

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Further Pure 4

MFP4

Wednesday 21 May 2008 1.30 pm to 3.00 pm

For this paper you must have:

- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{bmatrix} 7 & 12 \\ 12 & 0 \end{bmatrix}$. (6 marks)

- 2 The vectors **a**, **b** and **c** are given by

$$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \mathbf{c} = -2\mathbf{i} + t\mathbf{j} + 6\mathbf{k}$$

where t is a scalar constant.

- (a) Determine, in terms of t where appropriate:

(i) $\mathbf{a} \times \mathbf{b}$; (2 marks)

(ii) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$; (2 marks)

(iii) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$. (2 marks)

- (b) Find the value of t for which **a**, **b** and **c** are linearly dependent. (2 marks)

- (c) Find the value of t for which **c** is parallel to $\mathbf{a} \times \mathbf{b}$. (2 marks)

- 3 The matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 4 & 3 & k \end{bmatrix}$, where k is a constant.

Determine, in terms of k where appropriate:

(a) $\det \mathbf{A}$; (2 marks)

(b) $\mathbf{A}^{-1}$. (5 marks)

4 Two planes have equations

$$\mathbf{r} \cdot \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} = 12 \quad \text{and} \quad \mathbf{r} \cdot \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = 7$$

- (a) Find, to the nearest 0.1° , the acute angle between the two planes. (4 marks)
- (b) (i) The point $P(0, a, b)$ lies in both planes. Find the value of a and the value of b . (3 marks)
- (ii) By using a vector product, or otherwise, find a vector which is parallel to both planes. (2 marks)
- (iii) Find a vector equation for the line of intersection of the two planes. (2 marks)

5 A plane transformation is represented by the 2×2 matrix $\mathbf{M}$. The eigenvalues of $\mathbf{M}$ are 1 and 2, with corresponding eigenvectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ respectively.

- (a) State the equations of the invariant lines of the transformation and explain which of these is also a line of invariant points. (3 marks)
- (b) The diagonalised form of $\mathbf{M}$ is $\mathbf{M} = \mathbf{U} \mathbf{D} \mathbf{U}^{-1}$, where $\mathbf{D}$ is a diagonal matrix.
- (i) Write down a suitable matrix $\mathbf{D}$ and the corresponding matrix $\mathbf{U}$. (2 marks)
- (ii) Hence determine $\mathbf{M}$. (4 marks)
- (iii) Show that $\mathbf{M}^n = \begin{bmatrix} 1 & f(n) - 1 \\ 0 & f(n) \end{bmatrix}$ for all positive integers n , where $f(n)$ is a function of n to be determined. (3 marks)

Turn over for the next question

Turn over ►

6 Three planes have equations

$$\begin{aligned}x + y - 3z &= b \\2x + y + 4z &= 3 \\5x + 2y + az &= 4\end{aligned}$$

where a and b are constants.

- (a) Find the coordinates of the single point of intersection of these three planes in the case when $a = 16$ and $b = 6$. (5 marks)
- (b) (i) Find the value of a for which the three planes do not meet at a single point. (3 marks)
- (ii) For this value of a , determine the value of b for which the three planes share a common line of intersection. (5 marks)

7 A transformation T of three-dimensional space is given by the matrix $\mathbf{W} = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix}$.

- (a) (i) Evaluate $\det \mathbf{W}$, and describe the geometrical significance of the answer in relation to T . (2 marks)
- (ii) Determine the eigenvalues of $\mathbf{W}$. (6 marks)

(b) The plane H has equation $\mathbf{r} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 0$.

- (i) Write down a cartesian equation for H . (1 mark)
- (ii) The point P has coordinates (a, b, c) . Show that, whatever the values of a, b and c , the image of P under T lies in H . (4 marks)

8 By considering the determinant

$$\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$$

show that $(x + y + z)$ is a factor of $x^3 + y^3 + z^3 - kxyz$ for some value of the constant k to be determined. (3 marks)

END OF QUESTIONS