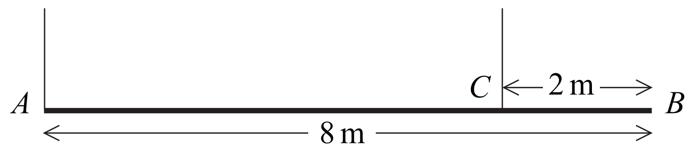
Answer **all** questions.

1 A uniform plank, AB, is 8 m long and has mass 30 kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes. One of the ropes is attached to the plank at A and the other rope to the point C, where $BC = 2$ m, as shown in the diagram.



Find the tension in each rope. *(5 marks)*

2 A car of mass 1500 kg is travelling along a straight horizontal road. When the car is travelling at a speed of $v \text{ m s}^{-1}$, it experiences a resistance force of magnitude $35v$ newtons.

(a) On this road, the car has a maximum speed of 50 m s^{-1}.

Show that the maximum power of the car is 87 500 watts. *(4 marks)*

(b) Find the maximum possible acceleration of the car when its speed on the road is 30 m s^{-1}. *(5 marks)*

3 A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j}) \text{ m s}^{-1}$. The velocity of the particle at time t is $\mathbf{v} \text{ m s}^{-1}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

4 An elastic string of natural length 1.5 metres has one end attached to a fixed point *O*. A particle of mass 4 kg is attached to the other end of the string. The particle is released from rest at *O*.

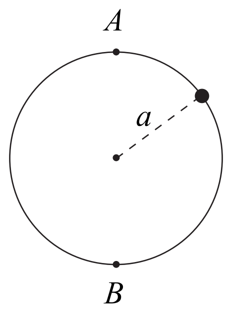
(a) Find the kinetic energy of the particle when the string becomes taut. *(2 marks)*

(b) The particle first comes to rest when it is 3.5 metres below *O*.

Show that the modulus of elasticity of the string is 103 N, correct to three significant figures. *(4 marks)*

(c) Find the speed of the particle when it is 2.7 metres below *O*. *(5 marks)*

5 A bead of mass *m* moves on a smooth circular ring of radius *a* which is fixed in a vertical plane, as shown in the diagram. Its speed at *A*, the highest point of its path, is *v* and its speed at *B*, the lowest point of its path, is 7*v*.



(a) Show that $v = \sqrt{\frac{ag}{12}}$. *(5 marks)*

(b) Find the reaction of the ring on the bead, in terms of *m* and *g*, when the bead is at *A*. *(4 marks)*

Turn over for the next question

6 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude $\lambda m v$, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v$$ *(2 marks)*

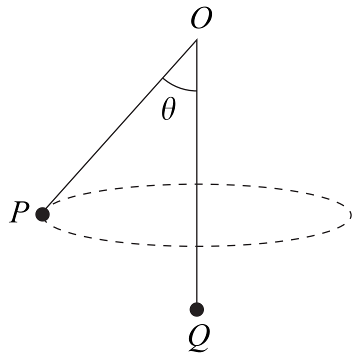
(b) The initial speed of the stone is U.

Show that

$$v = U\mathrm{e}^{-\lambda t}$$ *(4 marks)*

7 A particle, P, of mass 3 kg is attached to one end of a light inextensible string. The string passes through a smooth fixed ring, O, and a second particle, Q, of mass 5 kg is attached to the other end of the string. The particle Q hangs at rest vertically below the ring and the particle P moves with speed $4\,\mathrm{m\,s^{-1}}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



(a) Explain why the tension in the string is 49 N. *(2 marks)*

(b) Find θ. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 A hot air balloon moves vertically upwards with a constant velocity. When the balloon is at a height of 30 metres above ground level, a box of mass 5 kg is released from the balloon.

After the box is released, it initially moves vertically upwards with speed $10\,\text{m s}^{-1}$.

(a) Find the initial kinetic energy of the box. *(2 marks)*

(b) Show that the kinetic energy of the box when it hits the ground is 1720 J. *(3 marks)*

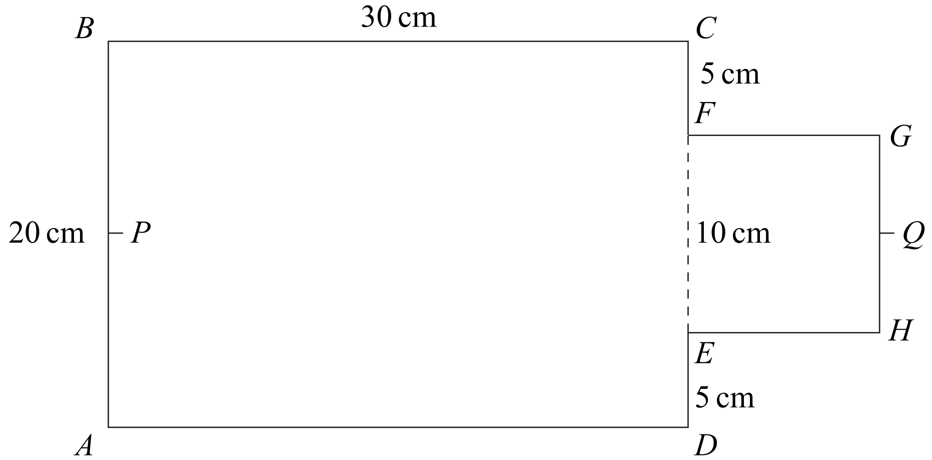
(c) Hence find the speed of the box when it hits the ground. *(3 marks)*

(d) State **two** modelling assumptions which you have made. *(2 marks)*

2 A uniform lamina is in the shape of a rectangle *ABCD* and a square *EFGH*, as shown in the diagram.

The length *AB* is 20 cm, the length *BC* is 30 cm, the length *DE* is 5 cm and the length *EF* is 10 cm.

The point *P* is the midpoint of *AB* and the point *Q* is the midpoint of *HG*.



(a) Explain why the centre of mass of the lamina lies on *PQ*. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from *AB*. *(4 marks)*

(c) The lamina is freely suspended from *A*.

Find, to the nearest degree, the angle between *AD* and the vertical when the lamina is in equilibrium. *(4 marks)*

3 A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j})\,\mathrm{m\,s^{-1}}$. The velocity of the particle at time t is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

4 A uniform plank is 10 m long and has mass 15 kg. It is placed on horizontal ground at the edge of a vertical river bank, so that 2 m of the plank is projecting over the edge, as shown in the diagram below.

(a) A woman of mass 50 kg stands on the part of the plank which projects over the river.

Find the greatest distance from the river bank at which she can safely stand. *(3 marks)*

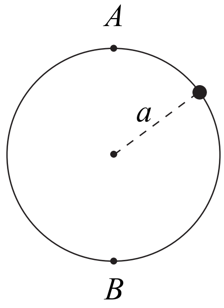
(b) The woman wishes to stand safely at the end of the plank which projects over the river.

Find the minimum mass which she should place on the other end of the plank so that she can do this. *(4 marks)*

(c) State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

(d) State one other modelling assumption which you have made. *(1 mark)*

5 A bead of mass m moves on a smooth circular ring of radius a which is fixed in a vertical plane, as shown in the diagram. Its speed at A, the highest point of its path, is v and its speed at B, the lowest point of its path, is $7v$.



(a) Show that $v = \sqrt{\dfrac{ag}{12}}$. *(5 marks)*

(b) Find the reaction of the ring on the bead, in terms of m and g, when the bead is at A. *(4 marks)*

6 An elastic string has one end attached to a point O, fixed on a horizontal table. The other end of the string is attached to a particle of mass 5 kilograms. The elastic string has natural length 2 metres and modulus of elasticity 200 newtons. The particle is pulled so that it is 2.5 metres from the point O and it is then released from rest on the table.

(a) Calculate the elastic potential energy when the particle is 2.5 m from the point O. *(2 marks)*

(b) If the table is smooth, show that the speed of the particle when the string becomes slack is $\sqrt{5}$ m s^{-1}. *(3 marks)*

(c) The table is, in fact, rough and the coefficient of friction between the particle and the table is 0.4 .

Find the speed of the particle when the string becomes slack. *(7 marks)*

7 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v$$ *(2 marks)*

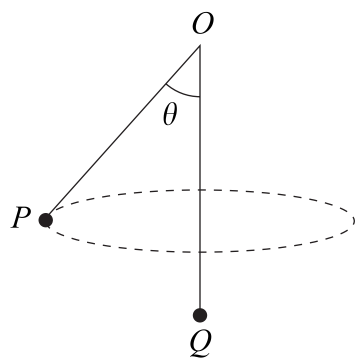
(b) The initial speed of the stone is U.

Show that

$$v = U\mathrm{e}^{-\lambda t}$$ *(4 marks)*

8 A particle, P, of mass 3 kg is attached to one end of a light inextensible string. The string passes through a smooth fixed ring, O, and a second particle, Q, of mass 5 kg is attached to the other end of the string. The particle Q hangs at rest vertically below the ring and the particle P moves with speed $4\,\mathrm{m\,s^{-1}}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



(a) Explain why the tension in the string is 49 N. *(2 marks)*

(b) Find θ. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

END OF QUESTIONS

Practice 3

Leave blank

1. A cyclist and his bicycle have a combined mass of 90 kg. He rides on a straight road up a hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{21}$. He works at a constant rate of 444 W and cycles up the hill at a constant speed of 6 m s^{-1}.

Find the magnitude of the resistance to motion from non-gravitational forces as he cycles up the hill.

(4)

Leave blank

2. A particle P of mass 0.5 kg moves under the action of a single force $\mathbf{F}$ newtons. At time t seconds, the velocity $\mathbf{v}$ m s^{-1} of P is given by

$$\mathbf{v} = 3t^2\mathbf{i} + (1 - 4t)\mathbf{j}.$$

Find

(a) the acceleration of P at time t seconds,

(2)

(b) the magnitude of $\mathbf{F}$ when $t = 2$.

(4)

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3.

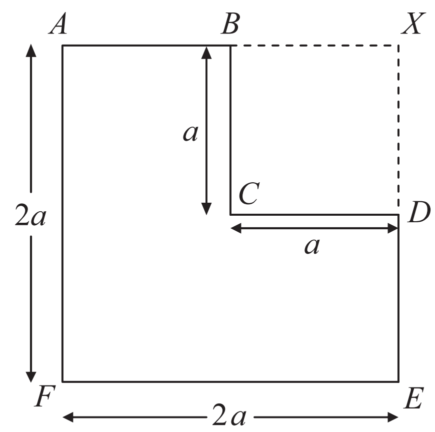

Figure 1

A uniform lamina *ABCDEF* is formed by taking a uniform sheet of card in the form of a square *AXEF*, of side $2a$, and removing the square *BXDC* of side a, where *B* and *D* are the mid-points of *AX* and *XE* respectively, as shown in Figure 1.

(a) Find the distance of the centre of mass of the lamina from *AF*.

(4)

The lamina is freely suspended from *A* and hangs in equilibrium.

(b) Find, in degrees to one decimal place, the angle which *AF* makes with the vertical.

(4)

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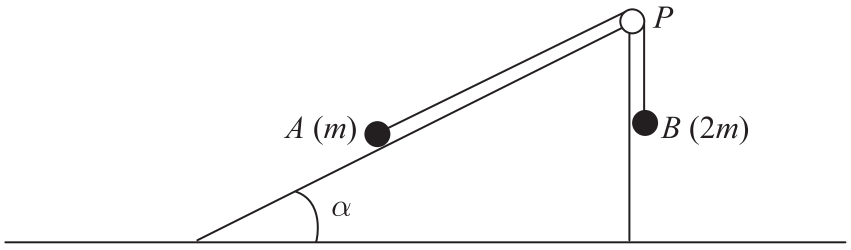

Figure 2

Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P, as shown in Figure 2. The particles are released from rest with the string taut and the section of the string from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{5}{8}$. When each particle has moved a distance h, B has not reached the ground and A has not reached P.

(a) Find an expression for the potential energy lost by the system when each particle has moved a distance h.

(2)

When each particle has moved a distance h, they are moving with speed v. Using the work-energy principle,

(b) find an expression for v^2, giving your answer in the form kgh, where k is a number.

(5)

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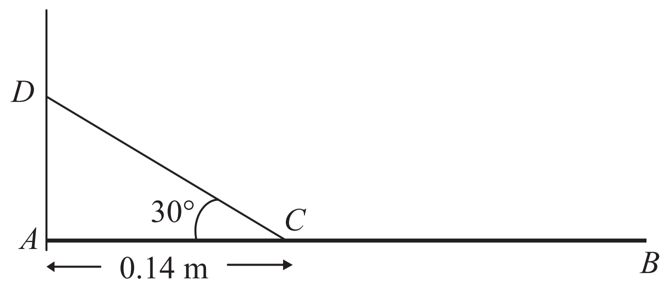

Figure 3

A uniform beam AB of mass 2 kg is freely hinged at one end A to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point C on the beam, where $AC = 0.14$ m. The rope is attached to the point D on the wall vertically above A, where $\angle ACD = 30°$, as shown in Figure 3. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

Find

(a) the length of AB,

(4)

(b) the magnitude of the resultant reaction of the hinge on the beam at A.

(5)

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6.

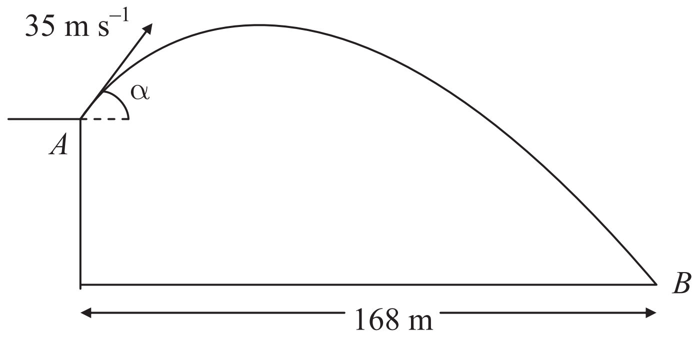

Figure 4

A golf ball P is projected with speed 35 m s^{-1} from a point A on a cliff above horizontal ground. The angle of projection is α to the horizontal, where $\tan \alpha = \frac{4}{3}$. The ball moves freely under gravity and hits the ground at the point B, as shown in Figure 4.

(a) Find the greatest height of P above the level of A.

(3)

The horizontal distance from A to B is 168 m.

(b) Find the height of A above the ground.

(6)

By considering energy, or otherwise,

(c) find the speed of P as it hits the ground at B.

(3)

Leave blank

7. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(a) (i) Show that the speed of P immediately after the collision is $\frac{u}{6}(5e-1)$.

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

(6)

Three small spheres A, B and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(b) Show that, after B and C have collided, there is a collision between B and A.

(3)

(c) Determine whether, after B and A have collided, there is a further collision between B and C.

(4)

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8. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where v is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \leqslant t \leqslant 4, \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, P is at the origin O.

Find

(a) the greatest speed of P in the interval $0 \leqslant t \leqslant 4$, **(4)**

(b) the distance of P from O when $t = 4$, **(3)**

(c) the time at which P is instantaneously at rest for $t > 4$, **(1)**

(d) the total distance travelled by P in the first 10 s of its motion. **(8)**

1 A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of $35°$ with the horizontal and the tension in the rope is 40 N. Calculate the work done in moving the sack 100 m. [3]

2 Calculate the range on a horizontal plane of a small stone projected from a point on the plane with speed $12\,\mathrm{m\,s^{-1}}$ at an angle of elevation of $27°$. [4]

3 A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from $100\,\mathrm{m\,s^{-1}}$ to $150\,\mathrm{m\,s^{-1}}$ in a time t seconds.

(i) Calculate the value of t. [4]

(ii) Calculate the acceleration of the rocket at the instant when its speed is $120\,\mathrm{m\,s^{-1}}$. [4]

4 A ball is projected from a point O on the edge of a vertical cliff. The horizontal and vertically upward components of the initial velocity are $7\,\mathrm{m\,s^{-1}}$ and $21\,\mathrm{m\,s^{-1}}$ respectively. At time t seconds after projection the ball is at the point (x, y) referred to horizontal and vertically upward axes through O. Air resistance may be neglected.

(i) Express x and y in terms of t, and hence show that $y = 3x - \frac{1}{10}x^2$. [5]

The ball hits the sea at a point which is 25 m below the level of O.

(ii) Find the horizontal distance between the cliff and the point where the ball hits the sea. [3]

5 A cyclist and her bicycle have a combined mass of 70 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of $4\,\mathrm{m\,s^{-1}}$ at B. The level of B is 6 m above the level of A. For the cyclist's motion from A to B, find

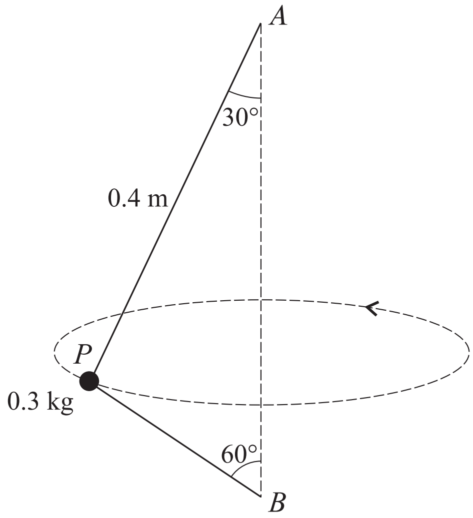
(i) the increase in kinetic energy, [2]

(ii) the increase in gravitational potential energy. [2]

During the ascent the resistance to motion is constant and has magnitude 60 N. The work done by the cyclist in moving from A to B is 8000 J.

(iii) Calculate the distance AB. [4]

6



A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings. The other end of the longer string is attached to a fixed point A and the other end of the shorter string is attached to a fixed point B, which is vertically below A. AP makes an angle of 30° with the vertical and is 0.4 m long. PB makes an angle of 60° with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut (see diagram). The tension in the string AP is 5 N.

Calculate

(i) the tension in the string PB, [3]

(ii) the angular speed of P, [3]

(iii) the kinetic energy of P. [3]

7 Two small spheres A and B, with masses 0.3 kg and m kg respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed $6\,\mathrm{m\,s^{-1}}$ and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are $1\,\mathrm{m\,s^{-1}}$ and $3\,\mathrm{m\,s^{-1}}$ respectively. The coefficient of restitution between A and B is e.

(i) Show that $m = 0.7$. [2]

(ii) Find e. [2]

B continues to move at $3\,\mathrm{m\,s^{-1}}$ and strikes a vertical wall at right angles. The coefficient of restitution between B and the wall is f.

(iii) Find the range of values of f for which there will be a second collision between A and B. [2]

(iv) Find, in terms of f, the magnitude of the impulse that the wall exerts on B. [3]

(v) Given that $f = \frac{3}{4}$, calculate the final speeds of A and B, correct to 1 decimal place. [7]

[Question 8 is printed overleaf.]

8

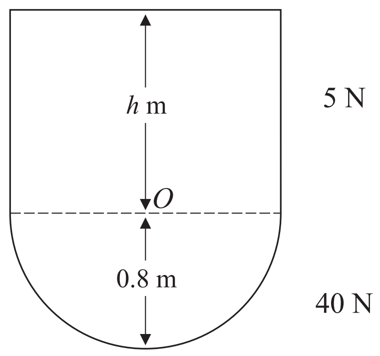


Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height h m. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point O (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from O.

(i) Show that $h = 1.2$. [6]

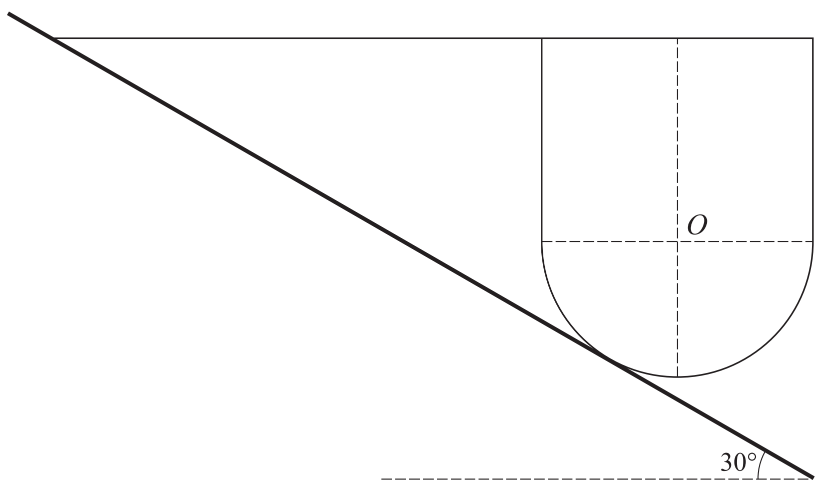


Fig. 2

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of 30° with the horizontal (see Fig. 2). The tension in the string is T N and the frictional force acting on the object is F N.

(ii) By taking moments about O, express F in terms of T. [4]

(iii) Find another equation connecting T and F. Hence calculate the tension and the frictional force. [6]

Answer **all** questions.

1 A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

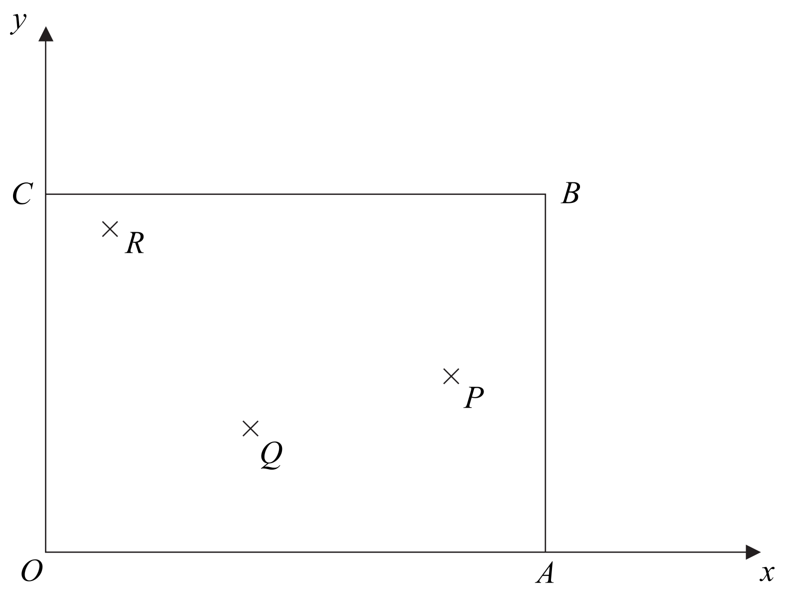
(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

2 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

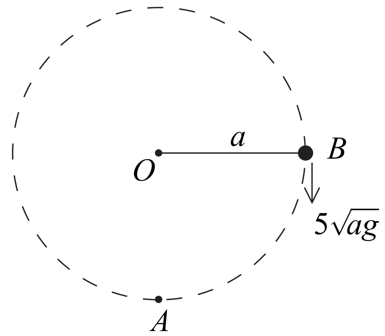
Particle P has mass 1 kg and is attached at the point (25, 10).
Particle Q has mass 4 kg and is attached at the point (12, 7).
Particle R has mass 5 kg and is attached at the point (4, 18).



Find the coordinates of the centre of mass of the three particles. *(4 marks)*

3 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end of the string. The particle is set into vertical circular motion with radius a and centre O.

When the particle is at B, on the same horizontal level as O, the string is taut and the particle is moving vertically downwards with speed $5\sqrt{ag}$.



(a) Find, in terms of a and g, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of m and g, the tension in the string when the particle is at A. *(3 marks)*

4 A particle moves on a horizontal plane in which the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

At time t seconds, the particle's position vector, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

5 A car, of mass *m*, is moving along a straight smooth horizontal road. At time *t*, the car has speed *v*. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

$$\frac{dv}{dt} = -0.05v$$ *(1 mark)*

(b) When $t = 0$, the speed of the car is $20\,\mathrm{m\,s^{-1}}$.

Show that $v = 20e^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(d) Find, in terms of *m*, the work done by the force in slowing the car from $20\,\mathrm{m\,s^{-1}}$ to $10\,\mathrm{m\,s^{-1}}$. *(3 marks)*

6 A van, of mass 1500 kg, has a maximum speed of $50\,\mathrm{m\,s^{-1}}$ on a straight horizontal road. When the van travels at a speed of $v\,\mathrm{m\,s^{-1}}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. *(2 marks)*

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is $25\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(c) The van starts to climb a hill which is inclined at 6° to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill. *(6 marks)*

7 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\dfrac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 2 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 245 J. *(2 marks)*

(iii) The particle is released from rest at the point P. Find the speed of the particle when it reaches O. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

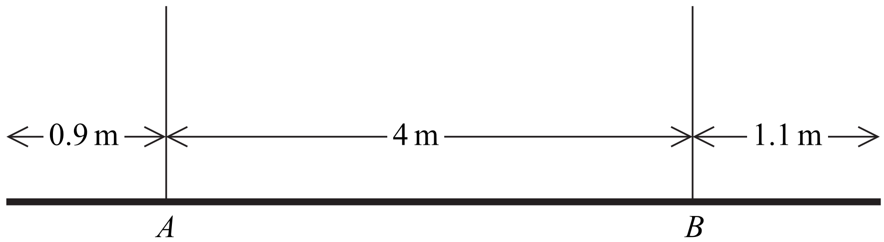
(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

(c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

2 A uniform plank, of length 6 metres, has mass 40 kg. The plank is held in equilibrium in a horizontal position by two vertical ropes attached to the plank at A and B, as shown in the diagram.



(a) Draw a diagram to show the forces acting on the plank. *(1 mark)*

(b) Show that the tension in the rope attached to the plank at B is $21g$ N. *(3 marks)*

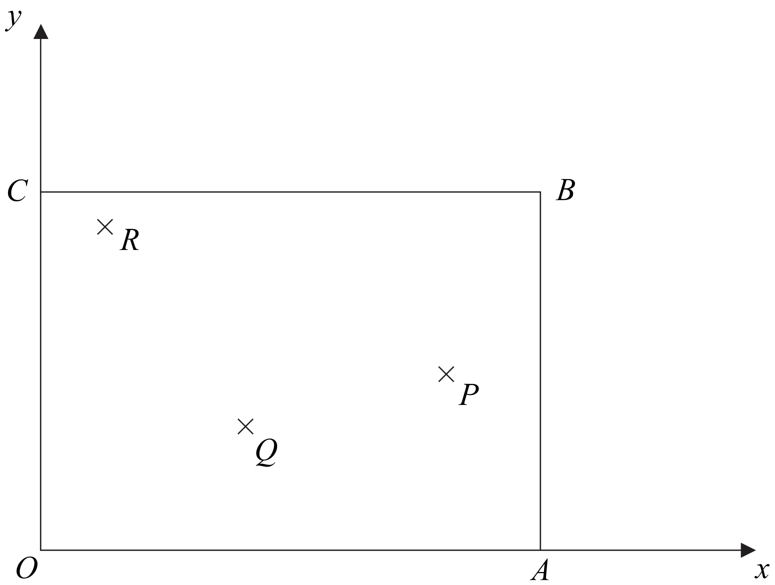
(c) Find the tension in the rope that is attached to the plank at A. *(2 marks)*

(d) State where in your solution you have used the fact that the plank is uniform. *(1 mark)*

3 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

Particle P has mass 1 kg and is attached at the point $(25, 10)$.
Particle Q has mass 4 kg and is attached at the point $(12, 7)$.
Particle R has mass 5 kg and is attached at the point $(4, 18)$.



Find the coordinates of the centre of mass of the three particles. *(4 marks)*

4 A van, of mass 1500 kg, has a maximum speed of $50\,\mathrm{m\,s^{-1}}$ on a straight horizontal road. When the van travels at a speed of $v\,\mathrm{m\,s^{-1}}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. *(2 marks)*

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is $25\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(c) The van starts to climb a hill which is inclined at $6°$ to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill. *(6 marks)*

5 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

6 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

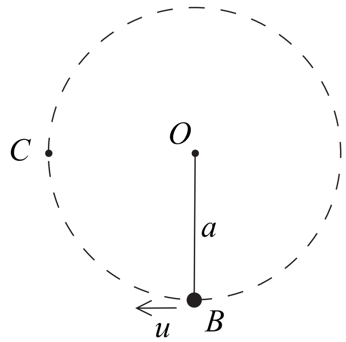
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.05v$$ *(1 mark)*

(b) When $t = 0$, the speed of the car is $20\,\mathrm{m\,s^{-1}}$.

Show that $v = 20\mathrm{e}^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\mathrm{m\,s^{-1}}$. *(3 marks)*

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string, of length a. The bead is then set into circular motion with the string taut at B, where B is vertically below O, with a horizontal speed u.



(a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*

(b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m, the tension in the string when the bead is at the point C, which is at the same horizontal level as O, as shown in the diagram. *(3 marks)*

(c) State one modelling assumption that you have made in your solution. *(1 mark)*

Turn over for the next question

8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\dfrac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 0.9 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. *(2 marks)*

(iii) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\text{m s}^{-1}$ when it is x metres **above *P***.

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2$$ *(7 marks)*

(iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(2 marks)*

END OF QUESTIONS

Practice 7

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1. A lorry of mass 2000 kg is moving down a straight road inclined at angle α to the horizontal, where $\sin \alpha = \frac{1}{25}$. The resistance to motion is modelled as a constant force of magnitude 1600 N. The lorry is moving at a constant speed of 14 m s^{-1}.

Find, in kW, the rate at which the lorry's engine is working.

(6)

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Question 1 continued

Q1

(Total 6 marks)

Turn over

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2. A particle A of mass $4m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass $3m$ moving with speed $2u$ in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is $4eu$.

(a) Show that $e = \frac{3}{4}$.

(5)

(b) Find the total kinetic energy lost in the collision.

(4)

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Question 2 continued

(Total 9 marks)

Q2

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3.

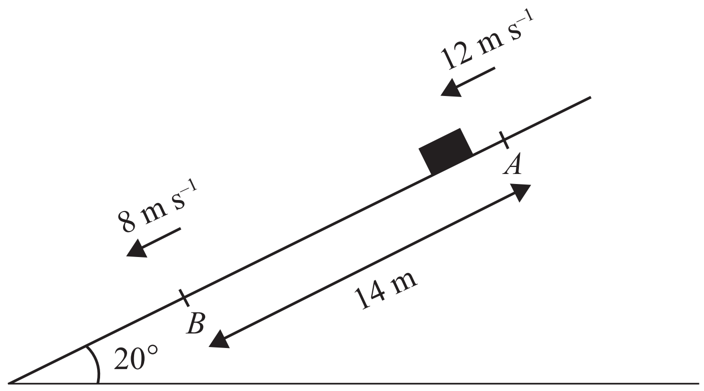

Figure 1

A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of 20° to the horizontal. The package slides down a line of greatest slope of the plane from a point A to a point B, where AB = 14 m. At A the package has speed 12 m s^{-1} and at B the package has speed 8 m s^{-1}, as shown in Figure 1. Find

(a) the total energy lost by the package in travelling from A to B,

(5)

(b) the coefficient of friction between the package and the ramp.

(5)

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Question 3 continued

(Total 10 marks)

Q3

Turn over

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4. A particle P of mass 0.5 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time t seconds,

$$\mathbf{F} = (6t - 5)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j}.$$

The velocity of P at time t seconds is $\mathbf{v}$ m s^{-1}. When $t = 0$, $\mathbf{v} = \mathbf{i} - 4\mathbf{j}$.

(a) Find $\mathbf{v}$ at time t seconds.

(6)

When $t = 3$, the particle P receives an impulse $(-5\mathbf{i} + 12\mathbf{j})$ N s.

(b) Find the speed of P immediately after it receives the impulse.

(6)

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5.

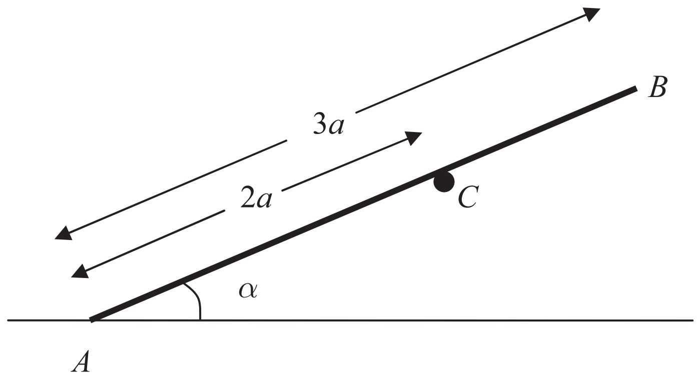

Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod *AB* and the pole as a smooth horizontal peg perpendicular to the vertical plane containing *AB*. The rod has length $3a$ and weight W and rests on the peg at C, where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3\cos^2 \alpha)W$.

(6)

Given that the rod is in limiting equilibrium and that $\cos \alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

(5)

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6.

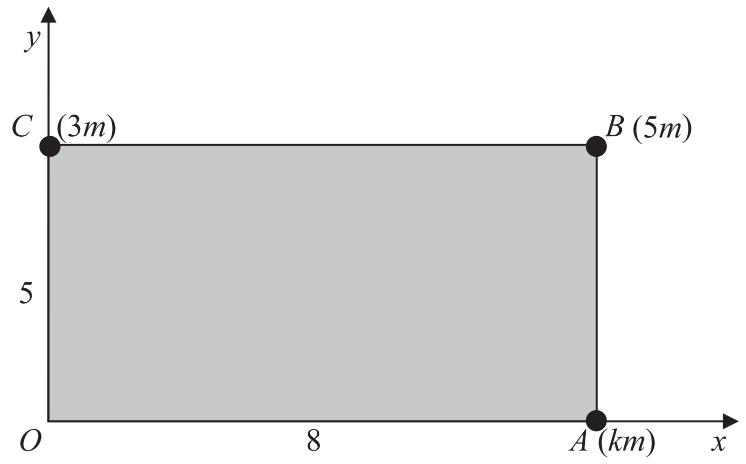

Figure 3

Figure 3 shows a rectangular lamina *OABC*. The coordinates of *O*, *A*, *B* and *C* are (0, 0), (8, 0), (8, 5) and (0, 5) respectively. Particles of mass km, $5m$ and $3m$ are attached to the lamina at *A*, *B* and *C* respectively.

The x-coordinate of the centre of mass of the three particles *without the lamina* is 6.4.

(a) Show that $k = 7$.

(4)

The lamina *OABC* is uniform and has mass $12m$.

(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina.

(6)

The combined system is freely suspended from *O* and hangs at rest.

(c) Find the angle between *OC* and the horizontal.

(3)

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7.

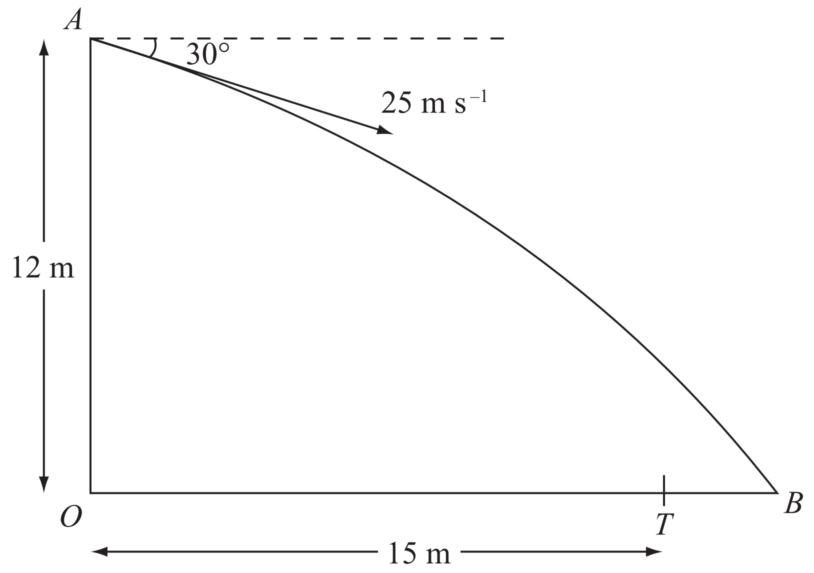

Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

(5)