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Answer **all** questions.

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1 (a) Simplify:

(i)  $x^{\frac{3}{2}} \times x^{\frac{1}{2}};$  (1 mark)

(ii)  $x^{\frac{3}{2}} \div x;$  (1 mark)

(iii)  $\left(x^{\frac{3}{2}}\right)^2.$  (1 mark)

(b) (i) Find  $\int 3x^{\frac{1}{2}} dx.$  (3 marks)

(ii) Hence find the value of  $\int_1^9 3x^{\frac{1}{2}} dx.$  (2 marks)

2 The  $n$ th term of a geometric sequence is  $u_n$ , where

$$u_n = 3 \times 4^n$$

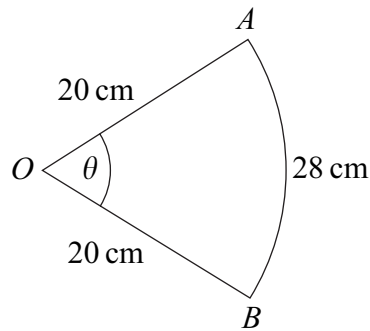
(a) Find the value of  $u_1$  and show that  $u_2 = 48.$  (2 marks)

(b) Write down the common ratio of the geometric sequence. (1 mark)

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is  $4^k - 4$ , where  $k$  is an integer. (3 marks)

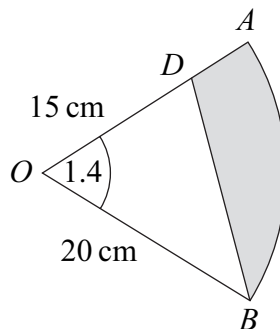
(ii) Hence find the value of  $\sum_{n=2}^{12} u_n.$  (1 mark)

- 3 The diagram shows a sector  $OAB$  of a circle with centre  $O$  and radius 20 cm. The angle between the radii  $OA$  and  $OB$  is  $\theta$  radians.



The length of the arc  $AB$  is 28 cm.

- Show that  $\theta = 1.4$ . (2 marks)
- Find the area of the sector  $OAB$ . (2 marks)
- The point  $D$  lies on  $OA$ . The region bounded by the line  $BD$ , the line  $DA$  and the arc  $AB$  is shaded.



The length of  $OD$  is 15 cm.

- Find the area of the shaded region, giving your answer to three significant figures. (3 marks)
- Use the cosine rule to calculate the length of  $BD$ , giving your answer to three significant figures. (3 marks)

- 4 An arithmetic series has first term  $a$  and common difference  $d$ .

The sum of the first 29 terms is 1102.

- (a) Show that  $a + 14d = 38$ . (3 marks)

- (b) The sum of the second term and the seventh term is 13.

Find the value of  $a$  and the value of  $d$ . (4 marks)

- 5 A curve is defined for  $x > 0$  by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point  $P$  lies on the curve where  $x = 2$ .

- (a) Find the  $y$ -coordinate of  $P$ . (1 mark)

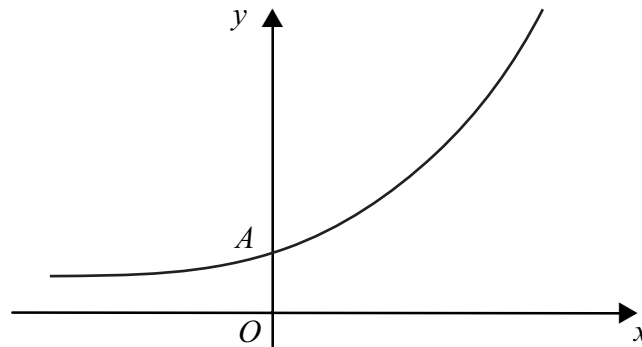
- (b) Expand  $\left(1 + \frac{2}{x}\right)^2$ . (2 marks)

- (c) Find  $\frac{dy}{dx}$ . (3 marks)

- (d) Hence show that the gradient of the curve at  $P$  is  $-2$ . (2 marks)

- (e) Find the equation of the normal to the curve at  $P$ , giving your answer in the form  $x + by + c = 0$ , where  $b$  and  $c$  are integers. (4 marks)

- 6 The diagram shows a sketch of the curve with equation  $y = 3(2^x + 1)$ .



The curve  $y = 3(2^x + 1)$  intersects the  $y$ -axis at the point  $A$ .

- (a) Find the  $y$ -coordinate of the point  $A$ . (2 marks)
- (b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for  $\int_0^6 3(2^x + 1) dx$ . (4 marks)
- (c) The line  $y = 21$  intersects the curve  $y = 3(2^x + 1)$  at the point  $P$ .
- (i) Show that the  $x$ -coordinate of  $P$  satisfies the equation  $2^x = 6$  (1 mark)
- (ii) Use logarithms to find the  $x$ -coordinate of  $P$ , giving your answer to three significant figures. (3 marks)

**Turn over for the next question**

- 7 (a) Sketch the graph of  $y = \tan x$  for  $0^\circ \leq x \leq 360^\circ$ . (3 marks)
- (b) Write down the **two** solutions of the equation  $\tan x = \tan 61^\circ$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (2 marks)
- (c) (i) Given that  $\sin \theta + \cos \theta = 0$ , show that  $\tan \theta = -1$ . (1 mark)
- (ii) Hence solve the equation  $\sin(x - 20^\circ) + \cos(x - 20^\circ) = 0$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (4 marks)
- (d) Describe the single geometrical transformation that maps the graph of  $y = \tan x$  onto the graph of  $y = \tan(x - 20^\circ)$ . (2 marks)
- (e) The curve  $y = \tan x$  is stretched in the  $x$ -direction with scale factor  $\frac{1}{4}$  to give the curve with equation  $y = f(x)$ . Write down an expression for  $f(x)$ . (1 mark)

- 8 (a) It is given that  $n$  satisfies the equation

$$\log_a n = \log_a 3 + \log_a (2n - 1)$$

Find the value of  $n$ . (3 marks)

- (b) Given that  $\log_a x = 3$  and  $\log_a y - 3 \log_a 2 = 4$ :

(i) express  $x$  in terms of  $a$ ; (1 mark)

(ii) express  $xy$  in terms of  $a$ . (4 marks)

**END OF QUESTIONS**

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## Q1

(4)

**(Total 4 marks)**

$$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(2)

(4)

**(Total 6 marks)**

**Q2**

3

**Turn over**



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**Question 3 continued**

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**Q3**

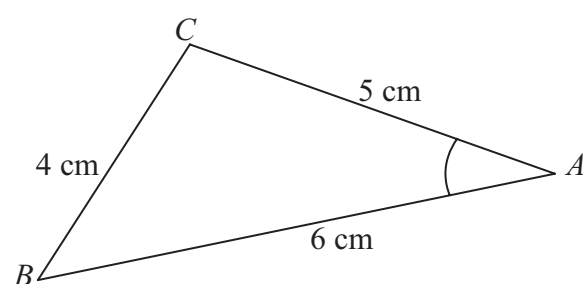
**(Total 6 marks)**

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**Figure 1**

Figure 1 shows the triangle  $ABC$ , with  $AB = 6$  cm,  $BC = 4$  cm and  $CA = 5$  cm.

- (a) Show that  $\cos A = \frac{3}{4}$ .

(3)

- (b) Hence, or otherwise, find the exact value of  $\sin A$ .

(2)

Q4

1

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5. The curve  $C$  has equation

$$y = x\sqrt[3]{(x^3 + 1)}, \quad 0 \leq x \leq 2.$$

(a) Complete the table below, giving the values of  $y$  to 3 decimal places at  $x = 1$  and  $x = 1.5$ .

$x$	0	0.5	1	1.5	2
$y$	0	0.530			6

(2)

(b) Use the trapezium rule, with all the  $y$  values from your table, to find an approximation for the value of  $\int_0^2 x\sqrt[3]{(x^3 + 1)}dx$ , giving your answer to 3 significant figures.

(4)

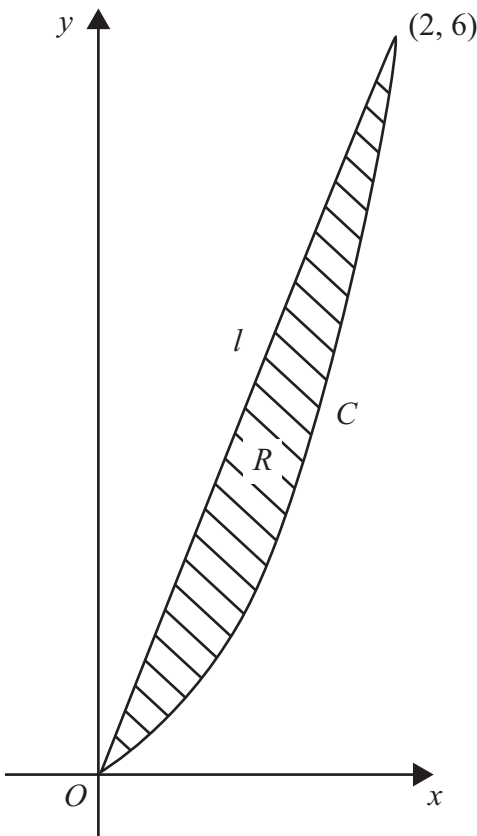


Figure 2

Figure 2 shows the curve  $C$  with equation  $y = x\sqrt[3]{(x^3 + 1)}$ ,  $0 \leq x \leq 2$ , and the straight line segment  $l$ , which joins the origin and the point  $(2, 6)$ . The finite region  $R$  is bounded by  $C$  and  $l$ .

(c) Use your answer to part (b) to find an approximation for the area of  $R$ , giving your answer to 3 significant figures.

(3)

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### Q5





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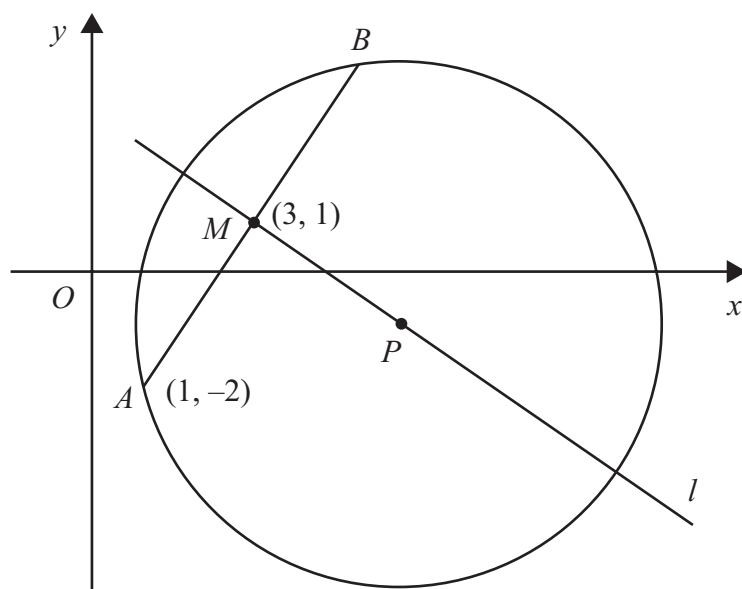
**Q6**

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7.



### Figure 3

The points  $A$  and  $B$  lie on a circle with centre  $P$ , as shown in Figure 3.  
The point  $A$  has coordinates  $(1, -2)$  and the mid-point  $M$  of  $AB$  has coordinates  $(3, 1)$ .  
The line  $l$  passes through the points  $M$  and  $P$ .

- (a) Find an equation for  $l$ . (4)

Given that the  $x$ -coordinate of  $P$  is 6,

- (b) use your answer to part (a) to show that the  $y$ -coordinate of  $P$  is  $-1$ , (1)

- (c) find an equation for the circle. **(4)**

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**Q7**

8. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio  $r$ ,  $r > 1$ .

The model therefore predicts that in 2007 (Year 2) a profit of £50 000 $r$  will be made.

(a) Write down an expression for the predicted profit in Year  $n$ .

(1)

The model predicts that in Year  $n$ , the profit made will exceed £200 000.

(b) Show that  $n > \frac{\log 4}{\log r} + 1$ .

(3)

Using the model with  $r = 1.09$ ,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

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**Q8**

**Turn over**



9. (a) Sketch, for  $0 \leq x \leq 2\pi$ , the graph of  $y = \sin\left(x + \frac{\pi}{6}\right)$ .

(2)

(b) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

(c) Solve, for  $0 \leq x \leq 2\pi$ , the equation
$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$
giving your answers in radians to 2 decimal places.

(5)

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**Question 9 continued**

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**Q9**

**(Total 10 marks)**

21

**Turn over**

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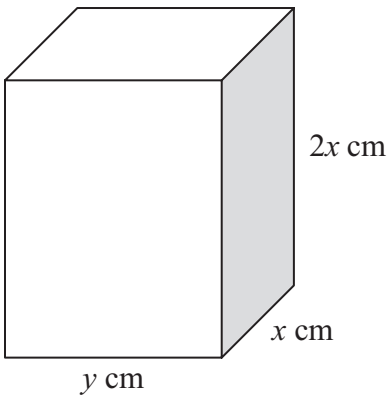


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring  $2x$  cm by  $x$  cm by  $y$  cm.  
The total surface area of the brick is  $600\text{ cm}^2$ .

(a) Show that the volume,  $V\text{ cm}^3$ , of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that  $x$  can vary,

(b) use calculus to find the maximum value of  $V$ , giving your answer to the nearest  $\text{cm}^3$ .  
(5)

(c) Justify that the value of  $V$  you have found is a maximum.  
(2)

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**Q10**

**TOTAL FOR PAPER: 75 MARKS**

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## Practice 3

2

- 1 A geometric progression  $u_1, u_2, u_3, \dots$  is defined by

$$u_1 = 15 \quad \text{and} \quad u_{n+1} = 0.8u_n \text{ for } n \geq 1.$$

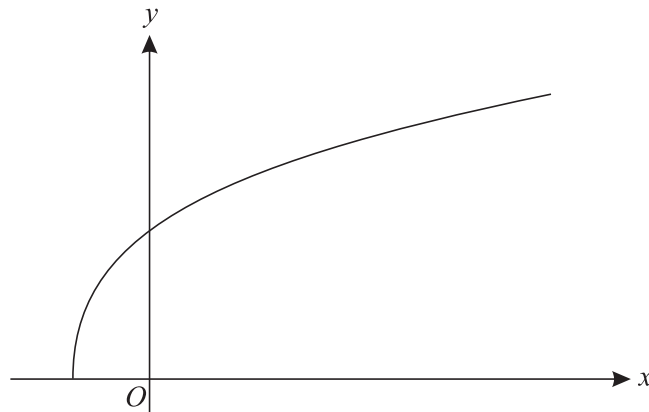
- (i) Write down the values of  $u_2, u_3$  and  $u_4$ . [2]

- (ii) Find  $\sum_{n=1}^{20} u_n$ . [3]

- 2 Expand  $\left(x + \frac{2}{x}\right)^4$  completely, simplifying the terms. [5]

- 3 Use logarithms to solve the equation  $3^{2x+1} = 5^{200}$ , giving the value of  $x$  correct to 3 significant figures. [5]

4



The diagram shows the curve  $y = \sqrt{4x + 1}$ .

- (i) Use the trapezium rule, with strips of width 0.5, to find an approximate value for the area of the region bounded by the curve  $y = \sqrt{4x + 1}$ , the  $x$ -axis, and the lines  $x = 1$  and  $x = 3$ . Give your answer correct to 3 significant figures. [4]

- (ii) State with a reason whether this approximation is an under-estimate or an over-estimate. [2]

- 5 (i) Show that the equation

$$3 \cos^2 \theta = \sin \theta + 1$$

can be expressed in the form

$$3 \sin^2 \theta + \sin \theta - 2 = 0. \quad [2]$$

- (ii) Hence solve the equation

$$3 \cos^2 \theta = \sin \theta + 1,$$

giving all values of  $\theta$  between  $0^\circ$  and  $360^\circ$ . [5]

6 (a) (i) Find  $\int x(x^2 - 4) dx$ . [3]

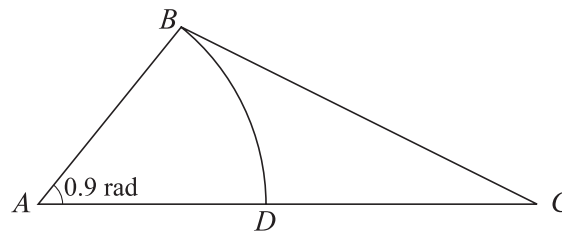
(ii) Hence evaluate  $\int_1^6 x(x^2 - 4) dx$ . [2]

(b) Find  $\int \frac{6}{x^3} dx$ . [3]

7 (a) In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

(b) In a geometric progression, the second term is  $-4$  and the sum to infinity is 9. Find the common ratio. [7]

8



The diagram shows a triangle  $ABC$ , where angle  $BAC$  is 0.9 radians.  $BAD$  is a sector of the circle with centre  $A$  and radius  $AB$ .

(i) The area of the sector  $BAD$  is  $16.2 \text{ cm}^2$ . Show that the length of  $AB$  is 6 cm. [2]

(ii) The area of triangle  $ABC$  is twice the area of sector  $BAD$ . Find the length of  $AC$ . [3]

(iii) Find the perimeter of the region  $BCD$ . [6]

9 The polynomial  $f(x)$  is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) (a) Show that  $(x + 1)$  is a factor of  $f(x)$ . [1]

(b) Hence find the exact roots of the equation  $f(x) = 0$ . [6]

(ii) (a) Show that the equation

$$2 \log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

can be written in the form  $f(x) = 0$ . [5]

(b) Explain why the equation

$$2 \log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

has only one real root and state the exact value of this root. [2]

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Answer **all** questions.

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- 1 (a) Write  $\sqrt{x^3}$  in the form  $x^k$ , where  $k$  is a fraction. (1 mark)

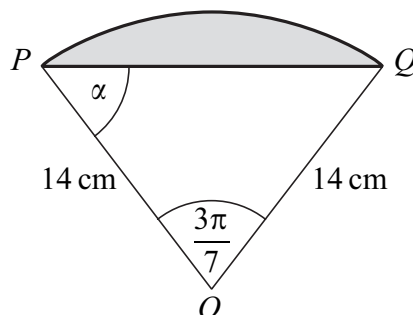
- (b) A curve, defined for  $x \geq 0$ , has equation

$$y = x^2 - \sqrt{x^3}$$

- (i) Find  $\frac{dy}{dx}$ . (3 marks)

- (ii) Find the equation of the tangent to the curve at the point where  $x = 4$ , giving your answer in the form  $y = mx + c$ . (5 marks)

- 2 The diagram shows a shaded segment of a circle with centre  $O$  and radius 14 cm, where  $PQ$  is a chord of the circle.



In triangle  $OPQ$ , angle  $POQ = \frac{3\pi}{7}$  radians and angle  $OPQ = \alpha$  radians.

- (a) Find the length of the arc  $PQ$ , giving your answer as a multiple of  $\pi$ . (2 marks)
- (b) Find  $\alpha$  in terms of  $\pi$ . (2 marks)
- (c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. (2 marks)

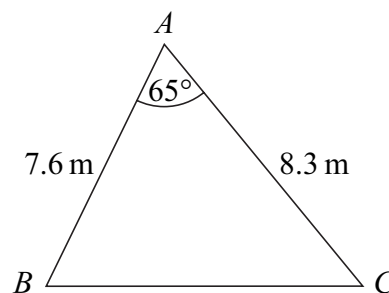


3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \dots$$

- (a) Find the common ratio of the series. (1 mark)
- (b) Find the sum to infinity of the series. (2 marks)
- (c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. (2 marks)
- (d) Prove that the  $n$ th term of the series is  $25 \times 0.8^n$ . (2 marks)

4 The diagram shows a triangle  $ABC$ .



The size of angle  $BAC$  is  $65^\circ$ , and the lengths of  $AB$  and  $AC$  are 7.6 m and 8.3 m respectively.

- (a) Show that the length of  $BC$  is 8.56 m, correct to three significant figures. (3 marks)
- (b) Calculate the area of triangle  $ABC$ , giving your answer in  $\text{m}^2$  to three significant figures. (2 marks)
- (c) The perpendicular from  $A$  to  $BC$  meets  $BC$  at the point  $D$ .

Calculate the length of  $AD$ , giving your answer to the nearest 0.1 m. (3 marks)

5 (a) Write down the value of:

(i)  $\log_a 1$ ; (1 mark)

(ii)  $\log_a a$ . (1 mark)

- (b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of  $x$ . (3 marks)

- 6 The  $n$ th term of a sequence is  $u_n$ .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where  $p$  and  $q$  are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \quad u_2 = 8 \quad u_3 = 4$$

- (a) Show that  $q = 6$  and find the value of  $p$ . (5 marks)
- (b) Find the value of  $u_4$ . (1 mark)
- (c) The limit of  $u_n$  as  $n$  tends to infinity is  $L$ .
- (i) Write down an equation for  $L$ . (1 mark)
- (ii) Hence find the value of  $L$ . (2 marks)

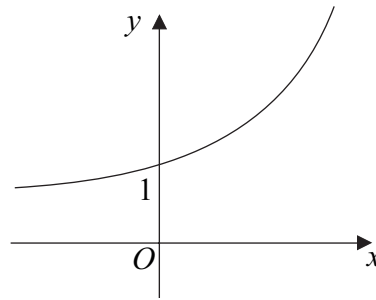
- 7 (a) The expression  $\left(1 + \frac{4}{x^2}\right)^3$  can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers  $p$  and  $q$ . (3 marks)

- (b) (i) Hence find  $\int \left(1 + \frac{4}{x^2}\right)^3 dx$ . (4 marks)
- (ii) Hence find the value of  $\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 dx$ . (2 marks)

- 8 The diagram shows a sketch of the curve with equation  $y = 6^x$ .



- (a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for  $\int_0^2 6^x \, dx$ , giving your answer to three significant figures. (4 marks)
- (ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of  $\int_0^2 6^x \, dx$ . (2 marks)
- (b) (i) Describe a single geometrical transformation that maps the graph of  $y = 6^x$  onto the graph of  $y = 6^{3x}$ . (2 marks)
- (ii) The line  $y = 84$  intersects the curve  $y = 6^{3x}$  at the point  $A$ . By using logarithms, find the  $x$ -coordinate of  $A$ , giving your answer to three decimal places. (4 marks)
- (c) The graph of  $y = 6^x$  is translated by  $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$  to give the graph of the curve with equation  $y = f(x)$ . Write down an expression for  $f(x)$ . (2 marks)
- 9 (a) Solve the equation  $\sin 2x = \sin 48^\circ$ , giving the values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ . (4 marks)
- (b) Solve the equation  $2 \sin \theta - 3 \cos \theta = 0$  in the interval  $0^\circ \leq \theta < 360^\circ$ , giving your answers to the nearest  $0.1^\circ$ . (4 marks)

**END OF QUESTIONS**

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**1.**

(a) Use the factor theorem to show that  $(x + 4)$  is a factor of  $f(x)$ .

(b) Factorise  $f(x)$  completely.

(4)

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**Q1**

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## Q2

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3. (a) Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 + ax)^{10}$ , where  $a$  is a non-zero constant. Give each term in its simplest form. (4)

Given that, in this expansion, the coefficient of  $x^3$  is double the coefficient of  $x^2$ ,

- (b) find the value of  $a$ . (2)

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Q4

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5. The circle  $C$  has centre  $(3, 1)$  and passes through the point  $P(8, 3)$ .

(a) Find an equation for  $C$ .

(4)

(b) Find an equation for the tangent to  $C$  at  $P$ , giving your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers.

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**Q5**

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**Q6**

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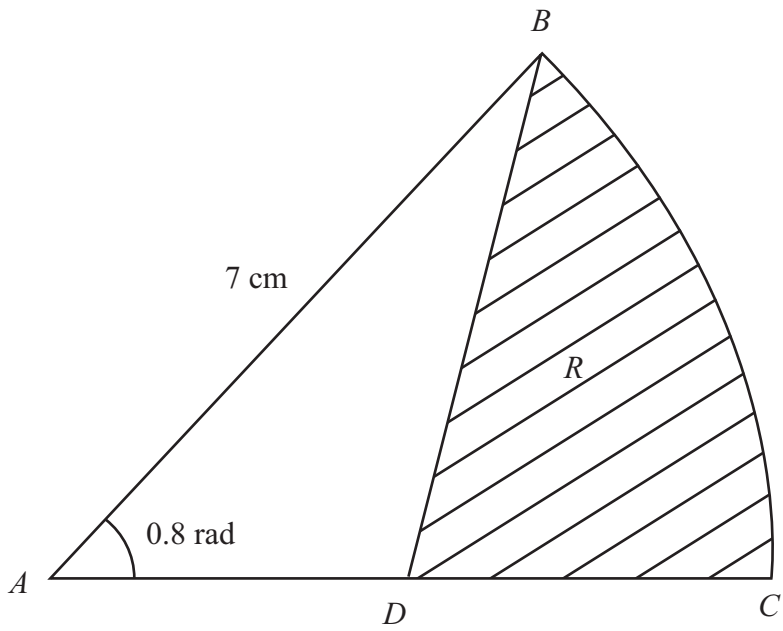


Figure 1

Figure 1 shows  $ABC$ , a sector of a circle with centre  $A$  and radius 7 cm.

Given that the size of  $\angle BAC$  is exactly 0.8 radians, find

- (a) the length of the arc  $BC$ , (2)
- (b) the area of the sector  $ABC$ . (2)

The point  $D$  is the mid-point of  $AC$ . The region  $R$ , shown shaded in Figure 1, is bounded by  $CD$ ,  $DB$  and the arc  $BC$ .

Find

- (c) the perimeter of  $R$ , giving your answer to 3 significant figures, (4)
- (d) the area of  $R$ , giving your answer to 3 significant figures. (4)

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**Q7**

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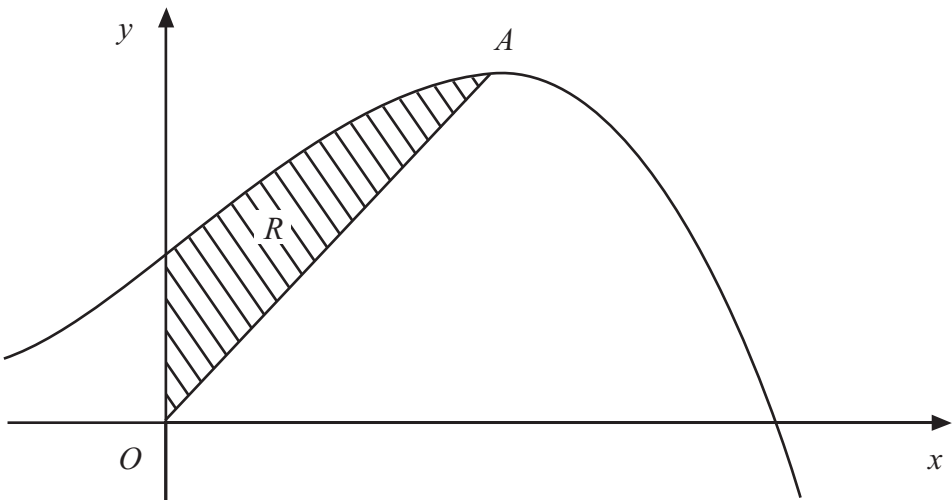


Figure 2

Figure 2 shows a sketch of part of the curve with equation  $y = 10 + 8x + x^2 - x^3$ .

The curve has a maximum turning point  $A$ .

- (a) Using calculus, show that the  $x$ -coordinate of  $A$  is 2. (3)

The region  $R$ , shown shaded in Figure 2, is bounded by the curve, the  $y$ -axis and the line from  $O$  to  $A$ , where  $O$  is the origin.

- (b) Using calculus, find the exact area of  $R$ . (8)

**Question 8 continued**

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**Question 8 continued**

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**Q8**

**(Total 11 marks)**

23

**Turn over**

9. Solve, for  $0 \leq x < 360^\circ$ ,

(a)  $\sin(x - 20^\circ) = \frac{1}{\sqrt{2}}$

(b)  $\cos 3x = -\frac{1}{2}$

(4)

(6)

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**Question 9 continued**

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This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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**Practice 6****2**

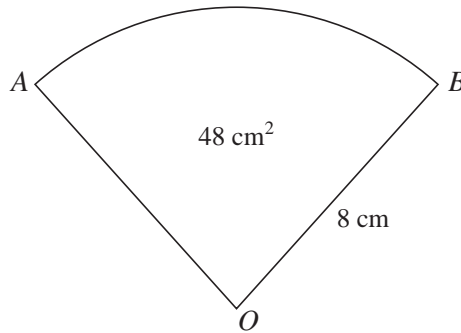
**1** Find and simplify the first three terms in the expansion of  $(2 - 3x)^6$  in ascending powers of  $x$ . [4]

**2** A sequence  $u_1, u_2, u_3, \dots$  is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = 1 - \frac{1}{u_n} \quad \text{for } n \geq 1.$$

(i) Write down the values of  $u_2, u_3$  and  $u_4$ . [3]

(ii) Describe the behaviour of the sequence. [1]

**3**

The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius 8 cm. The area of the sector is  $48 \text{ cm}^2$ .

(i) Find angle  $AOB$ , giving your answer in radians. [2]

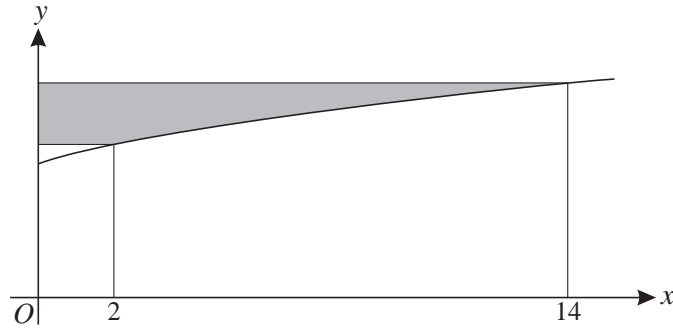
(ii) Find the area of the segment bounded by the arc  $AB$  and the chord  $AB$ . [3]

**4** The cubic polynomial  $ax^3 - 4x^2 - 7ax + 12$  is denoted by  $f(x)$ .

(i) Given that  $(x - 3)$  is a factor of  $f(x)$ , find the value of the constant  $a$ . [3]

(ii) Using this value of  $a$ , find the remainder when  $f(x)$  is divided by  $(x + 2)$ . [2]

5



The diagram shows the curve  $y = 3 + \sqrt{x+2}$ .

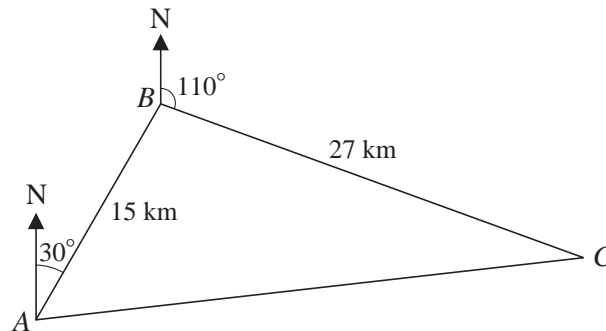
The shaded region is bounded by the curve, the  $y$ -axis, and two lines parallel to the  $x$ -axis which meet the curve where  $x = 2$  and  $x = 14$ .

(i) Show that the area of the shaded region is given by

$$\int_5^7 (y^2 - 6y + 7) dy. \quad [3]$$

(ii) Hence find the exact area of the shaded region. [4]

6



In the diagram, a lifeboat station is at point  $A$ . A distress call is received and the lifeboat travels 15 km on a bearing of  $030^\circ$  to point  $B$ . A second call is received and the lifeboat then travels 27 km on a bearing of  $110^\circ$  to arrive at point  $C$ . The lifeboat then travels back to the station at  $A$ .

(i) Show that angle  $ABC$  is  $100^\circ$ . [1]

(ii) Find the distance that the lifeboat has to travel to get from  $C$  back to  $A$ . [2]

(iii) Find the bearing on which the lifeboat has to travel to get from  $C$  to  $A$ . [4]

7 (a) Find  $\int x^3(x^2 - x + 5) dx$ . [4]

(b) (i) Find  $\int 18x^{-4} dx$ . [2]

(ii) Hence evaluate  $\int_2^\infty 18x^{-4} dx$ . [2]

[Turn over]

- 8 (i) Sketch the curve  $y = 2 \times 3^x$ , stating the coordinates of any intersections with the axes. [3]
- (ii) The curve  $y = 2 \times 3^x$  intersects the curve  $y = 8^x$  at the point  $P$ . Show that the  $x$ -coordinate of  $P$  may be written as

$$\frac{1}{3 - \log_2 3}. \quad [5]$$

- 9 (a) (i) Show that the equation

$$2 \sin x \tan x - 5 = \cos x$$

can be expressed in the form

$$3 \cos^2 x + 5 \cos x - 2 = 0. \quad [3]$$

- (ii) Hence solve the equation

$$2 \sin x \tan x - 5 = \cos x,$$

giving all values of  $x$ , in radians, for  $0 \leq x \leq 2\pi$ . [4]

- (b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \cos x \, dx,$$

where  $x$  is in radians. Give your answer correct to 3 significant figures. [4]

- 10 Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

- (i) Find how far Jamie runs on Day 15. [2]

- (ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

- (iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

- (iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]