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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2006 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)	For a 1-year period The number of A grades ~ $\mathrm{Po}(3)$ For a 5-year period Number of A grades ~ $\mathrm{Po}(15)$	B1		
	$\mathrm{P}(\text{Total A-grades} > 18)$ $= 1 - (\text{Total} \le 18)$ $= 1 - 0.8195$ $= 0.1805$	M1		
	$= 0.181$	A1	3	AWFW 0.180 to 0.181
(b)(i)	$X + Y \sim \mathrm{Po}(10)$	B1		
	$\mathrm{P}(X + Y \le 14) = 0.917$	M1A1	3	AWFW 0.916 to 0.917 incl
(ii)	X and Y are independent variables.	E1	1	
	Total		**7**	
2(a)	$\bar{x} = \frac{254}{5} = 50.8$	B1		
	$s = 4.55$	B1		
	$\nu = 5 - 1 = 4$	B1		
	$t_{\text{crit}} = 2.776$	B1		
	95% confidence interval $= 50.8 \pm 2.776 \times \frac{4.55}{\sqrt{5}}$	M1√		ft their values
	$= 50.8 \pm 5.648$ $= (45.2, 56.4)$	A1	6	
(b)	0.05	B1	1	
	Total		**7**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathrm{E}(R) = \sum_{\text{all } r} r\ \mathrm{P}(R = r)$			
	$= \left(1 \times \frac{7}{16}\right) + \left(2 \times \frac{5}{16}\right) + \left(3 \times \frac{3}{16}\right) + \left(4 \times \frac{1}{16}\right)$			
	$= \frac{30}{16}$			
	$= 1\frac{7}{8}$	B1		(1.875)
	$\mathrm{E}\left(R^2\right) = \sum_{\text{all } r} r^2\ \mathrm{P}(R = r)$			
	$= \frac{70}{16}$ or $4\frac{3}{8}$	B1		(4.375)
	$\mathrm{Var}(R) = 4\frac{3}{8} - \left(1\frac{7}{8}\right)^2$	M1		
	$= \frac{220}{256}$ or $\frac{55}{64}$	A1	4	(0.859375)
(b)(i)	$32 \times \frac{1}{4} = 8$	B1	1	
(ii)	$= \left(32 \times \frac{7}{16} \times \frac{1}{5}\right) + \left(32 \times \frac{5}{16} \times \frac{1}{2}\right) + 8 \times \frac{9}{10}$	M1		
	$= 2.8 + 5 + 7.2$			A0 if these numbers rounded before adding
	$= 15$	A1	2	
	Total		**7**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)				
		B1 B1	2	for A values for B values
(ii)	H_0 : no association between area and age profile H_1 : association between area and age profile	B1		At least H_0
		M1		Attempt at Row & Column totals
		M1		Attempt at E_i
		M1		Attempt at $\frac{(O_i - E_i)^2}{E_i}$
		M1		Attempt at χ^2
		A1		AWFW 12.5 to 12.6 provided correct method used
	$\nu = (3-1)(2-1) = 2$	B1		
	$\chi^2_{1\%}(2) = 9.210 < 12.554$	B1√		ft on their ν and χ^2
	Reject H_0 The evidence suggests that the area within which a school is situated seems to have an effect on the age-profile of the staff employed.	E1√	9	ft on χ^2 and calculated value depends on H_0 correct, if stated
(b)	There seems to be fewer staff employed in 22 - 34 age group than expected in school A and more than expected in school B	E1 E1	2	
	Total		**13**	

	A	B	Total
22-34	21	32	53
35-39	72	36	108
40-59	27	12	39
Total	120	80	200

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
24	31.8	3.6679
72	64.8	0.8000
24	23.4	0.5538
32	21.2	5.5019
36	43.2	1.2000
12	15.6	0.8308
$\sum O_i = 200$	$\sum E_i = 200$	$\chi^2 = 12.554$

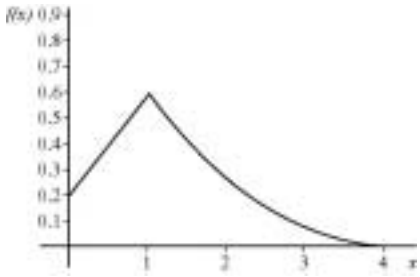
MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathrm{E}(X)=\frac{1}{2}b$	B1	1	
(ii)	$\mathrm{E}(X^2)=\int_0^b \frac{1}{b}x^2\,\mathrm{d}x$	M1		
	$=\frac{1}{b}\left[\frac{x^3}{3}\right]_0^b$	A1		For correct integration
	$=\frac{1}{b}\left(\frac{b^3}{3}\right)$			
	$=\frac{1}{3}b^2$	A1		OE
	$\mathrm{Var}(X)=\frac{1}{3}b^2-\left(\frac{b}{2}\right)^2$	m1		Depending on using integration to get $\mathrm{E}(X^2)$
	$=\frac{1}{3}b^2-\frac{1}{4}b^2$			
	$=\frac{1}{12}b^2$	A1	5	AG
(b)	$\mathrm{P}(\lvert T\rvert>0.02)=1-\mathrm{P}(-0.02<T<0.02)$	M1		
	$=1-0.04\times 5$	M1		
	$=0.8$	A1	3	
	Total		**9**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\bar{x} = \frac{471}{5} = 94.2$	B1		
	$s = 6.058$	B1		Or $s^2 = 36.7$
	$\nu = 4$ 1-tailed test	B1		
	$t_{\text{crit}} = -2.132$	B1		Or on diagram
	$H_0 : \mu = 100$ $H_1 : \mu < 100$	B1		
	$t = \frac{94.2 - 100}{6.058/\sqrt{5}} = -2.14$	M1A1		$\frac{\text{their } \bar{x} - 100}{(\text{their } s)/\sqrt{5}}$
	Reject H_0 at 5% level of significance	A1√		On their t and critical value
	Evidence at the 5% level of significance to support the members' belief that the batteries last less than 100 hours.	E1√	9	
(b)	$\bar{x} = \frac{8080}{80} = 101$			
	$s^2 = \frac{6399}{79} = 81$ (or $\frac{6399}{80} = 79.9875$) $s = 9$ (or $s = 8.944$)	B1		For s (or s^2) and $\bar{x}$
	$H_0 : \mu = 100$ $H_1 : \mu \neq 100$	B1		
	$\bar{X} \sim N\left(100, \frac{81 \text{ (or } 79.9875)}{80}\right)$ under H_0	B1		Or 100, $\frac{9}{\sqrt{80}}$ used
	$z = \frac{101 - 100}{9/\sqrt{80}} = 0.99$	M1 A1		Allow use of t method AWFW 0.99 to 1.00 (allow 1)
	2-tailed test $z_{\text{crit}} = \pm 1.96$	B1		Or z = 1.96
	Accept H_0 at 5% level of significance.	A1√		On their z and critical value Or t
	Sufficient evidence at the 5% level of significance to support the manufacturer's belief.	E1√	8	
	Total		**17**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)		B2	2	B1 for line segment (0,0.2) to (1,0.6) B1 for correctly shaped curve (1,0.6) to (4,0)
(b)(i)	for $0 \le x \le 1$			
	$\mathrm{F}(x) = \int_0^x \frac{1}{5}(2x+1)\,\mathrm{d}x$	M1		Ignore limits
	$= \left[\frac{1}{5}\left(x^2+x\right)\right]_0^x$	A1		Ignore limits
	$= \frac{1}{5}x(x+1)$	A1	3	
(ii)	$\mathrm{P}(X \le 1) = \mathrm{F}(1)$ $= \frac{2}{5}$	B1	1	
(iii)	$\mathrm{P}(X \ge x) = \frac{17}{20} \quad \Rightarrow \quad \mathrm{F}(x) = \frac{3}{20}$	M1		
	$\frac{1}{5}x(x+1) = \frac{3}{20}$	m1		
	$x(x+1) = \frac{3}{4}$			
	$x^2 + x - \frac{3}{4} = 0$	A1		
	$\left(x - \frac{1}{2}\right)\left(x + \frac{3}{2}\right) = 0$	m1		Any valid method attempted
	$x = \frac{1}{2}$	A1	5	CAO
(iv)	Since $\mathrm{F}(1) = 0.4$, q lies in $0 \le r \le 1$			
	$\mathrm{F}(q) = \frac{1}{5}\left(q^2+q\right) = 0.25$	M1		
	$\Rightarrow \quad q^2 + q = 1.25$			
	$q^2 + q - 1.25 = 0$	A1		
	$\Rightarrow \quad q = \frac{-1 \pm \sqrt{1 - 4 \times (-1.25)}}{2}$	m1		
	$q = \frac{1}{2}\left(\sqrt{6} - 1\right) \quad (q > 0)$	A1	4	AWFW (0.724 to 0.725)
	Total		**15**	
	TOTAL		**75**	