Answer **all** questions.

1 Two groups of patients, suffering from the same medical condition, took part in a clinical trial of a new drug. One of the groups was given the drug whilst the other group was given a placebo, a drug that has no physical effect on their medical condition.

The table shows the number of patients in each group and whether or not their condition improved.

	Placebo	**Drug**
Condition improved	20	46
Condition did not improve	55	29

Conduct a χ^2 test, at the 5% level of significance, to determine whether the condition of the patients at the conclusion of the trial is associated with the treatment that they were given.
(10 marks)

2 The number of telephone calls per day, X, received by Candice may be modelled by a Poisson distribution with mean 3.5 .

The number of e-mails per day, Y, received by Candice may be modelled by a Poisson distribution with mean 6.0 .

(a) For any particular day, find:

(i) $\mathrm{P}(X = 3)$; *(2 marks)*

(ii) $\mathrm{P}(Y \geqslant 5)$. *(2 marks)*

(b) (i) Write down the distribution of T, the total number of telephone calls and e-mails per day received by Candice. *(1 mark)*

(ii) Determine $\mathrm{P}(7 \leqslant T \leqslant 10)$. *(3 marks)*

(iii) Hence calculate the probability that, on each of three consecutive days, Candice will receive a total of at least 7 but at most 10 telephone calls and e-mails.
(2 marks)

3 David is the professional coach at the golf club where Becki is a member. He claims that, after having a series of lessons with him, the mean number of putts that Becki takes per round of golf will reduce from her present mean of 36.

After having the series of lessons with David, Becki decides to investigate his claim.

She therefore records, for each of a random sample of 50 rounds of golf, the number of putts, x, that she takes to complete the round. Her results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 1730 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 784$$

Using a z-test and the 1% level of significance, investigate David's claim. *(8 marks)*

4 Ten students each independently carried out the same experiment in order to measure, in $\mathrm{m\,s^{-2}}$, the value of g, the acceleration due to gravity, with the following results:

9.75	9.72	9.71	9.69	9.66	9.70	9.72	9.71	9.69	9.65

(a) Assuming that values from the experiment are normally distributed, with mean g, construct a 95% confidence interval for g. *(6 marks)*

(b) It was subsequently discovered that the equipment used in the experiment was faulty. As a consequence, each of the values above is $0.10\,\mathrm{m\,s^{-2}}$ less than the actual value.

Use this additional information to write down a revised 95% confidence interval for g. *(2 marks)*

Turn over for the next question

5 A discrete random variable X has probability distribution as given in the table.

x	1	2	3	4
$\mathbf{P}(X=x)$	p	p	p	$1-3p$

(a) Show that, for this to be a valid distribution, $0 \leqslant p \leqslant \frac{1}{3}$. *(3 marks)*

(b) (i) Find an expression, in terms of p, for $\mathrm{E}(X)$. *(1 mark)*

(ii) Show that $\mathrm{Var}(X) = 2p(7-18p)$. *(3 marks)*

(c) (i) Find the value of p for which $\mathrm{Var}(X)$ is a maximum. *(2 marks)*

(ii) Find the maximum value of the standard deviation of X. *(3 marks)*

6 The continuous random variable X has the probability density function given by

$$\mathrm{f}(x) = \begin{cases} 3x^2 & 0 < x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine:

(i) $\mathrm{E}\left(\frac{1}{X}\right)$; *(3 marks)*

(ii) $\mathrm{Var}\left(\frac{1}{X}\right)$. *(4 marks)*

(b) Hence, or otherwise, find the mean and the variance of $\left(\frac{5+2X}{X}\right)$. *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 Two groups of patients, suffering from the same medical condition, took part in a clinical trial of a new drug. One of the groups was given the drug whilst the other group was given a placebo, a drug that has no physical effect on their medical condition.

The table shows the number of patients in each group and whether or not their condition improved.

	Placebo	**Drug**
Condition improved	20	46
Condition did not improve	55	29

Conduct a χ^2 test, at the 5% level of significance, to determine whether the condition of the patients at the conclusion of the trial is associated with the treatment that they were given. *(10 marks)*

2 The number of telephone calls per day, X, received by Candice may be modelled by a Poisson distribution with mean 3.5 .

The number of e-mails per day, Y, received by Candice may be modelled by a Poisson distribution with mean 6.0 .

(a) For any particular day, find:

(i) $\mathrm{P}(X = 3)$; *(2 marks)*

(ii) $\mathrm{P}(Y \geqslant 5)$. *(2 marks)*

(b) (i) Write down the distribution of T, the total number of telephone calls and e-mails per day received by Candice. *(1 mark)*

(ii) Determine $\mathrm{P}(7 \leqslant T \leqslant 10)$. *(3 marks)*

(iii) Hence calculate the probability that, on each of three consecutive days, Candice will receive a total of at least 7 but at most 10 telephone calls and e-mails. *(2 marks)*

3 David is the professional coach at the golf club where Becki is a member. He claims that, after having a series of lessons with him, the mean number of putts that Becki takes per round of golf will reduce from her present mean of 36.

After having the series of lessons with David, Becki decides to investigate his claim.

She therefore records, for each of a random sample of 50 rounds of golf, the number of putts, x, that she takes to complete the round. Her results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 1730 \quad \text{and} \quad \sum (x - \bar{x})^2 = 784$$

Using a z-test and the 1% level of significance, investigate David's claim. *(8 marks)*

4 Students are each asked to measure the distance between two points to the nearest tenth of a metre.

(a) Given that the rounding error, X metres, in these measurements has a rectangular distribution, explain why its probability density function is

$$f(x) = \begin{cases} 10 & -0.05 < x \leqslant 0.05 \\ 0 & \text{otherwise} \end{cases}$$

(3 marks)

(b) Calculate $P(-0.01 < X < 0.02)$. *(2 marks)*

(c) Find the mean and the standard deviation of X. *(2 marks)*

Turn over for the next question

5 Members of a residents' association are concerned about the speeds of cars travelling through their village. They decide to record the speed, in mph, of each of a random sample of 10 cars travelling through their village, with the following results:

33	27	34	30	48	35	34	33	43	39

(a) Construct a 99% confidence interval for μ, the mean speed of cars travelling through the village, stating any assumption that you make. *(7 marks)*

(b) Comment on the claim that a 30 mph speed limit is being adhered to by most motorists. *(3 marks)*

6 The continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} 3x^2 & 0 < x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine:

(i) $\mathrm{E}\left(\dfrac{1}{X}\right)$; *(3 marks)*

(ii) $\mathrm{Var}\left(\dfrac{1}{X}\right)$. *(4 marks)*

(b) Hence, or otherwise, find the mean and the variance of $\left(\dfrac{5 + 2X}{X}\right)$. *(5 marks)*

7 On a multiple choice examination paper, each question has five alternative answers given, only one of which is correct. For each question, candidates gain 4 marks for a correct answer but lose 1 mark for an incorrect answer.

(a) James guesses the answer to each question.

(i) Copy and complete the following table for the probability distribution of X, the number of marks obtained by James for each question.

x	4	-1
$\mathrm{P}(X = x)$		

(1 mark)

(ii) Hence find $\mathrm{E}(X)$. *(2 marks)*

(b) Karen is able to eliminate two of the incorrect answers from the five alternative answers given for each question before guessing the answer from those remaining.

Given that the examination paper contains 24 questions, calculate Karen's expected total mark. *(4 marks)*

8 A jam producer claims that the mean weight of jam in a jar is 230 grams.

(a) A random sample of 8 jars is selected and the weight of jam in each jar is determined. The results, in grams, are

220 228 232 219 221 223 230 229

Assuming that the weight of jam in a jar is normally distributed, test, at the 5% level of significance, the jam producer's claim. *(9 marks)*

(b) It is later discovered that the mean weight of jam in a jar is indeed 230 grams.

Indicate whether a Type I error, a Type II error or neither has occurred in carrying out the hypothesis test in part (a). Give a reason for your answer. *(2 marks)*

END OF QUESTIONS

Practice 3

Leave blank

1. A string AB of length 5 cm is cut, in a random place C, into two pieces. The random variable X is the length of AC.

(a) Write down the name of the probability distribution of X and sketch the graph of its probability density function.
(3)

(b) Find the values of E(X) and Var(X).
(3)

(c) Find P(X >3).
(1)

(d) Write down the probability that AC is 3 cm long.
(1)

Leave blank

Question 1 continued

Q1

(Total 8 marks)

Leave blank

2. Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and finds that it contains 7 bacteria. Stating your hypotheses clearly test, at the 5% level of significance, the claim of the scientist.
(7)

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty.
Faulty components are detected at a rate of 1.5 per hour.

(a) Suggest a suitable model for the number of faulty components detected per hour.
(1)

(b) Describe, in the context of this question, two assumptions you have made in part (a) for this model to be suitable.
(2)

(c) Find the probability of 2 faulty components being detected in a 1 hour period.
(2)

(d) Find the probability of at least one faulty component being detected in a 3 hour period.
(3)

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. A bag contains a large number of coins:

75% are 10p coins,

25% are 5p coins.

A random sample of 3 coins is drawn from the bag.
Find the sampling distribution for the median of the values of the 3 selected coins.
(7)

Leave blank

Question 4 continued

(Total 7 marks)

Q4

Leave blank

5. (a) Write down the conditions under which the Poisson distribution may be used as an approximation to the Binomial distribution.
(2)

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01

(b) Find the probability that 2 consecutive calls will be connected to the wrong agent.
(2)

(c) Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent.
(3)

The call centre receives 1000 calls each day.

(d) Find the mean and variance of the number of wrongly connected calls.
(3)

(e) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent.
(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 12 marks)

Q5

Leave blank

6. Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times.

You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.

(7)

Leave blank

Question 6 continued

Q6

(Total 7 marks)

Leave blank

7. (a) (i) Write down two conditions for $X \sim \text{Bin}(n, p)$ to be approximated by a normal distribution $Y \sim \text{N}(\mu, \sigma^2)$.

(2)

(ii) Write down the mean and variance of this normal approximation in terms of n and p.

(2)

A factory manufactures 2000 DVDs every day. It is known that 3% of DVDs are faulty.

(b) Using a normal approximation, estimate the probability that at least 40 faulty DVDs are produced in one day.

(5)

The quality control system in the factory identifies and destroys every faulty DVD at the end of the manufacturing process. It costs £0.70 to manufacture a DVD and the factory sells non-faulty DVDs for £11.

(c) Find the expected profit made by the factory per day.

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 12 marks)

Leave blank

8. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x & 0 < x \leqslant 3 \\ 2 - \frac{1}{2}x & 3 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the probability density function of X.
(3)

(b) Find the mode of X.
(1)

(c) Specify fully the cumulative distribution function of X.
(7)

(d) Using your answer to part (c), find the median of X.
(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

1 A random sample of observations of a random variable X is summarised by

$$n = 100, \quad \Sigma x = 4830.0, \quad \Sigma x^2 = 249\,509.16.$$

(i) Obtain unbiased estimates of the mean and variance of X. [4]

(ii) The sample mean of 100 observations of X is denoted by $\overline{X}$. Explain whether you would need any further information about the distribution of X in order to estimate $\mathrm{P}(\overline{X} > 60)$. [You should not attempt to carry out the calculation.] [2]

2 It is given that on average one car in forty is yellow. Using a suitable approximation, find the probability that, in a random sample of 130 cars, exactly 4 are yellow. [5]

3 The proportion of adults in a large village who support a proposal to build a bypass is denoted by p. A random sample of size 20 is selected from the adults in the village, and the members of the sample are asked whether or not they support the proposal.

(i) Name the probability distribution that would be used in a hypothesis test for the value of p. [1]

(ii) State the properties of a random sample that explain why the distribution in part **(i)** is likely to be a good model. [2]

4 X is a continuous random variable.

(i) State two conditions needed for X to be well modelled by a normal distribution. [2]

(ii) It is given that $X \sim \mathrm{N}(50.0, 8^2)$. The mean of 20 random observations of X is denoted by $\overline{X}$. Find $\mathrm{P}(\overline{X} > 47.0)$. [4]

5 The number of system failures per month in a large network is a random variable with the distribution $\mathrm{Po}(\lambda)$. A significance test of the null hypothesis $\mathrm{H}_0 : \lambda = 2.5$ is carried out by counting R, the number of system failures in a period of 6 months. The result of the test is that H_0 is rejected if $R > 23$ but is not rejected if $R \leqslant 23$.

(i) State the alternative hypothesis. [1]

(ii) Find the significance level of the test. [3]

(iii) Given that $\mathrm{P}(R > 23) < 0.1$, use tables to find the largest possible actual value of λ. You should show the values of any relevant probabilities. [3]

6 In a rearrangement code, the letters of a message are rearranged so that the frequency with which any particular letter appears is the same as in the original message. In ordinary German the letter *e* appears 19% of the time. A certain encoded message of 20 letters contains one letter *e*.

(i) Using an exact binomial distribution, test at the 10% significance level whether there is evidence that the proportion of the letter *e* in the language from which this message is a sample is less than in German, i.e., less than 19%. [8]

(ii) Give a reason why a binomial distribution might not be an appropriate model in this context. [1]

7 Two continuous random variables S and T have probability density functions as follows.

$$S: \quad f(x) = \begin{cases} \frac{1}{2} & -1 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

$$T: \quad g(x) = \begin{cases} \frac{3}{2}x^2 & -1 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) *Sketch* on the same axes the graphs of $y = f(x)$ and $y = g(x)$. [You should not use graph paper or attempt to plot points exactly.] [3]

(ii) Explain in everyday terms the difference between the two random variables. [2]

(iii) Find the value of t such that $P(T > t) = 0.2$. [5]

8 A random variable Y is normally distributed with mean μ and variance 12.25. Two statisticians carry out significance tests of the hypotheses $H_0 : \mu = 63.0$, $H_1 : \mu > 63.0$.

(i) Statistician A uses the mean $\overline{Y}$ of a sample of size 23, and the critical region for his test is $\overline{Y} > 64.20$. Find the significance level for A's test. [4]

(ii) Statistician B uses the mean of a sample of size 50 and a significance level of 5%.

(a) Find the critical region for B's test. [3]

(b) Given that $\mu = 65.0$, find the probability that B's test results in a Type II error. [4]

(iii) Given that, when $\mu = 65.0$, the probability that A's test results in a Type II error is 0.1365, state with a reason which test is better. [2]

9 **(a)** The random variable G has the distribution $B(n, 0.75)$. Find the set of values of n for which the distribution of G can be well approximated by a normal distribution. [3]

(b) The random variable H has the distribution $B(n, p)$. It is given that, using a normal approximation, $P(H \geqslant 71) = 0.0401$ and $P(H \leqslant 46) = 0.0122$.

(i) Find the mean and standard deviation of the approximating normal distribution. [6]

(ii) Hence find the values of n and p. [4]

Answer **all** questions.

1 It is thought that the incidence of asthma in children is associated with the volume of traffic in the area where they live.

Two surveys of children were conducted: one in an area where the volume of traffic was heavy and the other in an area where the volume of traffic was light.

For each area, the table shows the number of children in the survey who had asthma and the number who did not have asthma.

	Asthma	**No asthma**	**Total**
Heavy traffic	52	58	110
Light traffic	28	62	90
Total	80	120	200

(a) Use a χ^2 test, at the 5% level of significance, to determine whether the incidence of asthma in children is associated with the volume of traffic in the area where they live. *(8 marks)*

(b) Comment on the number of children in the survey who had asthma, given that they lived in an area where the volume of traffic was heavy. *(1 mark)*

2 (a) The number of genuine telephone calls, X, made to the emergency services each hour may be modelled by a Poisson distribution with mean 6.5.

Determine the probability that, in a given hour, there will be fewer than 6 genuine telephone calls made to the emergency services. *(2 marks)*

(b) The number of bogus telephone calls, Y, made to the emergency services each hour may be modelled by a Poisson distribution with mean 1.5.

Calculate the probability that, in a given hour, there will be at least 1 bogus telephone call made to the emergency services. *(3 marks)*

(c) Assuming that X and Y are independent Poisson variables, find:

(i) $P(X + Y = 6)$; *(3 marks)*

(ii) $P(Y \geqslant 1 \mid X < 6)$. *(1 mark)*

3 Alan's company produces packets of crisps. The standard deviation of the weight of a packet of crisps is known to be 2.5 grams.

Alan believes that, due to the extra demand on the production line at a busy time of the year, the mean weight of packets of crisps is not equal to the target weight of 34.5 grams.

In an experiment set up to investigate Alan's belief, the weights of a random sample of 50 packets of crisps were recorded. The mean weight of this sample is 35.1 grams.

Investigate Alan's belief at the 5% level of significance. *(6 marks)*

4 The weight of fat in a digestive biscuit is known to be normally distributed.

Pat conducted an experiment in which she measured the weight of fat, x grams, in each of a random sample of 10 digestive biscuits, with the following results:

$$\sum x = 31.9 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 1.849$$

(a) (i) Construct a 99% confidence interval for the mean weight of fat in digestive biscuits. *(5 marks)*

(ii) Comment on a claim that the mean weight of fat in digestive biscuits is 3.5 grams. *(2 marks)*

(b) If 200 such 99% confidence intervals were constructed, how many would you expect **not** to contain the population mean? *(1 mark)*

5 The management of the Wellfit gym claims that the mean cholesterol level of those members who have held membership of the gym for more than one year is 3.8 .

A local doctor believes that the management's claim is too low and investigates by measuring the cholesterol levels of a random sample of 7 such members of the Wellfit gym, with the following results:

4.2 4.3 3.9 3.8 3.6 4.8 4.1

Is there evidence, at the 5% level of significance, to justify the doctor's belief that the mean cholesterol level is greater than the management's claim? State any assumption that you make. *(8 marks)*

6 (a) The number of text messages, N, sent by Peter each month on his mobile phone never exceeds 40.

When $0 \leqslant N \leqslant 10$, he is charged for 5 messages.
When $10 < N \leqslant 20$, he is charged for 15 messages.
When $20 < N \leqslant 30$, he is charged for 25 messages.
When $30 < N \leqslant 40$, he is charged for 35 messages.

The number of text messages, Y, that Peter is charged for each month has the following probability distribution:

y	5	15	25	35
$\mathbf{P}(Y = y)$	0.1	0.2	0.3	0.4

(i) Calculate the mean and the standard deviation of Y. *(4 marks)*

(ii) The Goodtime phone company makes a total charge for text messages, C pence, each month given by

$$C = 10Y + 5$$

Calculate $\mathrm{E}(C)$. *(1 mark)*

(b) The number of text messages, X, sent by Joanne each month on her mobile phone is such that

$$\mathrm{E}(X) = 8.35 \qquad \text{and} \qquad \mathrm{E}(X^2) = 75.25$$

The Newtime phone company makes a total charge for text messages, T pence, each month given by

$$T = 0.4X + 250$$

Calculate $\mathrm{Var}(T)$. *(4 marks)*

7 The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{k+1} & -1 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where k is a positive constant.

(a) Find, in terms of k, an expression for $\mathrm{P}(X < 0)$. *(2 marks)*

(b) (i) Show that the probability density function of X is defined by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{k+1} & -1 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

(2 marks)

(ii) Sketch the graph of f. *(2 marks)*

(c) Given that $\mathrm{Var}(X) = [\mathrm{E}(X)]^2$, show that $k = 2 \pm \sqrt{3}$. *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 It is thought that the incidence of asthma in children is associated with the volume of traffic in the area where they live.

Two surveys of children were conducted: one in an area where the volume of traffic was heavy and the other in an area where the volume of traffic was light.

For each area, the table shows the number of children in the survey who had asthma and the number who did not have asthma.

	Asthma	**No asthma**	**Total**
Heavy traffic	52	58	110
Light traffic	28	62	90
Total	80	120	200

(a) Use a χ^2 test, at the 5% level of significance, to determine whether the incidence of asthma in children is associated with the volume of traffic in the area where they live. *(8 marks)*

(b) Comment on the number of children in the survey who had asthma, given that they lived in an area where the volume of traffic was heavy. *(1 mark)*

2 (a) The number of telephone calls, X, received per hour for Dr Able may be modelled by a Poisson distribution with mean 6.

Determine $\mathrm{P}(X = 8)$. *(2 marks)*

(b) The number of telephone calls, Y, received per hour for Dr Bracken may be modelled by a Poisson distribution with mean λ and standard deviation 3.

(i) Write down the value of λ. *(1 mark)*

(ii) Determine $\mathrm{P}(Y > \lambda)$. *(2 marks)*

(c) (i) Assuming that X and Y are independent Poisson variables, write down the distribution of the **total** number of telephone calls received per hour for Dr Able and Dr Bracken. *(1 mark)*

(ii) Determine the probability that a total of at most 20 telephone calls will be received during any one-hour period. *(1 mark)*

(iii) The total number of telephone calls received during each of 6 one-hour periods is to be recorded. Calculate the probability that a total of at least 21 telephone calls will be received during exactly 4 of these one-hour periods. *(3 marks)*

3 Alan's company produces packets of crisps. The standard deviation of the weight of a packet of crisps is known to be 2.5 grams.

Alan believes that, due to the extra demand on the production line at a busy time of the year, the mean weight of packets of crisps is not equal to the target weight of 34.5 grams.

In an experiment set up to investigate Alan's belief, the weights of a random sample of 50 packets of crisps were recorded. The mean weight of this sample is 35.1 grams.

Investigate Alan's belief at the 5% level of significance. *(6 marks)*

4 The delay, in hours, of certain flights from Australia may be modelled by the continuous random variable T, with probability density function

$$f(t) = \begin{cases} \frac{2}{15}t & 0 \leqslant t \leqslant 3 \\ 1 - \frac{1}{5}t & 3 \leqslant t \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(3 marks)*

(b) Calculate:

(i) $P(T \leqslant 2)$; *(2 marks)*

(ii) $P(2 < T < 4)$. *(3 marks)*

(c) Determine $E(T)$. *(4 marks)*

5 The weight of fat in a digestive biscuit is known to be normally distributed.

Pat conducted an experiment in which she measured the weight of fat, x grams, in each of a random sample of 10 digestive biscuits, with the following results:

$$\sum x = 31.9 \quad \text{and} \quad \sum (x - \bar{x})^2 = 1.849$$

(a) (i) Construct a 99% confidence interval for the mean weight of fat in digestive biscuits. *(5 marks)*

(ii) Comment on a claim that the mean weight of fat in digestive biscuits is 3.5 grams. *(2 marks)*

(b) If 200 such 99% confidence intervals were constructed, how many would you expect **not** to contain the population mean? *(1 mark)*

6 The management of the Wellfit gym claims that the mean cholesterol level of those members who have held membership of the gym for more than one year is 3.8.

A local doctor believes that the management's claim is too low and investigates by measuring the cholesterol levels of a random sample of 7 such members of the Wellfit gym, with the following results:

4.2 4.3 3.9 3.8 3.6 4.8 4.1

Is there evidence, at the 5% level of significance, to justify the doctor's belief that the mean cholesterol level is greater than the management's claim? State any assumption that you make. *(8 marks)*

7 (a) The number of text messages, N, sent by Peter each month on his mobile phone never exceeds 40.

When $0 \leqslant N \leqslant 10$, he is charged for 5 messages.
When $10 < N \leqslant 20$, he is charged for 15 messages.
When $20 < N \leqslant 30$, he is charged for 25 messages.
When $30 < N \leqslant 40$, he is charged for 35 messages.

The number of text messages, Y, that Peter is charged for each month has the following probability distribution:

y	5	15	25	35
$\mathrm{P}(Y = y)$	0.1	0.2	0.3	0.4

(i) Calculate the mean and the standard deviation of Y. *(4 marks)*

(ii) The Goodtime phone company makes a total charge for text messages, C pence, each month given by

$$C = 10Y + 5$$

Calculate $\mathrm{E}(C)$. *(1 mark)*

(b) The number of text messages, X, sent by Joanne each month on her mobile phone is such that

$$\mathrm{E}(X) = 8.35 \qquad \text{and} \qquad \mathrm{E}(X^2) = 75.25$$

The Newtime phone company makes a total charge for text messages, T pence, each month given by

$$T = 0.4X + 250$$

Calculate $\mathrm{Var}(T)$. *(4 marks)*

8 The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{k+1} & -1 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where k is a positive constant.

(a) Find, in terms of k, an expression for $\mathrm{P}(X < 0)$. *(2 marks)*

(b) Determine an expression, in terms of k, for the lower quartile, q_1. *(3 marks)*

(c) Show that the probability density function of X is defined by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{k+1} & -1 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$ *(2 marks)*

(d) Given that $k = 11$:

(i) sketch the graph of f; *(2 marks)*

(ii) determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$; *(2 marks)*

(iii) show that $\mathrm{P}(q_1 < X < \mathrm{E}(X)) = 0.25$. *(2 marks)*

END OF QUESTIONS