

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Mechanics 5

MM05

Wednesday 18 June 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A simple pendulum of length 2 metres is set in motion.

- (a) (i) Show that the period of the motion is 2.84 seconds, correct to three significant figures. (2 marks)
- (ii) Show that the frequency of the motion is 0.352 cycles per second, correct to three significant figures. (1 mark)
- (b) The length of the pendulum is adjusted so that the period of its motion is 2.5 seconds. Find the adjusted length of the pendulum. (2 marks)

2 A particle moves in a straight line with simple harmonic motion such that its displacement at time t seconds relative to a fixed origin on this line is x metres. The motion of the particle satisfies the differential equation

$$\frac{d^2x}{dt^2} + 16x = 0$$

- (a) Verify that

$$x = A \cos 4t + B \sin 4t$$

where A and B are constants, is a solution to this differential equation. (4 marks)

- (b) When $t = 0$, the particle is momentarily at rest. Show that $B = 0$. (1 mark)
- (c) Given that $x = h$ ($h > 0$) when $t = \frac{\pi}{12}$, find A in terms of h . (2 marks)
- (d) Find the maximum speed of the particle in terms of h . (1 mark)
- (e) The mass of the particle is m kg. Find the magnitude of the maximum force acting on the particle during the motion. Give your answer in terms of h and m . (2 marks)

3 A particle P moves in a plane so that, at time t , its polar coordinates (r, θ) with respect to a fixed origin, O , are given by

$$r = t^2 \quad \theta = \frac{9}{\pi^2} \sin \frac{\pi t}{6}$$

- (a) Find the radial and transverse components of the velocity of P when $t = 3$. (4 marks)
- (b) Find the radial and transverse components of the acceleration of P when $t = 3$. (5 marks)
- (c) Determine the angle between the acceleration of P and OP when $t = 3$. (2 marks)

- 4 A rocket is launched from the ground so that it travels vertically upwards. The rocket ejects burnt fuel vertically downwards at a speed of 1400 m s^{-1} relative to the rocket at a constant rate of 10 kg s^{-1} .

The initial mass of the rocket and its fuel is 1000 kg .

The velocity of the rocket at time t seconds after it is launched is $v \text{ m s}^{-1}$.

It may be assumed that the only external force acting on the rocket is gravity. The acceleration due to gravity should be taken as constant.

- (a) Show that

$$\frac{dv}{dt} = -9.8 + \frac{1400}{100 - t} \quad (8 \text{ marks})$$

- (b) Given that $v = 0$ when $t = 0$, show that

$$v = -9.8t + 1400 \ln\left(\frac{100}{100 - t}\right) \quad (3 \text{ marks})$$

- (c) When $t = 80$, the fuel in the rocket has all been burnt. Find the total time taken for the rocket to reach its maximum height. (4 marks)

- 5 A particle, of mass 2 kg , is suspended from a fixed point O by a light spring of natural length 0.5 metres and modulus of elasticity 49 N .

- (a) Initially, the particle hangs at rest in equilibrium below O . Find the extension of the spring in this position. (2 marks)

- (b) A force, F newtons, is then applied to the particle in a vertically downwards direction. The displacement of the particle below its equilibrium position at time t seconds later is x metres. Given that $F = 12 \cos nt$, where n is a positive constant, show that

$$\frac{d^2x}{dt^2} + 49x = 6 \cos nt \quad (5 \text{ marks})$$

- (c) In the case where $n = 5$, find an expression for x at time t . (10 marks)

- (d) State the value of n for which resonance occurs. (1 mark)

Turn over for the next question

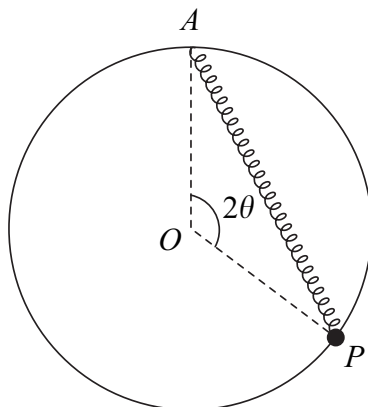
Turn over ►

- 6 A smooth circular wire, of radius a and centre O , is fixed in a vertical plane.

A small smooth bead, P , of mass m , can move freely on the wire.

The bead is attached to one end of a light spring, which has modulus of elasticity $4mg$ and natural length a . The other end of the spring is attached to A , the highest point on the wire.

The angle subtended by the spring at O is 2θ , as shown in the diagram, where $0 < \theta \leq \frac{\pi}{2}$.



- (a) (i) Show that the elastic potential energy stored in the spring in this position is given by

$$2mga(2 \sin \theta - 1)^2 \quad (3 \text{ marks})$$

- (ii) The gravitational potential energy is taken to be zero at the level of the lowest point on the wire. Show that the total potential energy, V , is given by

$$V = 2mga(3 \sin^2 \theta - 4 \sin \theta + 2) \quad (5 \text{ marks})$$

- (b) Find the two values of θ for which the bead is in equilibrium, giving your answers to two decimal places. (4 marks)
- (c) Determine, for each of these values, whether the bead is in stable or unstable equilibrium. (4 marks)

END OF QUESTIONS