Answer **all** questions.

1 The time T taken for a simple pendulum to make a single small oscillation is thought to depend only on its length l, its mass m and the acceleration due to gravity g.

By using dimensional analysis:

(a) show that T does **not** depend on m; *(3 marks)*

(b) express T in terms of l, g and k, where k is a dimensionless constant. *(4 marks)*

2 Three smooth spheres A, B and C of equal radii and masses m, m and $2m$ respectively lie at rest on a smooth horizontal table. The centres of the spheres lie in a straight line with B between A and C. The coefficient of restitution between any two spheres is e.

The sphere A is projected directly towards B with speed u and collides with B.

(a) Find, in terms of u and e, the speed of B immediately after the impact between A and B. *(5 marks)*

(b) The sphere B subsequently collides with C. The speed of C immediately after this collision is $\frac{3}{8}u$. Find the value of e. *(7 marks)*

3 A ball of mass 0.45 kg is travelling horizontally with speed $15\,\mathrm{m\,s^{-1}}$ when it strikes a fixed vertical bat directly and rebounds from it. The ball stays in contact with the bat for 0.1 seconds.

At time t seconds after first coming into contact with the bat, the force exerted on the ball by the bat is $1.4 \times 10^5(t^2 - 10t^3)$ newtons, where $0 \leqslant t \leqslant 0.1$.

In this simple model, ignore the weight of the ball and model the ball as a particle.

(a) Show that the magnitude of the impulse exerted by the bat on the ball is 11.7 N s, correct to three significant figures. *(4 marks)*

(b) Find, to two significant figures, the speed of the ball immediately after the impact. *(4 marks)*

(c) Give a reason why the speed of the ball immediately after the impact is different from the speed of the ball immediately before the impact. *(1 mark)*

4 The unit vectors **i** and **j** are directed due east and due north respectively.

Two cyclists, Aazar and Ben, are cycling on straight horizontal roads with constant velocities of $(6\mathbf{i} + 12\mathbf{j})\ \mathrm{km\,h^{-1}}$ and $(12\mathbf{i} - 8\mathbf{j})\ \mathrm{km\,h^{-1}}$ respectively. Initially, Aazar and Ben have position vectors $(5\mathbf{i} - \mathbf{j})$ km and $(18\mathbf{i} + 5\mathbf{j})$ km respectively, relative to a fixed origin.

(a) Find, as a vector in terms of **i** and **j**, the velocity of Ben relative to Aazar. *(2 marks)*

(b) The position vector of Ben relative to Aazar at time t hours after they start is **r** km.

Show that

$$\mathbf{r} = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$$

(4 marks)

(c) Find the value of t when Aazar and Ben are closest together. *(6 marks)*

(d) Find the closest distance between Aazar and Ben. *(2 marks)*

5 A football is kicked from a point O on a horizontal football ground with a velocity of $20\ \mathrm{m\,s^{-1}}$ at an angle of elevation of 30°. During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

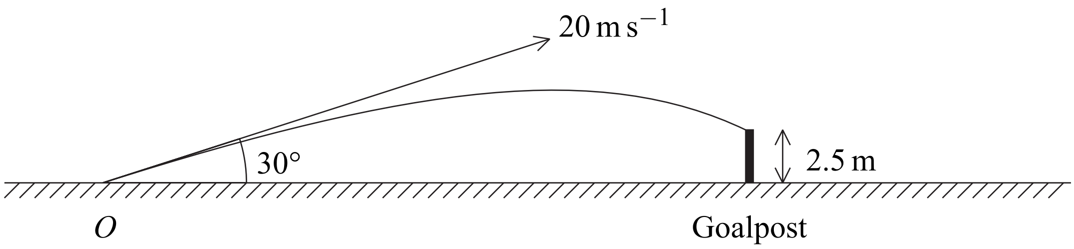
(a) Show that x and y satisfy the equation

$$y = x \tan 30° - \frac{gx^2}{800 \cos^2 30°}$$

(6 marks)

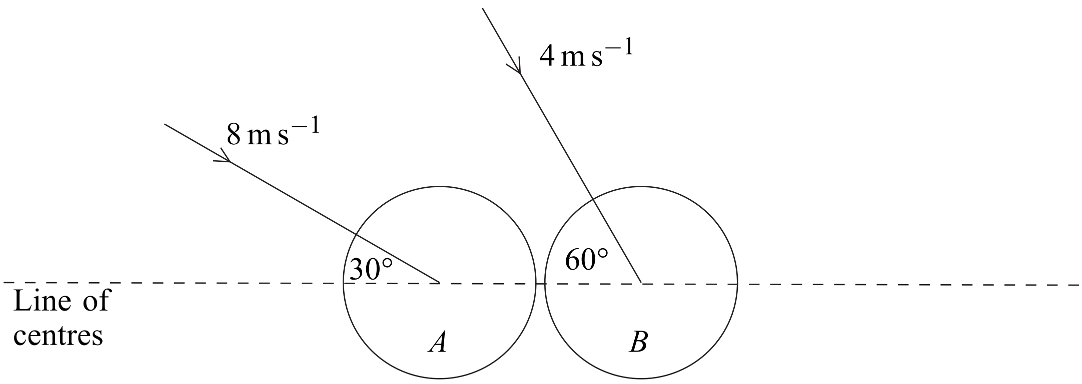
(b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal.

(4 marks)



(c) State **two** modelling assumptions that you have made. *(2 marks)*

6 Two smooth billiard balls A and B, of identical size and equal mass, move towards each other on a horizontal surface and collide. Just before the collision, A has velocity $8\,\text{m}\,\text{s}^{-1}$ in a direction inclined at 30° to the line of centres of the balls, and B has velocity $4\,\text{m}\,\text{s}^{-1}$ in a direction inclined at 60° to the line of centres, as shown in the diagram.



The coefficient of restitution between the balls is $\frac{1}{2}$.

(a) Find the speed of B immediately after the collision. *(9 marks)*

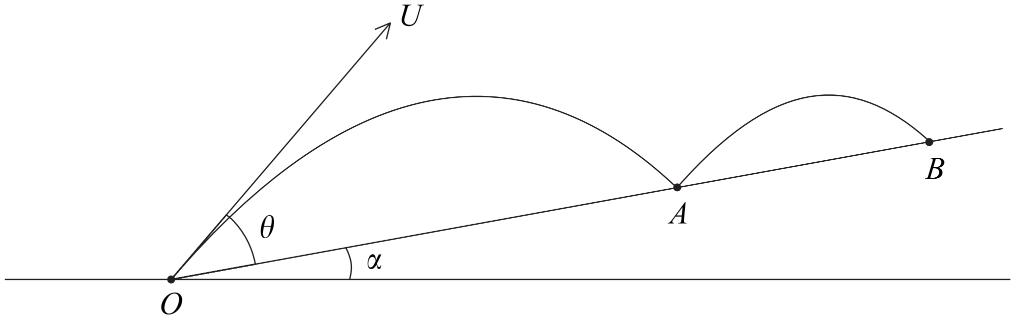
(b) Find the angle between the velocity of B and the line of centres of the balls immediately after the collision. *(2 marks)*

7 A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A. The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.

(a) (i) Find, in terms of U, g, α and θ, the time taken by the projectile to travel from O to A. *(3 marks)*

(ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A, is the same as it was initially at O. *(3 marks)*

(b) The projectile rebounds and strikes the hill again at a point B. The hill is smooth and the coefficient of restitution between the projectile and the hill is e.



Find the ratio of the time of flight from O to A to the time of flight from A to B. Give your answer in its simplest form. *(4 marks)*

END OF QUESTIONS

Practice 2

Leave blank

1. A light elastic string of natural length 0.4 m has one end A attached to a fixed point. The other end of the string is attached to a particle P of mass 2 kg. When P hangs in equilibrium vertically below A, the length of the string is 0.56 m.

(a) Find the modulus of elasticity of the string.

(3)

A horizontal force is applied to P so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(b) Find the angle θ.

(3)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal table. When P is a distance x metres from a fixed point O on the line, it experiences a force of magnitude $\frac{16}{5x^2}$ N away from O in the direction OP. Initially P is at a point 2 m from O and is moving towards O with speed 8 m s^{-1}.

Find the distance of P from O when P first comes to rest.

(8)

Leave blank

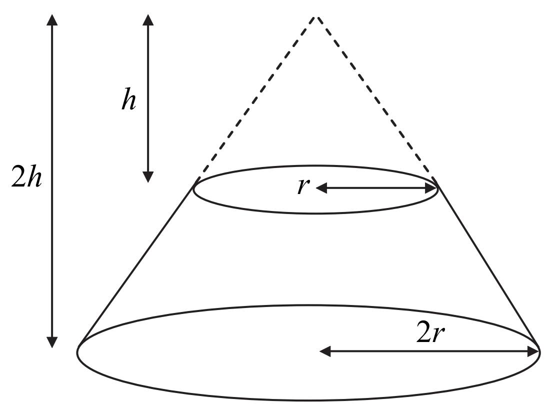
Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. **Figure 1**



A uniform solid S is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius r and height h, which has the same vertex as the original cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$.
(5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle $\theta°$ to the horizontal. The table is sufficiently rough to prevent S from slipping.
Given that $h = 2r$,

(b) find the greatest value of θ for which S does not topple.
(3)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. A particle P of mass m lies on a smooth plane inclined at an angle 30º to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$. The particle P is now projected from A down a line of greatest slope of the plane with speed V. It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

(a) find V in terms of a and g,

(6)

(b) find, in terms of a and g, the speed of P when the string first becomes slack.

(4)

Leave blank

Question 4 continued

Q4

(Total 10 marks)

Leave blank

5. A car of mass m moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s^{-1}. Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6.

(3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

(b) Find, as a multiple of mg, the normal reaction between the car and road as the car moves round this bend.

(4)

(c) Find the speed of the car as it goes round this bend.

(5)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

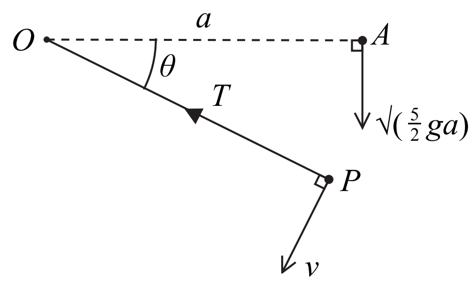
Q5

(Total 12 marks)

Leave blank

6.

Figure 2



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. At time $t = 0$, P is projected vertically downwards with speed $\sqrt{(\frac{5}{2}ga)}$ from a point A which is at the same level as O and a distance a from O. When the string has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in Figure 2.

(a) Show that $v^2 = \frac{ga}{2}(5 + 4\sin\theta)$.

(3)

(b) Find T in terms of m, g and θ.

(3)

The string becomes slack when $\theta = \alpha$.

(c) Find the value of α.

(3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A, the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O. The particle now moves on an arc of a circle centre B. Given that the particle reaches the point C, which is $\frac{1}{2}a$ vertically above the point B, without the string going slack,

(d) find the tension in the string when P is at the point C.

(6)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 15 marks)

Leave blank

7. A particle P of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point A. When P hangs freely below A in equilibrium, P is at the point E, 1.2 m below A. The particle is now pulled down to a point B which is 0.4 m vertically below E and released from rest.

(a) Prove that, while the string is taut, P moves with simple harmonic motion about E with period $\frac{2\pi}{7}$ s.

(5)

(b) Find the greatest magnitude of the acceleration of P while the string is taut.

(1)

(c) Find the speed of P when the string first becomes slack.

(3)

(d) Find, to 3 significant figures, the time taken, from release, for P to return to B for the first time.

(7)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 The magnitude of the gravitational force, F, between two planets of masses m_1 and m_2 with centres at a distance x apart is given by

$$F = \frac{Gm_1m_2}{x^2}$$

where G is a constant.

(a) By using dimensional analysis, find the dimensions of G. *(3 marks)*

(b) The lifetime, t, of a planet is thought to depend on its mass, m, its initial radius, R, the constant G and a dimensionless constant, k, so that

$$t = km^{\alpha}R^{\beta}G^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ. *(5 marks)*

2 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

Two helicopters, A and B, are flying with constant velocities of $(20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k})\,\mathrm{m\,s^{-1}}$ and $(30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k})\,\mathrm{m\,s^{-1}}$ respectively. At noon, the position vectors of A and B relative to a fixed origin, O, are $(8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k})\,\mathrm{m}$ and $(2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k})\,\mathrm{m}$ respectively.

(a) Write down the velocity of A relative to B. *(2 marks)*

(b) Find the position vector of A relative to B at time t seconds after noon. *(3 marks)*

(c) Find the value of t when A and B are closest together. *(5 marks)*

3 A particle P, of mass 2 kg, is initially at rest at a point O on a smooth horizontal surface. The particle moves along a straight line, OA, under the action of a horizontal force. When the force has been acting for t seconds, it has magnitude $(4t + 5)$ N.

(a) Find the magnitude of the impulse exerted by the force on P between the times $t = 0$ and $t = 3$. *(3 marks)*

(b) Find the speed of P when $t = 3$. *(2 marks)*

(c) The speed of P at A is $37.5\,\mathrm{m\,s^{-1}}$. Find the time taken for the particle to reach A. *(4 marks)*

4 Two small smooth spheres, A and B, of equal radii have masses 0.3 kg and 0.2 kg respectively. They are moving on a smooth horizontal surface directly towards each other with speeds $3\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively when they collide. The coefficient of restitution between A and B is 0.8 .

(a) Find the speeds of A and B immediately after the collision. *(6 marks)*

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the path of the sphere. The coefficient of restitution between B and the wall is 0.7 .

Show that B will collide again with A. *(3 marks)*

5 A ball is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$ *(6 marks)*

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P.

(i) By taking $g = 10\,\mathrm{m\,s^{-2}}$, show that R satisfies the equation

$$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$$ *(2 marks)*

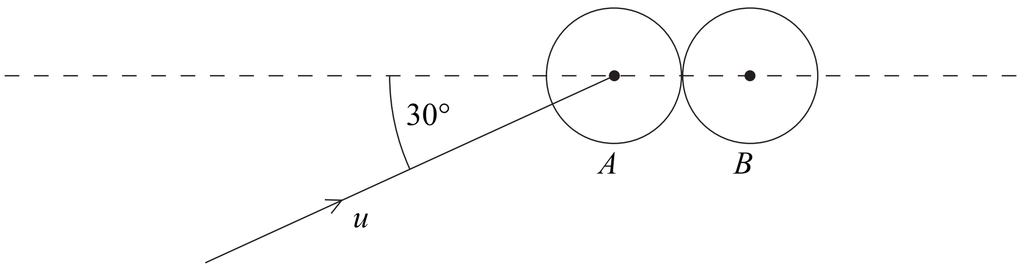
(ii) Hence, given that u and R are constants, show that, for $\tan\alpha$ to have real values, R must satisfy the inequality

$$R^2 \leqslant \frac{u^2(u^2 - 20)}{100}$$ *(2 marks)*

(iii) Given that $R = 5$, determine the minimum possible speed of projection. *(3 marks)*

6 A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table.

Just before the collision, the direction of motion of A makes an angle of 30° with the line of the centres of the two balls, as shown in the diagram.



The coefficient of restitution between A and B is e.

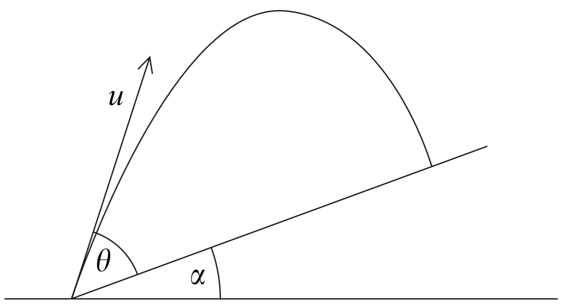
(a) Given that $\cos 30° = \frac{\sqrt{3}}{2}$, show that the speed of B immediately after the collision is

$$\frac{\sqrt{3}}{4}u(1+e)$$ *(5 marks)*

(b) Find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision. *(3 marks)*

(c) Given that $e = \frac{2}{3}$, find the angle that the velocity of A makes with the line of centres immediately after the collision. Give your answer to the nearest degree. *(3 marks)*

7 A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin\theta \cos(\theta+\alpha)}{g\cos^2\alpha}$$ *(8 marks)*

(b) Hence, using the identity $2\sin A \cos B = \sin(A+B) + \sin(A-B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$. *(3 marks)*

(c) Given that the particle strikes the plane at right angles, show that

$$2\tan\theta = \cot\alpha$$ *(4 marks)*

END OF QUESTIONS

Practice 4

Leave blank

1. A particle P of mass 3 kg is moving in a straight line. At time t seconds, $0 \leqslant t \leqslant 4$, the only force acting on P is a resistance to motion of magnitude $\left(9 + \frac{15}{(t+1)^2}\right)$ N. At time t seconds the velocity of P is v m s^{-1}. When $t = 4$, $v = 0$.

Find the value of v when $t = 0$.

(7)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Turn over

Leave blank

2.

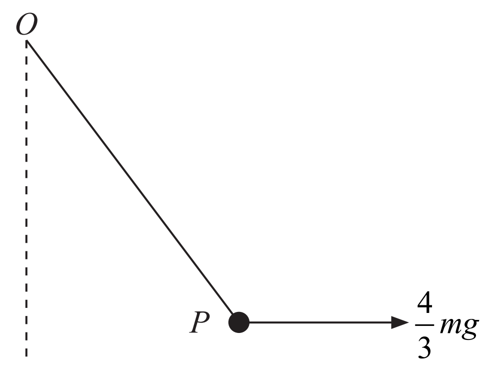


Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P.

This force acts in the vertical plane containing the string, as shown in Figure 1. Find

(a) the tension in the string,

(5)

(b) the elastic energy stored in the string.

(4)

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Turn over

Leave blank

3. A rough disc rotates about its centre in a horizontal plane with constant angular speed 80 revolutions per minute. A particle P lies on the disc at a distance 8 cm from the centre of the disc. The coefficient of friction between P and the disc is μ. Given that P remains at rest relative to the disc, find the least possible value of μ.

(7)

Leave blank

Question 3 continued

(Total 7 marks)

Q3

Leave blank

4. A small shellfish is attached to a wall in a harbour. The rise and fall of the water level is modelled as simple harmonic motion and the shellfish as a particle. On a particular day the minimum depth of water occurs at 10 00 hours and the next time that this minimum depth occurs is at 22 30 hours. The shellfish is fixed in a position 5 m above the level of the minimum depth of the water and 11 m below the level of the maximum depth of the water. Find

(a) the speed, in metres per hour, at which the water level is rising when it reaches the shellfish,
(7)

(b) the earliest time after 10 00 hours on this day at which the water reaches the shellfish.
(4)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 11 marks)

Leave blank

5.

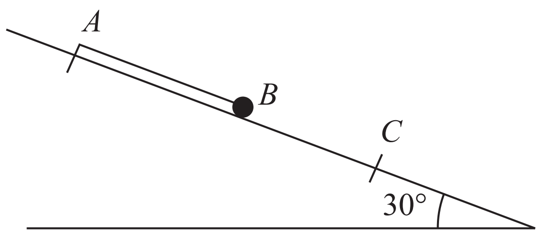


Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2. Find

(a) the length AC,

(5)

(b) the greatest speed attained by B as it moves from its initial position to C.

(7)

Leave blank

Question 5 continued

Turn over

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 12 marks)

Q5

Leave blank

6.

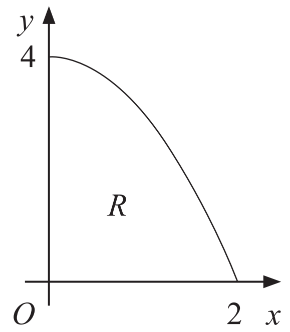


Figure 3

The region R is bounded by part of the curve with equation $y = 4 - x^2$, the positive x-axis and the positive y-axis, as shown in Figure 3. The unit of length on both axes is one metre. A uniform solid S is formed by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of S is $\frac{5}{8}$ m from O.

(10)

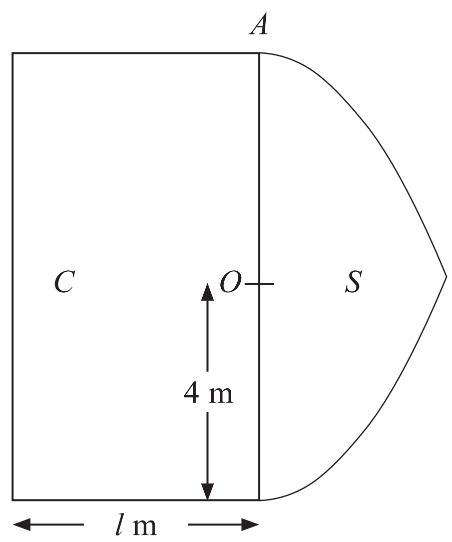


Figure 4

Figure 4 shows a cross section of a uniform solid P consisting of two components, a solid cylinder C and the solid S. The cylinder C has radius 4 m and length l metres. One end of C coincides with the plane circular face of S. The point A is on the circumference of the circular face common to C and S. When the solid P is freely suspended from A, the solid P hangs with its axis of symmetry horizontal.

(b) Find the value of l.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 14 marks)

Q6

Leave blank

7.

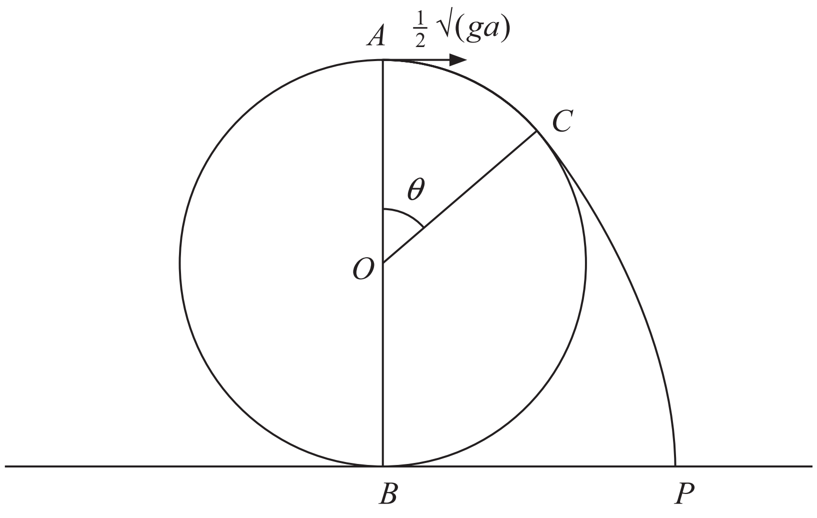


Figure 5

A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{(ga)}$. The particle leaves the surface of the sphere at the point C, where $\angle AOC = \theta$, and strikes the plane at the point P, as shown in Figure 5.

(a) Show that $\cos\theta = \frac{3}{4}$.

(7)

(b) Find the angle that the velocity of the particle makes with the horizontal as it reaches P.

(8)

Leave blank

Question 7 continued

Turn over

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 A ball of mass m is travelling vertically downwards with speed u when it hits a horizontal floor. The ball bounces vertically upwards to a height h.

It is thought that h depends on m, u, the acceleration due to gravity g, and a dimensionless constant k, such that

$$h = km^{\alpha}u^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

2 A particle is projected from a point O on a horizontal plane and has initial velocity components of $2\,\mathrm{m\,s^{-1}}$ and $10\,\mathrm{m\,s^{-1}}$ parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

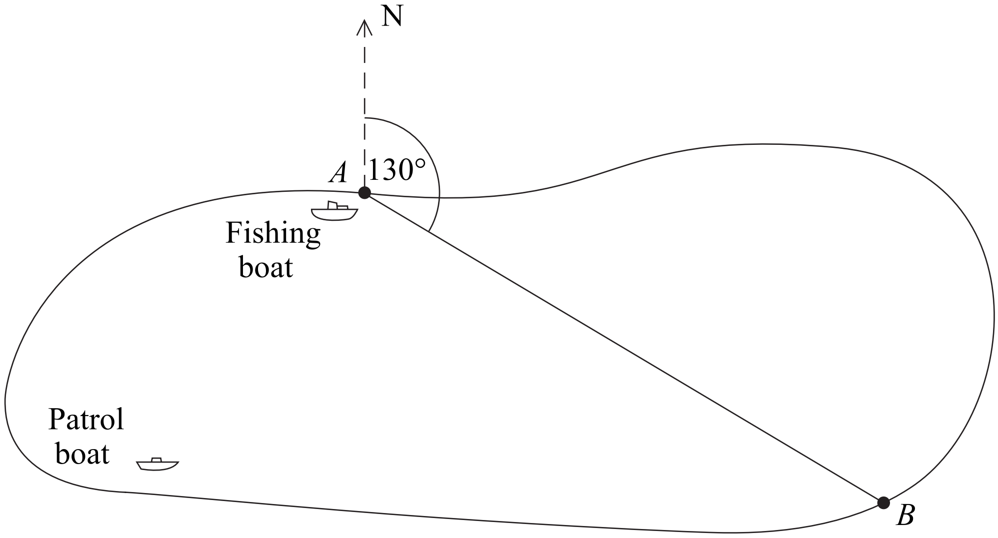
(a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x$$ *(4 marks)*

(b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. *(4 marks)*

(c) Hence find the time for which the particle is more than 1 metre above the plane. *(2 marks)*

3 A fishing boat is travelling between two ports, A and B, on the shore of a lake. The bearing of B from A is 130°. The fishing boat leaves A and travels directly towards B with speed $2\,\mathrm{m\,s^{-1}}$. A patrol boat on the lake is travelling with speed $4\,\mathrm{m\,s^{-1}}$ on a bearing of 040°.



(a) Find the velocity of the fishing boat relative to the patrol boat, giving your answer as a speed together with a bearing. *(5 marks)*

(b) When the patrol boat is 1500 m due west of the fishing boat, it changes direction in order to intercept the fishing boat in the shortest possible time.

(i) Find the bearing on which the patrol boat should travel in order to intercept the fishing boat. *(4 marks)*

(ii) Given that the patrol boat intercepts the fishing boat before it reaches B, find the time, in seconds, that it takes the patrol boat to intercept the fishing boat after changing direction. *(4 marks)*

(iii) State a modelling assumption necessary for answering this question, other than the boats being particles. *(1 mark)*

4 A particle of mass 0.5 kg is initially at rest. The particle then moves in a straight line under the action of a single force. This force acts in a constant direction and has magnitude $(t^3 + t)\,\mathrm{N}$, where t is the time, in seconds, for which the force has been acting.

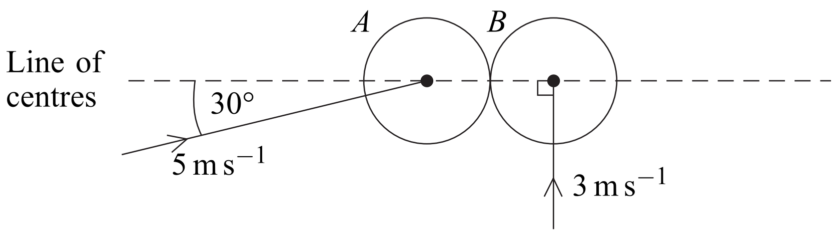
(a) Find the magnitude of the impulse exerted by the force on the particle between the times $t = 0$ and $t = 4$. *(3 marks)*

(b) Hence find the speed of the particle when $t = 4$. *(2 marks)*

(c) Find the time taken for the particle to reach a speed of $12\,\mathrm{m\,s^{-1}}$. *(5 marks)*

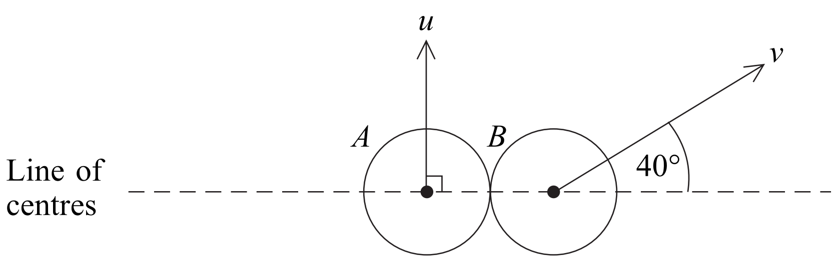
5 Two smooth spheres, A and B, of equal radii and different masses are moving on a smooth horizontal surface when they collide.

Just before the collision, A is moving with speed $5\,\text{m}\,\text{s}^{-1}$ at an angle of 30° to the line of centres of the spheres, and B is moving with speed $3\,\text{m}\,\text{s}^{-1}$ perpendicular to the line of centres, as shown in the diagram below.



Before collision

Immediately after the collision, A and B move with speeds u and v in directions which make angles of 90° and 40° respectively with the line of centres, as shown in the diagram below.



After collision

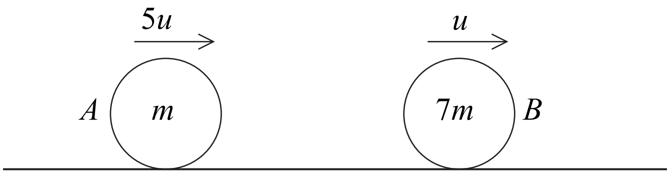
(a) Show that $v = 4.67\,\text{m}\,\text{s}^{-1}$, correct to three significant figures. *(3 marks)*

(b) Find the coefficient of restitution between the spheres. *(3 marks)*

(c) Given that the mass of A is 0.5 kg, show that the magnitude of the impulse exerted on A during the collision is 2.17 N s, correct to three significant figures. *(3 marks)*

(d) Find the mass of B. *(3 marks)*

6 A smooth sphere A of mass m is moving with speed $5u$ in a straight line on a smooth horizontal table. The sphere A collides directly with a smooth sphere B of mass $7m$, having the same radius as A and moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.



Before collision

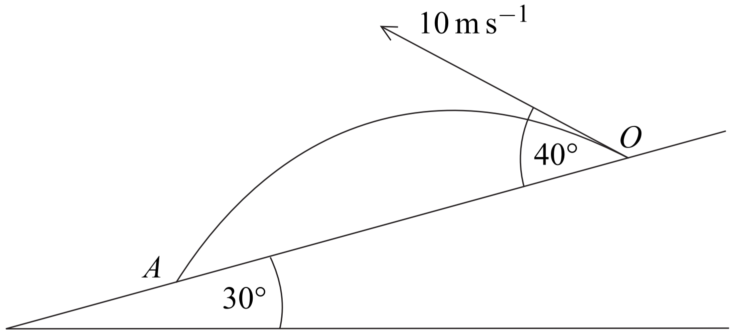
(a) Show that the speed of B after the collision is $\frac{u}{2}(e+3)$. *(5 marks)*

(b) Given that the direction of motion of A is reversed by the collision, show that $e > \frac{3}{7}$. *(4 marks)*

(c) Subsequently, B hits a wall fixed at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$. Given that after B rebounds from the wall both spheres move in the same direction and collide again, show also that $e < \frac{9}{13}$. *(4 marks)*

Turn over for the next question

7 A particle is projected from a point O on a smooth plane which is inclined at 30° to the horizontal. The particle is projected down the plane with velocity $10\,\mathrm{m\,s^{-1}}$ at an angle of 40° above the plane and first strikes it at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Show that the time taken by the particle to travel from O to A is

$$\frac{20\sin 40°}{g\cos 30°}$$ *(3 marks)*

(b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A. *(4 marks)*

(c) The coefficient of restitution between the slope and the particle is 0.5. Find the speed of the particle as it rebounds from the slope. *(4 marks)*

END OF QUESTIONS

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u\,\mathrm{m\,s^{-1}}$, at which the liquid escapes is given by

$$u = CV\rho g$$

where $V\,\mathrm{m^3}$ is the volume of the liquid in the tank, $\rho\,\mathrm{kg\,m^{-3}}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C. *(5 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

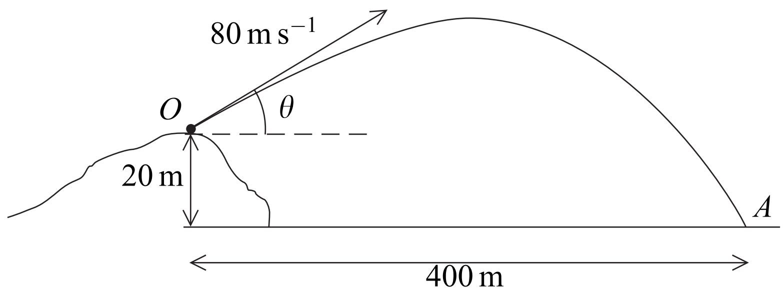
Do not write outside the box

2 A projectile is fired from a point O on top of a hill with initial velocity $80\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{gx^2}{12\,800}\left(1 + \tan^2\theta\right)$$ *(5 marks)*

(ii) The projectile hits a target A, which is 20 m vertically below O and 400 m horizontally from O.



Show that

$$49\tan^2\theta - 160\tan\theta + 41 = 0$$ *(2 marks)*

(b) (i) Find the two possible values of θ. Give your answers to the nearest 0.1°. *(3 marks)*

(ii) Hence find the shortest possible time of the flight of the projectile from O to A. *(2 marks)*

(c) State a necessary modelling assumption for answering part **(a)(i)**. *(1 mark)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

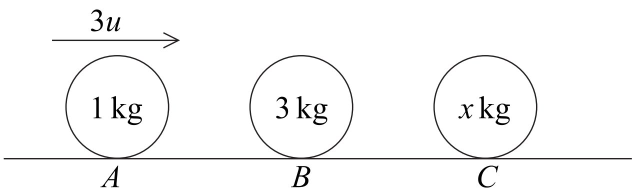
Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

3 Three smooth spheres, A, B and C, of equal radii have masses 1 kg, 3 kg and x kg respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed $3u$ directly towards B and collides with it.



The coefficient of restitution between each pair of spheres is $\frac{1}{3}$.

(a) Show that A is brought to rest by the impact and find the speed of B immediately after the collision in terms of u. *(6 marks)*

(b) Subsequently, B collides with C.

Show that the speed of C immediately after the collision is $\dfrac{4u}{3+x}$.

Find the speed of B immediately after the collision in terms of u and x. *(6 marks)*

(c) Show that B will collide with A again if $x > 9$. *(2 marks)*

(d) Given that $x = 5$, find the magnitude of the impulse exerted on C by B in terms of u. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

4 The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are directed east, north and vertically upwards respectively.

At time $t = 0$, the position vectors of two small aeroplanes, A and B, relative to a fixed origin O are $(-60\mathbf{i} + 30\mathbf{k})$ km and $(-40\mathbf{i} + 10\mathbf{j} - 10\mathbf{k})$ km respectively.

The aeroplane A is flying with constant velocity $(250\mathbf{i} + 50\mathbf{j} - 100\mathbf{k})\,\text{km h}^{-1}$ and the aeroplane B is flying with constant velocity $(200\mathbf{i} + 25\mathbf{j} + 50\mathbf{k})\,\text{km h}^{-1}$.

(a) Write down the position vectors of A and B at time t hours. *(3 marks)*

(b) Show that the position vector of A relative to B at time t hours is $((-20 + 50t)\mathbf{i} + (-10 + 25t)\mathbf{j} + (40 - 150t)\mathbf{k})$ km. *(2 marks)*

(c) Show that A and B do not collide. *(4 marks)*

(d) Find the value of t when A and B are closest together. *(6 marks)*

QUESTION PART REFERENCE

Do not write outside the box

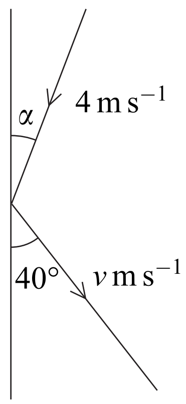
QUESTION PART REFERENCE

5 A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed $4\,\mathrm{m\,s^{-1}}$ and the angle between the sphere's direction of motion and the wall is α.

Immediately after the impact, the sphere is moving with speed $v\,\mathrm{m\,s^{-1}}$ and the angle between the sphere's direction of motion and the wall is 40°.

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



(a) Show that $\tan\alpha = \frac{3}{2}\tan 40°$. *(3 marks)*

(b) Find the value of v. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

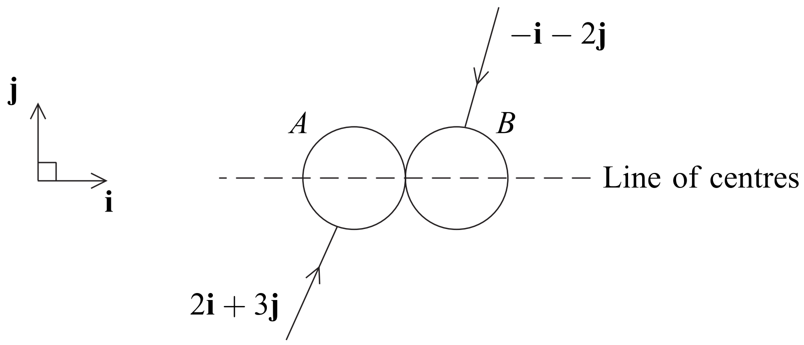
QUESTION PART REFERENCE

Turn over ▶

6 Two smooth spheres, A and B, have equal radii and masses 1 kg and 2 kg respectively.

The sphere A is moving with velocity $(2\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the sphere B is moving with velocity $(-\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ on the same smooth horizontal surface.

The spheres collide when their line of centres is parallel to the unit vector $\mathbf{i}$, as shown in the diagram.



(a) Briefly state why the components of the velocities of A and B parallel to the unit vector $\mathbf{j}$ are not changed by the collision. *(1 mark)*

(b) The coefficient of restitution between the spheres is 0.5 .

Find the velocities of A and B immediately after the collision. *(6 marks)*

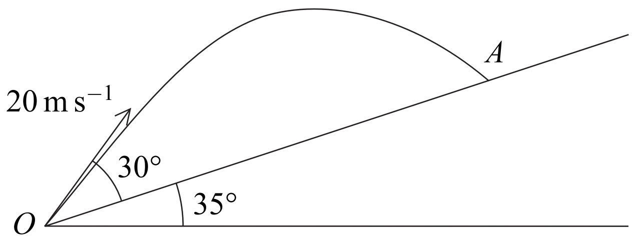
QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

7 A ball is projected from a point O on a smooth plane which is inclined at an angle of $35°$ above the horizontal. The ball is projected with velocity $20\,\mathrm{m\,s^{-1}}$ at an angle of $30°$ above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A.



(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A. *(7 marks)*

(b) On striking the plane at A, the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A. *(6 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

END OF QUESTIONS