Practice 1

Leave blank

1. Simplify $(3+\sqrt{5})(3-\sqrt{5})$.

(2)

Q1

(Total 2 marks)

Leave blank

2. (a) Find the value of $8^{\frac{4}{3}}$.

(2)

(b) Simplify $\dfrac{15x^{\frac{4}{3}}}{3x}$.

(2)

Q2

(Total 4 marks)

Turn over

Leave blank

3. Given that $y = 3x^2 + 4\sqrt{x}, \quad x > 0$, find

(a) $\dfrac{dy}{dx}$,

(2)

(b) $\dfrac{d^2y}{dx^2}$,

(2)

(c) $\int y \, dx$.

(3)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Turn over

Leave blank

4. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.
(3)

(b) Calculate her total savings over the complete 200 week period.
(3)

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Turn over

Leave blank

5.

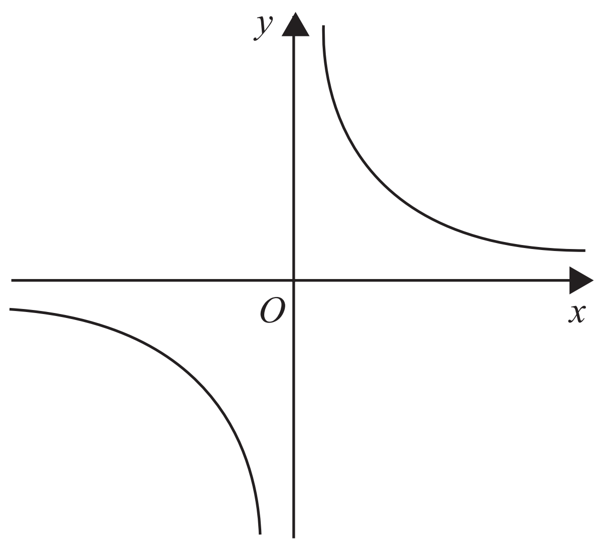


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}, \ x \neq 0$.

(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}, \ x \neq -2$,

showing the coordinates of any point at which the curve crosses a coordinate axis.
(3)

(b) Write down the equations of the asymptotes of the curve in part (a).
(2)

Leave blank

Question 5 continued

Q5

(Total 5 marks)

Leave blank

6. (a) By eliminating y from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0.$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

Leave blank

Question 6 continued

Q6

(Total 7 marks)

Turn over

Leave blank

7. The equation $x^2 + kx + (k+3) = 0$, where k is a constant, has different real roots.

(a) Show that $k^2 - 4k - 12 > 0$.

(2)

(b) Find the set of possible values of k.

(4)

Leave blank

Question 7 continued

Q7

(Total 6 marks)

Leave blank

8. A sequence $a_1, a_2, a_3, ...$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k. **(1)**

(b) Show that $a_3 = 9k + 20$. **(2)**

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10. **(4)**

Leave blank

Question 8 continued

Q8

(Total 7 marks)

Leave blank

9. The curve C with equation $y = \mathrm{f}(x)$ passes through the point (5, 65).

Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$,

(a) use integration to find $\mathrm{f}(x)$.

(4)

(b) Hence show that $\mathrm{f}(x) = x(2x+3)(x-4)$.

(2)

(c) In the space provided on page 17, sketch C, showing the coordinates of the points where C crosses the x-axis.

(3)

Leave blank

Question 9 continued

Q9

(Total 9 marks)

Leave blank

10. The curve C has equation $y = x^2(x-6) + \frac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(4)

(b) Show that the tangents to C at P and Q are parallel.

(5)

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

Question 11 continued

Q11

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

1 Simplify $(2x+5)^2-(x-3)^2$, giving your answer in the form ax^2+bx+c. [3]

2 **(a)** On separate diagrams, sketch the graphs of

(i) $y=\frac{1}{x}$, [2]

(ii) $y=x^4$. [1]

(b) Describe a transformation that transforms the curve $y=x^3$ to the curve $y=8x^3$. [2]

3 Simplify the following, expressing each answer in the form $a\sqrt{5}$.

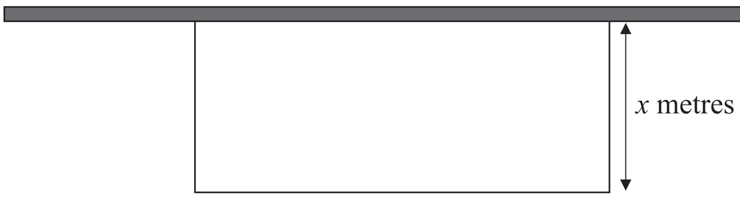
(i) $3\sqrt{10}\times\sqrt{2}$ [2]

(ii) $\sqrt{500}+\sqrt{125}$ [3]

4 **(i)** Find the discriminant of kx^2-4x+k in terms of k. [2]

(ii) The quadratic equation $kx^2-4x+k=0$ has equal roots. Find the possible values of k. [3]

5



The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres.

(i) Show that the enclosed area, A m^2, is given by

$$A=20x-2x^2.$$ [2]

(ii) Use differentiation to find the maximum value of A. [4]

6 By using the substitution $y=(x+2)^2$, find the real roots of the equation

$$(x+2)^4+5(x+2)^2-6=0.$$ [6]

7 **(a)** Given that $\mathrm{f}(x)=x+\frac{3}{x}$, find $\mathrm{f}'(x)$. [4]

(b) Find the gradient of the curve $y=x^{\frac{5}{2}}$ at the point where $x=4$. [5]

8 **(i)** Express $x^2 + 8x + 15$ in the form $(x + a)^2 - b$. [3]

(ii) Hence state the coordinates of the vertex of the curve $y = x^2 + 8x + 15$. [2]

(iii) Solve the inequality $x^2 + 8x + 15 > 0$. [4]

9 The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 4.

(i) Find the centre of the circle and the value of k. [4]

The points A (3, a) and B (−1, 0) lie on the circumference of the circle, with $a > 0$.

(ii) Calculate the length of AB, giving your answer in simplified surd form. [5]

(iii) Find an equation for the line AB. [3]

10 **(i)** Solve the equation $3x^2 - 14x - 5 = 0$. [3]

A curve has equation $y = 3x^2 - 14x - 5$.

(ii) Sketch the curve, indicating the coordinates of all intercepts with the axes. [3]

(iii) Find the value of c for which the line $y = 4x + c$ is a tangent to the curve. [6]

Answer **all** questions.

1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

2 (a) Express $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. *(3 marks)*

(b) Express $\frac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7} + q$, where p and q are integers. *(4 marks)*

3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t\,, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

5 A circle with centre C has equation $(x + 3)^2 + (y - 2)^2 = 25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates (2, 6).

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

Turn over for the next question

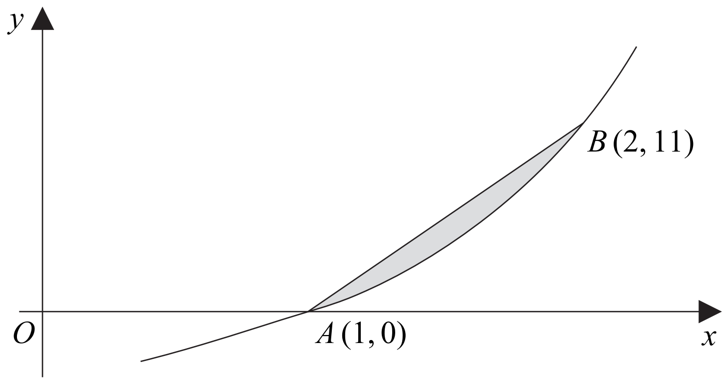
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

7 The quadratic equation

$$(2k-3)x^2 + 2x + (k-1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

END OF QUESTIONS

Practice 4

Leave blank

1. Find $\int (2 + 5x^2)\,dx$.

(3)

Q1

(Total 3 marks)

Leave blank

2. Factorise completely

$$x^3 - 9x.$$

(3)

Q2

(Total 3 marks)

Turn over

Leave blank

3.

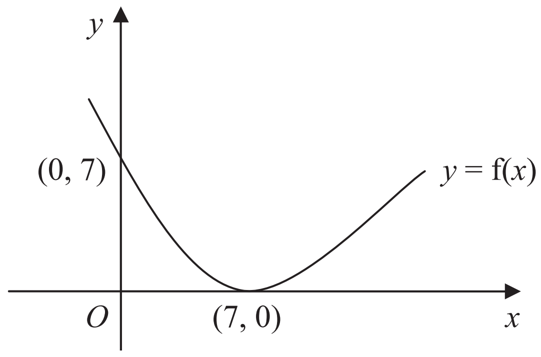


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the point (0, 7) and has a minimum point at (7, 0).

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 3$,

(3)

(b) $y = \mathrm{f}(2x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

Leave blank

Question 3 continued

Q3

(Total 5 marks)

Turn over

Leave blank

4. $$f(x) = 3x + x^3, \qquad x > 0.$$

(a) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(b) find the value of x.

(3)

Leave blank

Question 4 continued

Q4

(Total 5 marks)

Leave blank

5. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geqslant 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a.

(3)

Leave blank

Question 5 continued

Q5

(Total 6 marks)

Leave blank

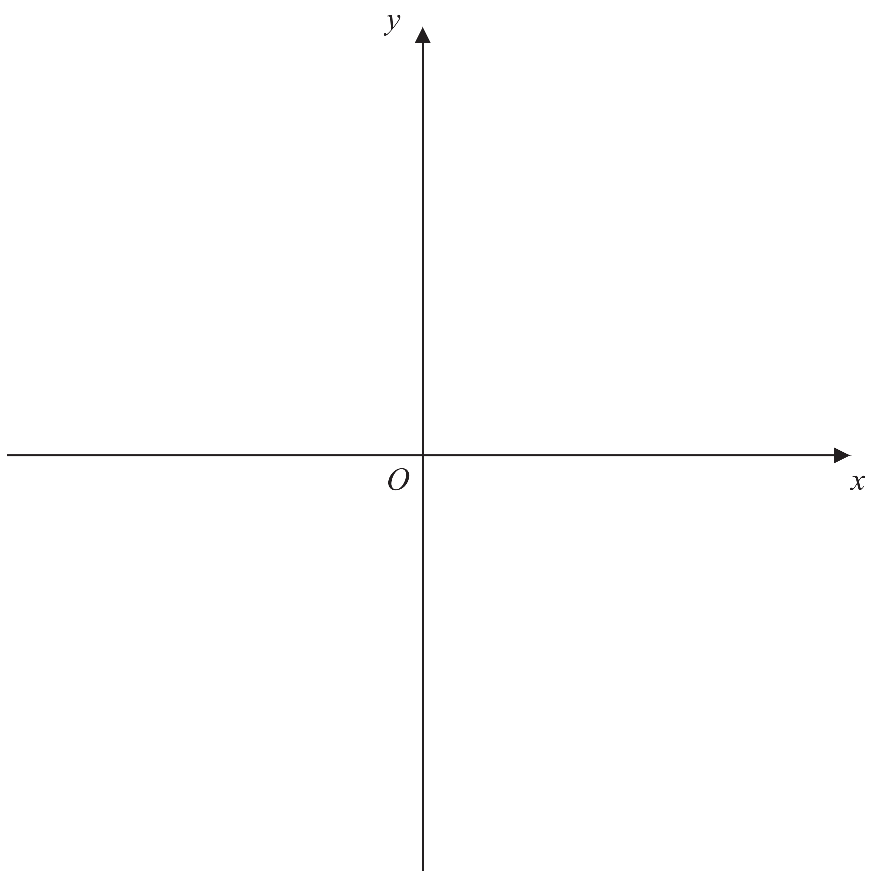
6. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(a) On the axes below, sketch the graphs of C and l, indicating clearly the coordinates of any intersections with the axes.

(3)

(b) Find the coordinates of the points of intersection of C and l.

(6)



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Question 6 continued

(Total 9 marks)

Q6

Leave blank

7. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(a) Show that on the 4th Saturday of training she runs 11 km.
(1)

(b) Find an expression, in terms of n, for the length of her training run on the nth Saturday.
(2)

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.
(3)

On the nth Saturday Sue runs 43 km.

(d) Find the value of n.
(2)

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.
(2)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 10 marks)

Turn over

Leave blank

8. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

Leave blank

Question 8 continued

Q8

(Total 5 marks)

Turn over

Leave blank

9. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\frac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y-coordinate of A.

(2)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 8 marks)

Leave blank

10.

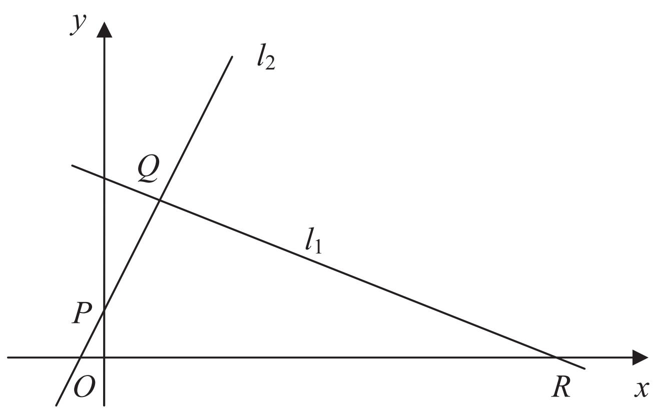


Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(a) Find the value of a. **(3)**

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2.

Find

(b) an equation for l_2, **(5)**

(c) the coordinates of P, **(1)**

(d) the area of ΔPQR. **(4)**

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 13 marks)

Turn over

Leave blank

11. The gradient of a curve C is given by $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

Leave blank

Question 11 continued

Turn over

Leave blank

Question 11 continued

Q11

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

END

1 Express each of the following in the form 4^n:

(i) $\frac{1}{16}$, [1]

(ii) 64, [1]

(iii) 8. [2]

2 **(i)** The curve $y = x^2$ is translated 2 units in the positive x-direction. Find the equation of the curve after it has been translated. [2]

(ii) The curve $y = x^3 - 4$ is reflected in the x-axis. Find the equation of the curve after it has been reflected. [1]

3 Express each of the following in the form $k\sqrt{2}$, where k is an integer:

(i) $\sqrt{200}$, [1]

(ii) $\dfrac{12}{\sqrt{2}}$, [1]

(iii) $5\sqrt{8} - 3\sqrt{2}$. [2]

4 Solve the equation $2x - 7x^{\frac{1}{2}} + 3 = 0$. [5]

5 Find the gradient of the curve $y = 8\sqrt{x} + x$ at the point whose x-coordinate is 9. [5]

6 **(i)** Expand and simplify $(x-5)(x+2)(x+5)$. [3]

(ii) Sketch the curve $y = (x-5)(x+2)(x+5)$, giving the coordinates of the points where the curve crosses the axes. [3]

7 Solve the inequalities

(i) $8 < 3x - 2 < 11$, [3]

(ii) $y^2 + 2y \geqslant 0$. [4]

8 The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Given that there is a stationary point when $x = 1$, find the value of k. [3]

(iii) Determine whether this stationary point is a minimum or maximum point. [2]

(iv) Find the x-coordinate of the other stationary point. [3]

9 **(i)** Find the equation of the circle with radius 10 and centre (2, 1), giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. [3]

(ii) The circle passes through the point (5, k) where $k > 0$. Find the value of k in the form $p + \sqrt{q}$. [3]

(iii) Determine, showing all working, whether the point (−3, 9) lies inside or outside the circle. [3]

(iv) Find an equation of the tangent to the circle at the point (8, 9). [5]

10 **(i)** Express $2x^2 - 6x + 11$ in the form $p(x + q)^2 + r$. [4]

(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 6x + 11$. [2]

(iii) Calculate the discriminant of $2x^2 - 6x + 11$. [2]

(iv) State the number of real roots of the equation $2x^2 - 6x + 11 = 0$. [1]

(v) Find the coordinates of the points of intersection of the curve $y = 2x^2 - 6x + 11$ and the line $7x + y = 14$. [5]

Answer **all** questions.

1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x+3)(x-1)$$

(a) Sketch on the same axes the line L and the curve C, showing the values of the intercepts on the x-axis and the y-axis. *(5 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

(a) xy ; *(1 mark)*

(b) $\dfrac{y}{x}$; *(2 marks)*

(c) $(x+y)^2$. *(3 marks)*

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\dfrac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

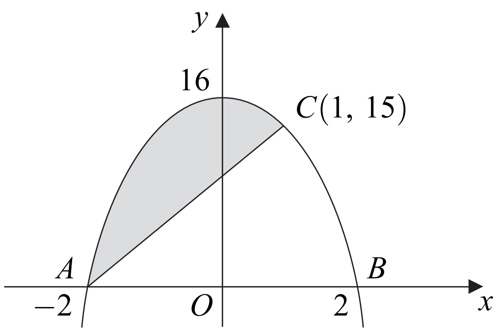
(iii) Find the maximum value of V. *(1 mark)*

4 (a) Express $x^2 - 3x + 4$ in the form $(x - p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)*

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\, dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

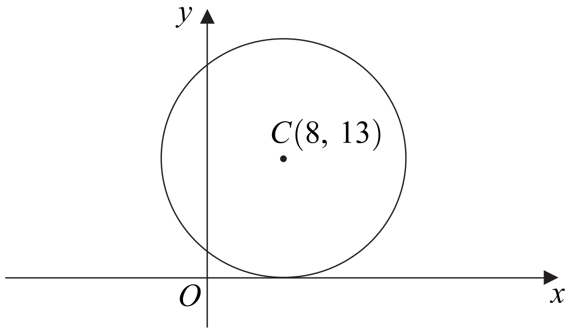
(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

Turn over for the next question

7 The circle S has centre $C(8, 13)$ and touches the x-axis, as shown in the diagram.



(a) Write down an equation for S, giving your answer in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(2 marks)*

(b) The point P with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through P and C. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle S at the point P, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(4 marks)*

(iii) The point Q also lies on the circle S, and the length of PQ is 10. Calculate the shortest distance from C to the chord PQ. *(3 marks)*

8 The quadratic equation $(k+1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS