable assumption. (o.e.)	B1 **(1)**
		[12]
	Notes	
(a)	M1 for use of a correct formula Allow $F = \frac{s_A{}^2}{s_B{}^2} = \frac{4.24^2}{4.37^2} = 0.941...$ with 0.1295..	
(b)	B1 if *A* and *B* not used it must be clear which is *A* and which is *B* 1st M1 for attempt to calculate s_p or s^2_p 2nd M1 for attempt correct test statistic 2nd A1 ft need **archaeologist's or compression**	
(c)	Need to refer to 'allows us to assume variances the same' and this is needed in for test. oe	

Question Number	Scheme	Marks
4. (a)	$E(X)=\mu=\frac{2a-a}{2}=\frac{a}{2}$; $E(\overline{X})=\mu=\frac{a}{2}$ so biased estimator for a	M1;A1
	Bias = $\frac{a}{2}-a=-\frac{a}{2}$	B1(accept $\underline{+}$)
		(3)
(b)	k = 2	B1 **(1)**
(c)	$\text{Var}(X)=\sigma^2=\frac{(2a--a)^2}{12}=\frac{9a^2}{12}=\frac{3a^2}{4}$; $\text{Var}(\overline{X})=\frac{\sigma^2}{2}$	B1;B1
	Var(Y) = $k^2\text{Var}(\overline{X})=4,\times\frac{\sigma^2}{2}=4\times\frac{3a^2}{4\times 2}=\frac{3}{2}a^2$	M1,A1 **(4)**
(d)	E(M) = $\int\frac{2x(x+a)}{9a^2}\,dx=\left[\frac{2x^3}{27a^2}+\frac{ax^2}{9a^2}\right]_{-a}^{2a},=\left(\frac{16a}{27}+\frac{4a}{9}\right)-\left(-\frac{2a}{27}+\frac{a}{9}\right)$ [$=a$]	M1A1,M1d
	So E(M) = a and therefore M is an unbiased estimator for a	A1cso **(4)**
(e)	Var(M) = $\frac{3}{2}a^2-a^2=\frac{1}{2}a^2$	B1 **(1)**
(f)	Var(M) < Var(Y) , so M is the better estimator of a	M1, A1 **(2)**
(g)	Maximum value = **<u>5</u>**	B1ft **(1)** **[16]**
	Notes	
(a)	M1 for use of formula or integration or symmetry to find E(X)	
(c)	1st B1 for use of formula for variance 2nd B1 for use of $\frac{\sigma^2}{n}$ formula M1 for $k^2\text{Var}(\overline{X})$ and ft their k	
(d)	1st M1 for attempt at correct integration of correct expression 1st A1 for correct integration 2nd M1d dependent on previous M, for attempting to use correct limits 2nd A1 need statement that M is therefore unbiased	
(f)	M1 for comparison of their Var(Y) and their Var(M)	
(g)	B1ft for calculation of their estimate based on their choice in (f). If they choose Y answer is 4 (or twice their k)	

Question Number	Scheme	Marks
5. (a)	Y = no. of organisms in 20 ml. $Y \sim \text{Po}(2\lambda)$ Size = P($Y \geq 4$| $Y \sim$Po(2)) , = 1 - P($Y \leq 3$) = 1- 0.8571 = **<u>0.1429</u>**	M1, A1 **(2)**
(b)	P(Type II error) = 1 - P($Y \geq 4$| $Y \sim$Po(5)) , = P($Y \leq 3$) = **<u>0.2650</u>**	M1, A1 **(2)**
(c)	X = no. of organisms in 10 ml. $X \sim \text{Po}(\lambda)$ Power = P($X \geq 2$) + P(X =1) × P($X \geq 2$) = P($X \geq 2$) [1+ P(X = 1)] = $\left[1-\mathrm{e}^{-\lambda}(1+\lambda)\right]\times\left[1+\lambda\mathrm{e}^{-\lambda}\right]$ $= 1-\mathrm{e}^{-\lambda}-\lambda\mathrm{e}^{-\lambda}+\lambda\mathrm{e}^{-\lambda}-\lambda(1+\lambda)\mathrm{e}^{-2\lambda} = 1-\mathrm{e}^{-\lambda}-\lambda(1+\lambda)\mathrm{e}^{-2\lambda}$	 M1 M1A1 A1cso **(4)**
(d)	r = 0.92	B1 **(1)**
(e)	See Graph paper 	B1B1 **(2)**

Question Number	Scheme	Marks
(f)	Expected time for statistician's test: $30\times\mathrm{P}(X=1)+15\times\left[1-\mathrm{P}(X=1)\right]$	M1

	$= 30\lambda e^{-\lambda} + 15\left(1 - \lambda e^{-\lambda}\right) = 15\left(1 + \lambda e^{-\lambda}\right)$	A1
	slower if: $15\left(1 + \lambda e^{-\lambda}\right) > 20, \quad \Rightarrow \lambda e^{-\lambda} > \frac{1}{3}$	M1,A1cso **(4)**
(g)	$\lambda e^{-\lambda}$ with $\lambda = 1$ is 0.36…, with $\lambda = 2$ is 0.27…so second(statisticians) test is slower if $\lambda = 1$ but faster for $\lambda = 2$. Second test is more powerful for all λ	B1
	Choose second test - more powerful and faster for $\lambda \geq 2$	B1 **(2)** **[17]**
	Notes	
(a)	M1 for correct expression for size using Po(2)	
(b)	M1 for correct expression using Po(5)	
(c)	1st M1 for a correct expression in terms of probabilities Alternate answer 1 – [P(X = 0) + P(X = 1) x P($X \leq 1$)] 2nd M1 for an attempt at a correct equation in λ 1st A1 for a correct expression in λ	
(e)	1st B1 points 2nd B1 curve (or straight lines)	
(f)	1st M1 for an attempt to calculate expected time Alternate method 15 + 15x P(X = 1) 1st A1 for a correct expression in terms of λ 2nd M1 for attempt at correct inequality	
(g)	1st B1 for a comment about power & timings 2nd B1 for selecting second test	

Question Number	Scheme	Marks
6. (a)	$H_0 : \sigma^2{}_A = \sigma^2{}_B \qquad H_1 : \sigma^2{}_A \neq \sigma^2{}_B$	B1
	$s_A{}^2 = (0.25)^2 = 0.0625 \qquad s_B{}^2 = (0.178885...)^2 = 0.032$	B1B1

	$F = \frac{0.0625}{0.032} = 1.953...$	M1A1
	Critical Value: $F_{3,5} = 5.41$	B1
	not sig, samples come from populations with common variance	A1cso **(7)**
(b)	$s_p^{\ 2} = \frac{3\times 0.25^2 + 5\times 0.032}{8} = 0.04343... = (0.0284...)^2$	M1A1
	Use $\frac{8s_p^{\ 2}}{\sigma^2} \sim \chi_8^{\ 2}$	M1
	$1.344 < \frac{8\times 0.0434...}{\sigma^2} < 21.955$	B1,B1
	99% confidence interval is **<u>(0.0158, 0.259)</u>**	A1 **(6)** **[13]**

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