Edexcel-AS/A-Level Physics

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AS-Unit-1-Physics-on-the-go

1 Mechanics

Chapter 1 Scalars and Vectors

1.1 Definition of scalars and vectors

Scalar: quantity has direction only.

Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

Examples of vectors: force, acceleration, displacement, velocity, momentum…

Representation of vectors: any vectors can be represented by a straight line with an arrow whose length represents the magnitude of the vectors, and the direction of the arrow gives the direction of the vectors.

Vector Notation: use an arrow $\vec{A}$, $\vec{S}$, $\vec{B}$…

Or use the bold letter **A, B, S**…

When considering the magnitude of a vector only, we can use the italic letter *A, B, S*…

1.2 Addition of vectors:

When adding vectors, the units of the vectors must be the same, the direction must be taken into account.

Addition Principles:

i : If two vectors are in the same direction: the magnitude of the resultant vector is equal to the sum of their magnitudes, in the same direction.

ii : If two vectors are in the opposite direction: the magnitude of the resultant vector is equal to the difference of the magnitude of the two vectors and is in the direction of the greater vector.

iii: If two vectors are placed tail-to-tail at an angle θ, it can also be represented as a closed triangle (**Fig. 1.1**).

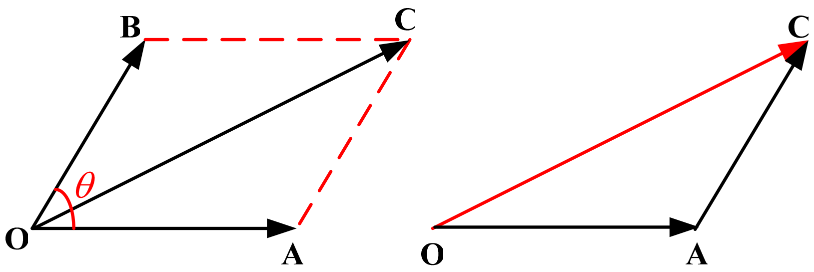


Fig. 1.1 addition principles of vectors

$\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OC}$ Because $\overrightarrow{OB}=\overrightarrow{AC}$

$\overrightarrow{OA}$ *and* $\overrightarrow{OB}$ are placed tail to tail to form two adjacent sides of a parallelogram and the diagonal $\overrightarrow{OC}$ gives the sum of the vectors $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$. This is also called as 'parallelogram rule of vector addition.

Addition Methods:

(i): Graphical Methods----using scale drawings

For example:

$\mathbf{F_1}$ and $\mathbf{F_2}$ are at right angle, and $\mathbf{F_1}$ = 3 N, $\mathbf{F_2}$ = 4 N, determine the resultant force **F** (**Fig. 1.2**).

Let 1cm=1N

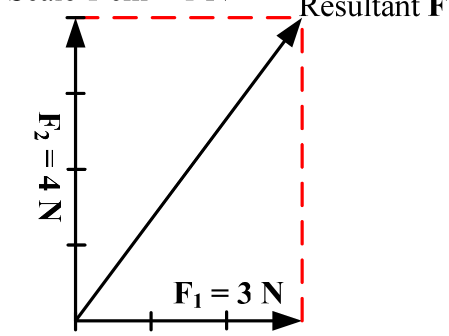


Fig. 1.2 addition methods—scale drawings

Measure the length of the resultant vector, we get length = 5cm, then resultant force, F = 5 N.

(ii) Algebraic Methods

For example:

$\mathbf{F_1}$ and $\mathbf{F_2}$ are at right angle, and $\mathbf{F_1}$ = 3 N, $\mathbf{F_2}$ = 4 N, determine the resultant force **F (Fig. 1.3)**.

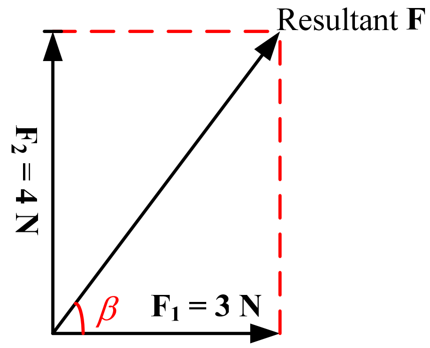


Fig. 1.3 Algebraic methods

Using the Pythagorean Theorem:

Magnitude of the resultant force, $F = \sqrt{F_1^2 + F_2^2} = \sqrt{3^2 + 4^2} = 5N$

The angle β between F and F_1 is given by:

$$\tan\beta = \frac{F_2}{F_1} = \frac{4}{3}$$

Or

$$\sin\beta = \frac{F_2}{F} = \frac{4}{5}$$

Or

$$\cos\beta = \frac{F_1}{F} = \frac{3}{5}$$

1.3 Resolving a vector into two perpendicular components

For example, for a vector $\overrightarrow{OC}$, θ is known, resolving it horizontally and vertically (**Fig. 1.4**).

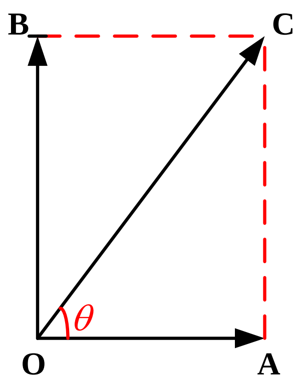


Fig. 1.4 Resolving a vector into two perpendicular components

Magnitude of Horizontally component $OA = OC\cos\theta$

Magnitude of vertically component $OB = OC\sin\theta$

Thus, a force can be resolved into two perpendicular components (**Fig. 1.5**):

F and θ are known.

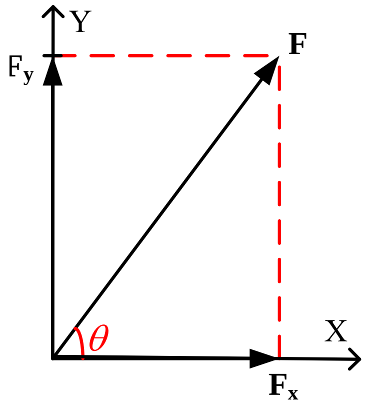


Fig. 1.5 Resolving a force into two perpendicular components

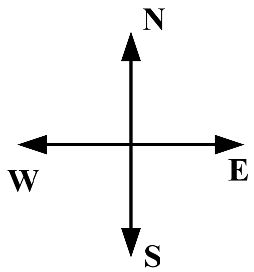
$F_x = F\cos\theta$ $F_y = F\sin\theta$

1.4 10 Worked examples

1. Representation of vectors:

(i) A displacement of 500 m due east

Represent the displacement:



Let scale: $1cm = 100m$

Then

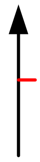
Note: of course you can also let scale: $1cm = 250m$

Then:

(ii) A force of $\vec{F} = 100N$ (or **F**=100N) due north.

Let scale: $1cm = 50N$

Then



2. Addition of the vectors

(i) are in the same direction.

Magnitude of the resultant $\mathrm{F} = F_1 + F_2 = 11N$

Direction: the same direction of $\vec{F}_1$ *and* $\vec{F}_2$

(ii) $\vec{F}_1$ *and* $\vec{F}_2$ are in the opposite direction.

Magnitude of the resultant $\mathrm{F} = F_2 - F_1 = 4N$

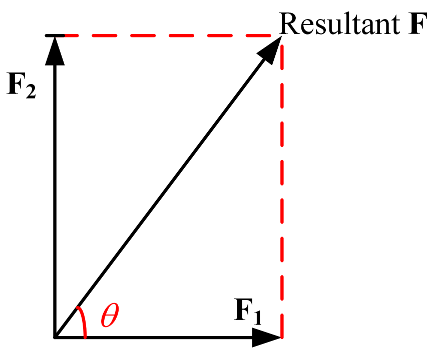
Direction: the same direction of $\vec{F}_2$

(iii) $\vec{F}_1$ *and* $\vec{F}_2$ are at right angles to each other.

Using the algebraic methods:

Magnitude of the resultant:

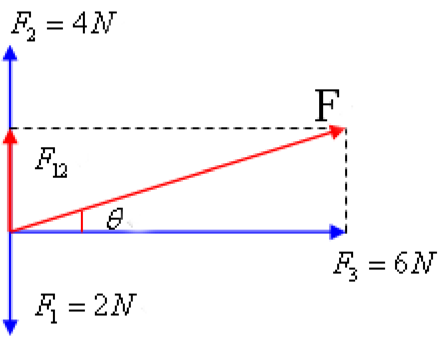
$$\mathrm{F} = \sqrt{F_1^{\,2} + F_2^{\,2}} = \sqrt{12.25 + 68.5} = 9N$$



Direction:

$$\tan\theta = \frac{F_2}{F_1} = \frac{7.5}{3.5} = 2.14 \quad Then \quad \theta = \arctan 2.14$$

3. Calculate the resultant force of F_1, F_2, F_3



Strategy: ① calculate the resultant of F_1 *and* F_2

$F_{12} = F_2 - F_1 = 2N$

② calculate the resultant force of F_{12} *and* F_3, that is the resultant force of F_1, F_2 *and* F_3

Magnitude of resultant force:

$$F = \sqrt{F_{12}^2 + F_3^2} = \sqrt{2^2 + 6^2} = 6.32N$$

Direction:

$$\tan\theta = \frac{F_{12}}{F_3} = \frac{2}{6} = \frac{1}{3}$$

$$\theta = \arctan\frac{1}{3}$$

4. A crane is used to raise one end of a steel girder off the ground, as shown in **Fig. 4.1**. When the cable attached to the end of the girder is at 20°to the vertical, the force of the cable on the girder is 6.5kN. Calculate the horizontal and vertical components of this force.

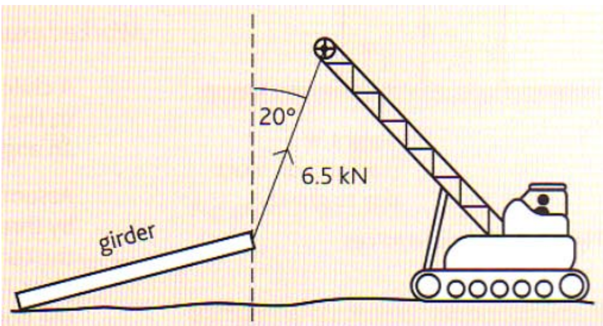
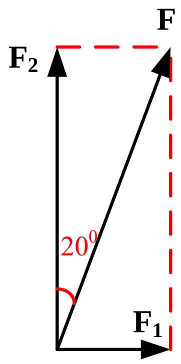


Fig. 4.1

Strategy:

Resolving the force F = 6.5 kN



$F_1 = F\sin 20^o = 6.5\sin 20^o = 2.2kN$ (Horizontal components of the force)

$F_2 = F\cos 20^o = 6.5\cos 20^o = 6.1kN$ (Vertical components of the force)

5. (a) (i) State what is meant by a scalar quantity.

Scalar quantity: quantity has direction only.

(ii) State two examples of scalar quantities.

Example 1: mass

Example 2: temperatures

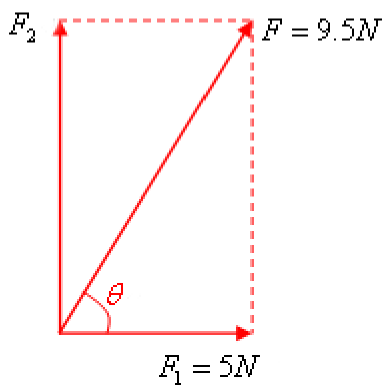
(b) An object is acted upon by two forces at right angles to each other. One of the forces has a magnitude of 5.0 N and the resultant force produced on the object is 9.5 N.

Determine

(i) The magnitude of the other force,

Strategy: adding of vectors, using the Algebraic Methods

Draw the forces below:



And $F_1^{\,2} + F_2^{\,2} = F^2$

$5^2 + F_2^{\,2} = 9.5^2$

So, $F_2 = 8.1N$

(ii) The angle between the resultant force and the 5.0 N force.

$$\cos\theta = \frac{F_1}{F} = \frac{5}{9.5} = 0.53$$

So $\theta = \arccos 0.53 = 58^0$

6. (a) State the difference between vector and scalar quantities.

Answers: Vector quantities have direction and scalar quantities do not.

(b) State one example of a vector quantity (other than force) and one example of a scalar quantity.

Vector quantity: velocity, acceleration.

Scalar quantity: mass, temperature.

(c) A 12.0 N force and a 8.0 N force act on a body of mass 6.5 kg at the same time. For this body, calculate

(i) The maximum resultant acceleration that it could experience,

Strategy: by the Newton's second law, $F = ma$, the maximum resultant acceleration when the body has the maximum resultant force. And when the two forces are at the same direction, the body has the maximum resultant force.

So, resultant force, $F = F_1 + F_2 = 12 + 8 = 20N$

So the maximum resultant acceleration, $a = \frac{F}{m} = \frac{20}{6.5} = 3.1ms^{-2}$

(ii) The minimum resultant acceleration that it could experience.

Strategy: by the Newton's second law, $F = ma$, the minimum resultant acceleration when the body has the minimum resultant force. And when the two forces are at the opposite direction, the body has the minimum resultant force.

That is, resultant force, $F = F_1 - F_2 = 12 - 8 = 4N$

So the minimum resultant acceleration, $a = \frac{F}{m} = \frac{4}{6.5} = 0.62ms^{-2}$

7. **Figure 7.1** shows a uniform steel girder being held horizontally by a crane. Two cables are attached to the ends of the girder and the tension in each of these cables is T.

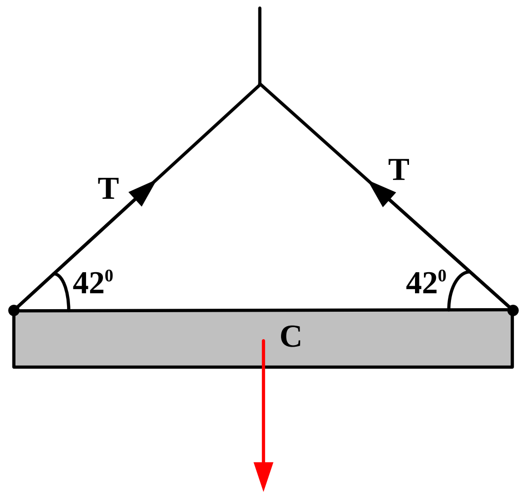


Fig. 7.1

(a) If the tension, T, in each cable is 850 N, calculate

(i) The horizontal component of the tension in each cable,

Answers: $T_h = T\cos 42 = 850 \times \cos 42 = 632\ N$

(ii) The vertical component of the tension in each cable,

$T_v = T\sin 42 + T\sin 42 = 1138N$

(iii) The weight of the girder.

Strategy: the girder is at a uniform state, so the weight of the girder is equal to the vertical component of the tension.

So weight, $W = T_v = 1138N$

(b) On **Figure** 1.**7** draw an arrow to show the line of action of the weight of the girder.

8. Which of the following contains three scalar quantities?

A	Mass	Charge	Speed
B	Density	Weight	Mass
C	Speed	Weight	Charge
D	Charge	Weight	Density

Solution:

Scalar: quantity has direction only.

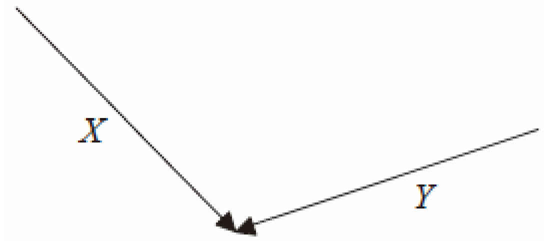
Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

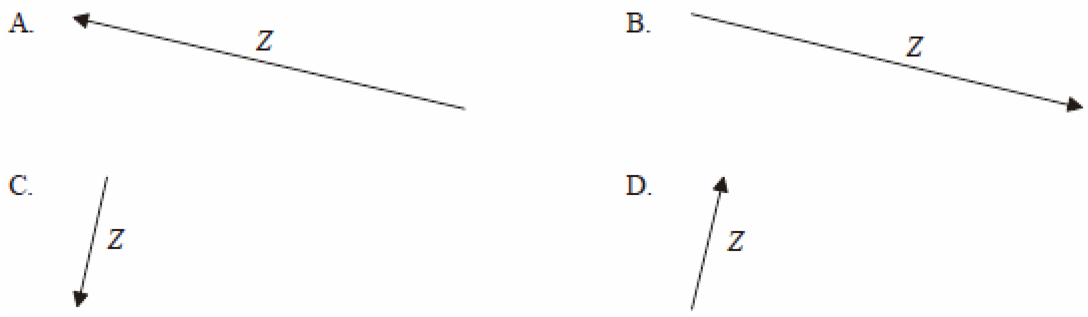
Examples of vectors: force, acceleration, displacement, velocity, momentum…

And $weight = m \cdot \vec{g}$ is a vector. Thus choose (A)

9. The diagram below shows two vectors **X** and **Y**.

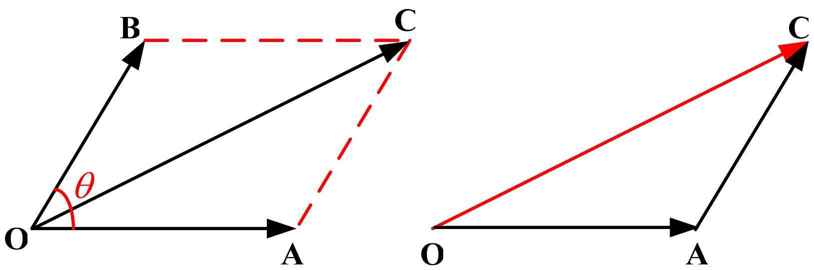


Which of the following best represents the vector **Z** = **X** – **Y**.



Strategy:

If two vectors are placed tail-to-tail at an angle θ, it can also be represented as a closed triangle.

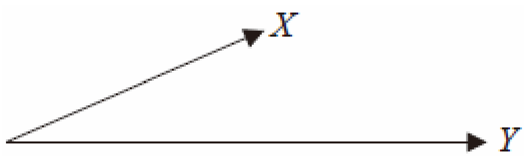


$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$ Because $\overrightarrow{OB} = \overrightarrow{AC}$

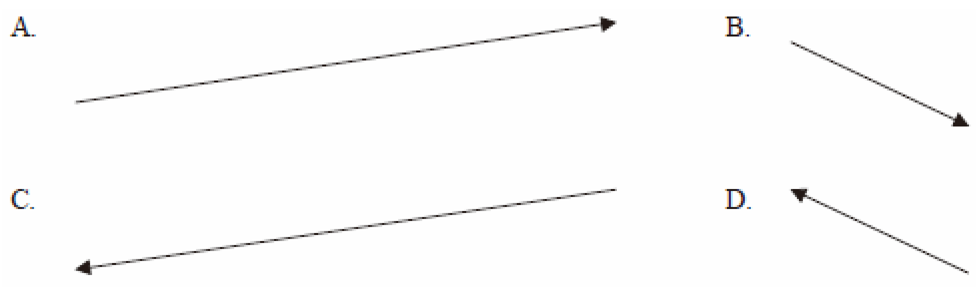
Solution:

And **X** = **Z** + **Y, thus choose (B)**

10. The magnitude and direction of two vectors **X** and **Y** are represented by the vector diagram below.

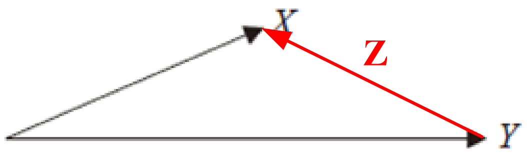


Which of the following best represents the vector (**X**–**Y**)?



Solution:

Let **X** minus **Y** to be **Z**: **X–Y = Z,** thus **X** = **Z** + **Y**



Choose (**D**):

Chapter 2 Rectilinear motion

2.1 Displacement and velocity

Distance: is the magnitude of the path covered, is a scalar.

SI unit: metre (m)

Displacement: the change in position between the starting point and the end point.

SI unit: metre (m)

Displacement is a vector; its direction is from the starting point to end point.

For example:

(i) An ant crawl along the arc that start from O to A (**Fig. 2.1**),

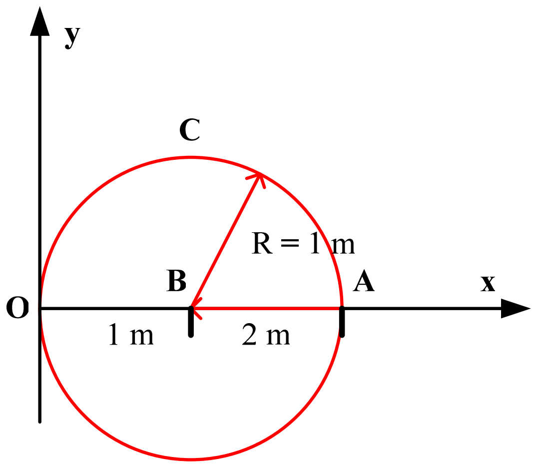


Fig. 2.1

Then:

Distance $= \pi R = \pi = 3.14m$

Displacement $= \overrightarrow{OA} = 2m$

(ii) The ant now goes on crawling from A to B,

Distance $= \overset{\frown}{OCA} + AB = \pi R + 1 = 4.14m$

Displacement $= \overrightarrow{OB} = 1m$

(iii) The ant now goes back from B to O,

Note: the ant start from O then go back to O. that is starting point is O, the end point is O.

Distance $= \overset{\frown}{OCA} + AO = \pi R + 2 = 5.14m$

Displacement $= \overrightarrow{OO} = 0m$

Speed: the distance traveled by a moving object over a period of time.

Constant speed: the moving object doesn't change its speed.

$$(average)speed = \frac{distance}{time\ taken}$$

$$v = \frac{s}{t}$$

Unit: $\mathrm{m/s}$ or ms^{-1}

Velocity: the speed in a given direction.

Average velocity: the change in position (displacement) over a period of time.

$$\vec{v}_{average} = \frac{change\ in\ position}{time\ taken} = \frac{displacement}{time\ taken} = \frac{\Delta\vec{x}}{t} = \frac{\vec{s}}{t}$$

Where $\vec{s}$ is displacement

Unit: $\mathrm{m/s}$ or ms^{-1}

Velocity is a vector; the direction is the same as the direction of the displacement.

Instantaneous velocity: the velocity that the moving object has at any one instance

2.2 Acceleration

Changing velocity (non-uniform) means an acceleration is present.

We can define acceleration as the change of velocity per unit time.

Uniform acceleration: the acceleration is constant, means the velocity of the moving object changes the same rate.

Average acceleration: change in velocity over a period of time.

$$\text{Average acceleration} = \frac{change\ in\ velocity}{time\ taken}$$

In symbol:

$$\vec{a}_{average} = \frac{\Delta \vec{v}}{t} = \frac{v-u}{t}$$

Where, v is the final velocity, u is the initial velocity.

SI unit: Meters per second squared (m/s^2)

Acceleration is a vector; the direction is the same as the direction of the change of velocity.

2.3 Equations for uniform acceleration

Consider a body is moving along a straight line with uniform acceleration, and its velocity increases from *u* (initial velocity) to *v* (final velocity) in time *t*.

First equation:

$$\text{acceleration} = \frac{change\ in\ velocity}{time\ taken}$$

$$a = \frac{v-u}{t}$$

So

$$at = v - u \quad \text{or} \quad v = u + at \ldots\ldots \text{①}$$

Second equation:

$$average\ velocity = \frac{change\ in\ position}{time\ taken} = \frac{displacement}{time\ taken}$$

$$\bar{v} = \frac{\Delta \vec{x}}{t} = \frac{\vec{s}}{t}$$

Because the body is moving along a straight line in one direction, the magnitude of the displacement is equal to the distance.
And for the acceleration is uniform,

$$the\ average\ velocity, \bar{v} = \frac{v+u}{2}$$

So

$$\bar{v} = \frac{v+u}{2} = \frac{s}{t} \quad \text{or} \quad s = \frac{(v+u)}{2}t \;\ldots\ldots ②$$

Third equation:

From equation ①, $v = u + at$ and equation ②, $s = \frac{(v+u)}{2}t$

$$s = \frac{(u+at+u)}{2}t = ut + \frac{1}{2}at^2 \;\ldots\ldots ③$$

Fourth equation:

From equation, $v = u + at$

We get:

$v^2 = (u+at)^2$

$v^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

But $s = ut + \frac{1}{2}at^2$

So

$$v^2 = u^2 + 2as \quad \ldots\ldots ④$$

2.4 Displacement—time graphs with uniform acceleration

Note: for a body moving along a straight line, we can only draw the displacement—time graphs (**Fig. 2.2**)

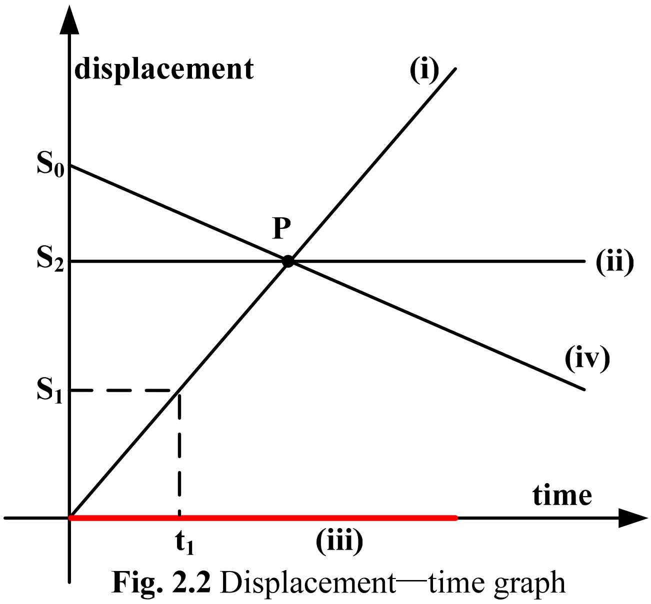


Fig. 2.2 Displacement—time graph

(i) Represents the body moving along a straight line with constant velocity; And the slope or gradient of the displacement—time graph represents the velocity of the body.

(ii) The body keeps rest with displacement S_2.

(iii) The body keeps rest with zero displacement.

(iv) The body moving along the opposite direction with constant velocity and initial displacement S_0.

(v) The point P means the displacement when the objects meeting with each other.

(vi) Displacement of the body is S_1 at time t_1.

2.5 Velocity—time graphs with uniform acceleration

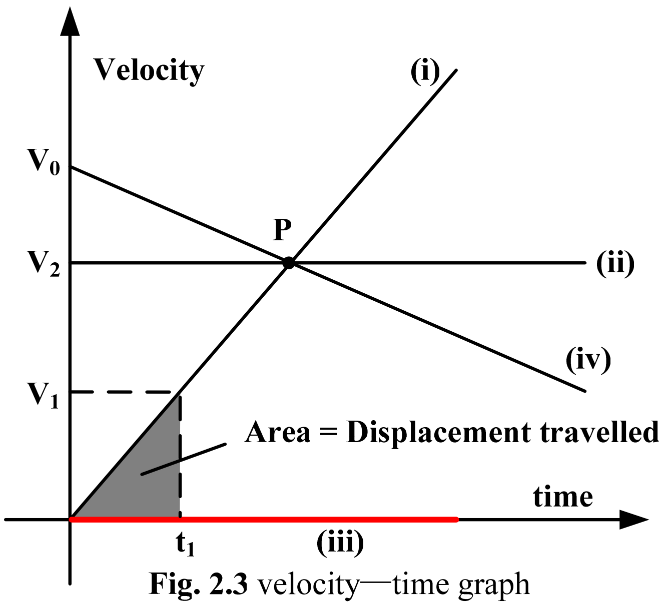


Fig. 2.3 velocity—time graph

(i) represents the body moving along a straight line with constant acceleration; And the slope or gradient of the velocity—time graph represents the acceleration of the body.

(ii) The body moving with constant velocity V_2.

(iii) The body keeps rest with zero velocity.

(iv)The body moving along a straight line with constant deceleration with initial velocity V_0; and the slope or gradient of the velocity—time graph represents the deceleration of the body

(v)The point P means the same velocity when the objects meeting with each other.

(vi)Velocity of the body is V_1 at time t_1 and the area under a velocity—time graph measures the displacement traveled.

2-6 Free-fall motion

6.1 Free-fall motion

The motion of a body that is only acted on by gravity and falls down from rest is called **free-fall motion.** This motion can occur only in a space without air. If air resistance is quite small and neglectable, the falling of a body in the air can also be referred to as a free-fall motion.

Galileo pointed out: free-fall motion is a uniformly accelerated rectilinear motion with zero initial velocity.

6.2 Acceleration of free-fall body

All bodies in free-fall motion have the same acceleration. This acceleration is called **free-fall acceleration** or **gravitational acceleration**. It is usually denoted by g.

The magnitude of gravitational acceleration g / ($m \cdot s^{-2}$)

Standard value: $g = 9.80665 m/s^2$

The direction of gravitational acceleration g is always vertically downward. Its magnitude can be measured through experiments.

Precise experiments show that the magnitude of g varies in different places on the earth. For example, at the equator $g = 9.780 m/s^2$. We take $9.81 m/s^2$ for g in general calculations. In rough calculations, $10 m/s^2$ is used.

As free-fall motion is uniformly accelerated rectilinear motion with zero initial velocity, the fundamental equations and the deductions for uniformly accelerated rectilinear motion are applicable for free-fall motion. What is only needed is to take zero for the initial velocity (u) and replace acceleration a with g.

2.7 Projectile motion

If you throw an object which acted upon only by the force of gravity (neglecting air resistance) with a certain initial velocity, the body will move with only the force of gravity acting on it. Such motion is called **projectile motion**.

(i) Vertical projection

Such an object moves vertically as it has no horizontal motion. Its acceleration is $9.8ms^{-2}$ downwards. Using the direction code '+ is upwards, − is downwards', its displacement, y, and velocity, v, after time t are given by:

$v = u - gt$

$y = ut - \frac{1}{2}gt^2$

Where u is the initial velocity

(ii) Horizontal projection

Think about a particle sent off in a horizontal direction and subject to a vertical gravitational force (its weight). Air resistance is neglected. We will analyze the motion in terms of the horizontal and vertical components of velocity. The particle is projected at time t = 0 at the origin of a system of x-y co-ordinates (**Fig. 2.4**) with velocity u_x in the x direction. Think first about the particle's vertical motion (in the y-direction). Throughout the motion it has an acceleration of g (the acceleration of free fall) in the y-direction. The initial value of the vertical component of velocity is $u_y = 0$. The vertical component increases continuously under the uniform acceleration g. Using v = u + at, its value v_y at time t is given by $v_y = gt$. Also at time t, the vertical displacement y downwards is given by $y = \frac{1}{2}gt^2$. Now for the horizontal motion（in the x-direction）: Here the acceleration is zero, so the horizontal component of velocity remains constant at u_x. at time t the horizontal displacement x is given by $x = u_x t$. To find the velocity of the particle at any time t, the two components v_x and v_y must be added vectorially. The direction of the resultant vector is the direction of motion of the particle the curve traced out by a particle subject to a constant force in one direction is a parabola.

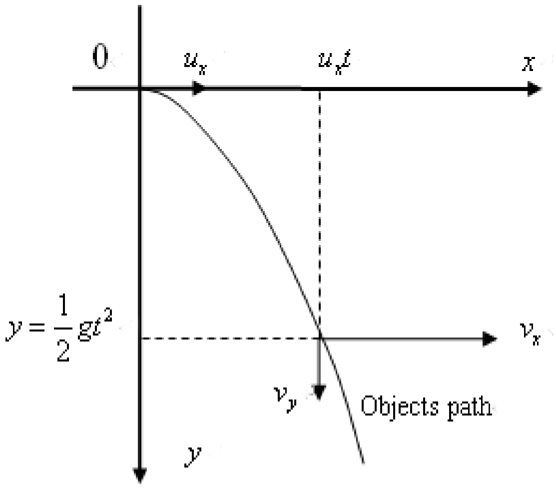


Fig. 2.4

2.8 20 Worked examples

1. An aero plane taking off accelerates uniformly on a runway from a velocity of $3ms^{-1}$ to a velocity of $90ms^{-1}$ in 45s.

Calculate:

(i) Its acceleration.

(ii) The distance on the runway.

Solution: data: $u = 3ms^{-1}$ $v = 93ms^{-1}$ $t = 45s$

Strategy: $v = u + at \Rightarrow a = \frac{v-u}{t}$, $s = ut + \frac{1}{2}at^2$

Answers:

$$a = \frac{v-u}{t} = \frac{93-3}{45} = 2ms^{-1}$$

$$s = ut + \frac{1}{2}at^2 = 3\times 45 + \frac{1}{2}\times 2\times 45^2 = 2160m = 2.16km$$

2. A car accelerates uniformly from a velocity of $15ms^{-1}$ to a velocity of $25ms^{-1}$ with a distance of 125m.

Calculate:

(i) Its acceleration

(ii) The time taken

Solution:

Data: $u = 15ms^{-1}$ $v = 25ms^{-1}$ $s = 125m$

Strategy: $v^2 = u^2 + 2as \Rightarrow a = \dfrac{(v^2 - u^2)}{2s}$

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

Answers: $a = \dfrac{(v^2 - u^2)}{2s} = \dfrac{25^2 - 15^2}{2 \times 125} = 1.6ms^{-2}$

$$v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{25-15}{1.6} = 6.25s$$

3. A racing car starts from rest and accelerates uniformly at $2ms^{-2}$ in 30seconds, it then travels at a constant speed for 2min and finally decelerates at $3ms^{-2}$ until it stops, determine the maximum speed in km/h and the total distance in km it covered.

Strategy:

First stage: $u = 0ms^{-1}$ $a = 2ms^{-2}$ $t = 30s$, $v = u + at = 60ms^{-1}$

Second stage: moving with a constant speed $60ms^{-1}$ for 2min.

Third stage: $u = 60ms^{-1}$ $v = 0ms^{-1}$ $a = -3ms^{-2}$ (*deceleration*)

Answers:

First stage: $v = u + at = 60ms^{-1}$

$$s_1 = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 30^2 = 900m$$

Second stage: the final speed of the first stage is the constant speed of the second stage.

$$s_2 = vt = 60 \times 2\min = 60 \times 2 \times 60 = 7200m$$

$(1\min = 60s)$

Third stage: $v^2 = u^2 + 2as \Rightarrow s_3 = \dfrac{v^2 - u^2}{2a} = \dfrac{0 - 60^2}{2 \times (-3)} = 600m$

So

Maximum speed = $60ms^{-1} = \dfrac{60}{1000} \times 60 \times 60 = 216km/h$

Total distance = $s_1 + s_2 + s_3 = 900 + 7200 + 600 = 8700m = 8.7km$

4. **Figure 4.1** shows the shuttle spacecraft as it is launched into space.

Fig. 4.1 shuttle spacecraft
launching into space

During the first 5 minutes of the launch the average acceleration of the shuttle is $14.5ms^{-2}$.

a. Calculate the speed of the shuttle after the first 5 minutes.

b. Calculate how far the shuttle travels in the first 5 minutes.

Data: $u = 0ms^{-1}$, $\overline{a} = 14.5ms^{-2}$, $t = 5\min = 300\sec$

Strategy: $v = u + at$, $s = ut + \frac{1}{2}at^2$

Answers: a. $v = u + at = 0 + 14.5 \times 300 = 4350m = 4.35km$

b. $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 14.5 \times 300^2 = 652500m = 652.5km$

5. Figure 5.1 shows an incomplete velocity—time graph for a boy running a distance of 100m.

a. What is his acceleration during the first 4 seconds?

b. How far does the boy travel during (i) the first 4 seconds, (ii) the next 9 seconds?

c. Copy and complete the graph showing clearly at what time he has covered the distance of 100m. Assume his speed remains constant at the value shown by the horizontal portion of the graph.

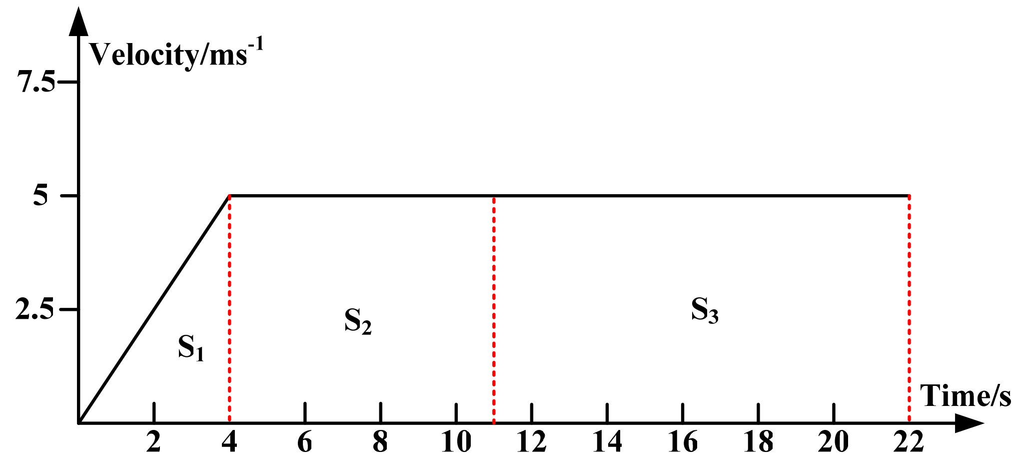


Fig. 5.1 velocity—time graph

Solution:

a. the gradient of the velocity—time graph represents the acceleration of the body.

During the first 4 seconds, $\text{gradient} = \frac{5}{4} = 1.25$

$\text{acceleration} = 1.25ms^{-2}$

b. (i) the area under a velocity—time graph measures the displacement traveled.

$area\ S_1 = \frac{1}{2} \times 4 \times 5 = 10$

$\text{Displacement} = 10m$

(ii) the next 9 seconds, $\text{area } S_2 = 9 \times 5 = 45$

$\text{Displacement} = 45m$

c. during the first 13 seconds, the distance covered is 10 + 45 = 55m,

The area needed $S_3 = 100 - 55 = 45$

So from 13s to 22 s, he covers $S_3 = 45$ m.

6. A constant resultant horizontal force of 1.8×10^3 N acts on a car of mass 900 kg, initially at rest on a level road.

(a) Calculate

(i) The acceleration of the car,

Strategy: by the Newton's second law, $F = ma$, $a = \frac{F}{m}$

So $a = \frac{F}{m} = \frac{1.8 \times 10^3}{900Kg} = 2ms^{-2}$

(ii) The speed of the car after 8.0 s,

Strategy: initial velocity, $u = 0$, $t = 8.0s$, $a = 2ms^{-2}$. And from the equation: $v = u + at$, gives

$v = 0 + 2 \times 8 = 16ms^{-1}$

(iii) The momentum of the car after 8.0 s,

Strategy: The product of an object's mass m and velocity v is called its momentum:

$momentum = mv = 900 \times 16 = 1.44 \times 10^4 kgms^{-1}$

(iv) The distance traveled by the car in the first 8.0 s of its motion,

Strategy: $s = ut + \frac{1}{2}at^2$

$S = 0 + \frac{1}{2} \times 2 \times 8^2 = 64m$

(v) The work done by the resultant horizontal force during the first 8.0 s.

Strategy: Work done=force × distance moved in direction of force.

$W = FS = 1.8 \times 10^3 \times 64 = 115.2kJ$

(b) On the axes below (**Fig. 6.1**) sketch the graphs for speed, *v*, and distance traveled, *s*, against time, *t*, for the first 8.0 s of the car's motion.

Strategy: for the first 8.0 s, the car is moving with constant acceleration, $a = 2ms^{-2}$, so the gradient of the v—t graph is equal to $2ms^{-2}$

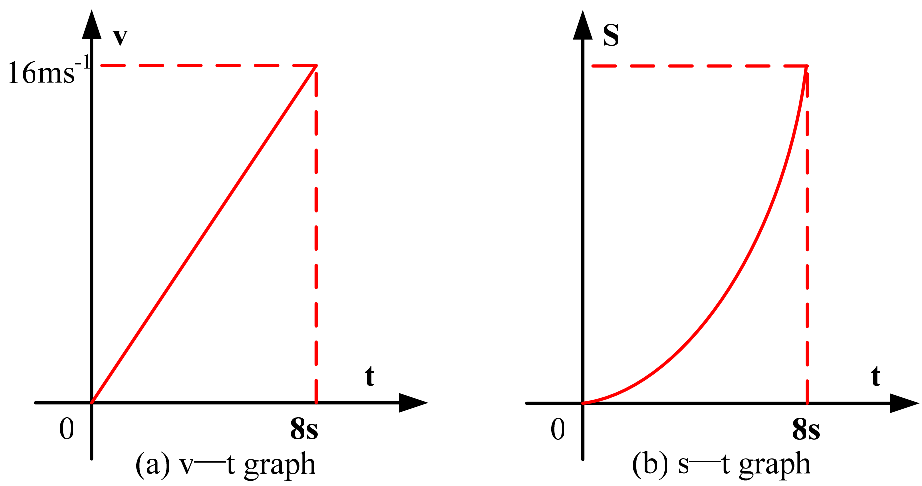


Fig. 6.1 (a) v—t graph (b) s—t graph

(c) In practice the resultant force on the car changes with time. Air resistance is one factor that affects the resultant force acting on the vehicle.

You may be awarded marks for the quality of written communication in your answer.

(i) Suggest, with a reason, how the resultant force on the car changes as its speed increases.

Answers: the resultant force decreases as its speed increases, because the air resistance increases as its speed increases, and the engine force of the car is constant, so the constant force decreases.

(ii) Explain, using Newton's laws of motion, why the vehicle has a maximum speed.

As the velocity increases, the air resistance increases, so the resultant force decreases, which means the acceleration of the car decreases, but the velocity is still increasing till the resultant force is zero (acceleration of the car is zero), according to the Newton's first law, then the vehicle has a maximum speed.

7. Figure 7.1 represents the motion of two cars, A and B, as they move along a straight, horizontal road.

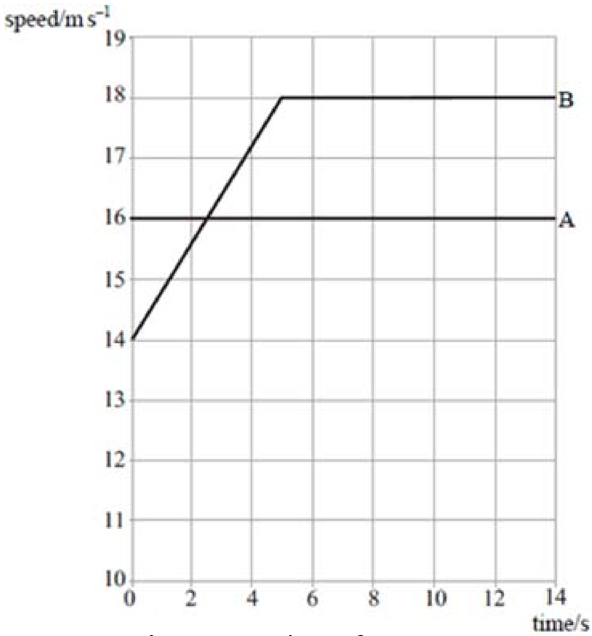


Fig. 7.1 motion of two cars

(a) Describe the motion of each car as shown on the graph.

(i) Car A: is moving with constant speed $16ms^{-1}$

(ii) Car B: accelerates in the first 5 seconds, and then moving with constant speed $18ms^{-1}$.

(b) Calculate the distance traveled by each car during the first 5.0 s.

(i) Car A:

Strategy: car A moving with constant speed, so distance of car A, So $S_A = ut = 16 \times 5 = 80m$

(ii) Car B:

Strategy: in the first 5 seconds, car B accelerates, and from the graph, the gradient of the v—t graph for B is $\frac{18-14}{5} = 0.8$, that is the acceleration of B is $a = 0.8ms^{-2}$

So $S_B = ut + \frac{1}{2}at^2 = 14 \times 5 + \frac{1}{2} \times 0.8 \times 5^2 = 80m$

(c) At time t = 0, the two cars are level. Explain why car A is at its maximum distance ahead of B at t = 2.5 s

Because car A is faster than car B at the first 2.5s, so for the first 2.5s, the distance between them increases till they have the same speed at 2.5s. After 2.5s, car B is faster than car A, so the distance then decreases. So at the time 2.5s, car A is at its maximum distance ahead of B.

8. A car accelerates from rest to a speed of $26ms^{-1}$. **Table 8.1** shows how the speed of the car varies over the first 30 seconds of motion.

Table 8.1

Time/s	0	5.0	10.0	15.0	20.0	25.0	30.0
Speed/ms^{-1}	0	16.5	22.5	24.5	25.5	26.0	26.0

(a) Draw a graph of speed against time on the grid provided (**Fig. 8.1**).

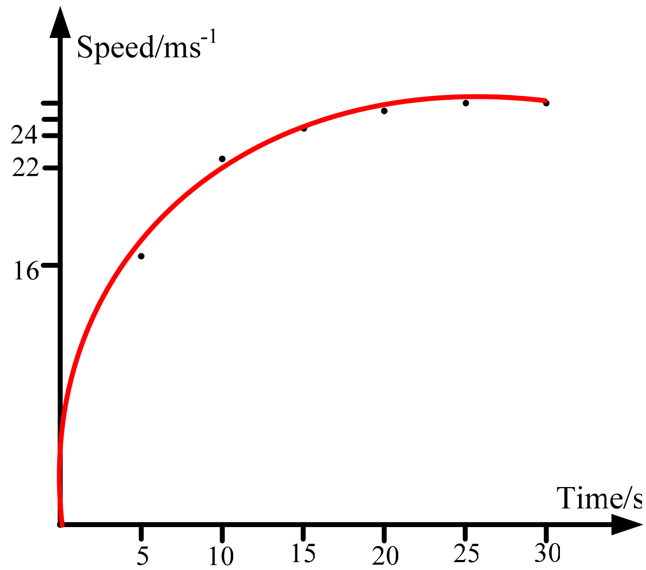


Fig. 8.1 speed—time graph

Note: you must draw the right scales and the six points are correctly plotted, and it is a trend line not a straight line.

(b) Calculate the average acceleration of the car over the first 25 s.

Strategy: $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{26}{25} = 1.04ms^{-2}$

(c) Use your graph to estimate the distance traveled by the car in the first 25 s.

Strategy: area under the v—t graph represents the distance traveled.

So from the graph, its distance is 510m

(d) Using the axes below, sketch a graph to show how the resultant force acting on the car varies over the first 30 s of motion.

Solution:

From table 3.1, the rate of change of speed decreases to zero, thus the resultant force decreases to zero. As shown in **Fig. 8.2**.

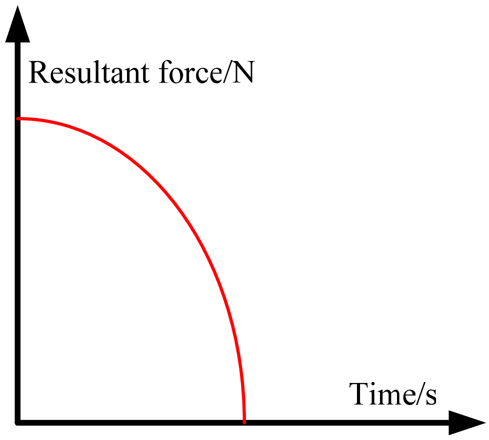


Fig. 8.2 resultant force—time graph

(e) Explain the shape of the graph you have sketched in part (d), with reference to the graph you plotted in part (a).

Because the first graph shows that the gradient of the car decreases, which means that the acceleration of the car decreases, and by the Newton's second law, $F = ma$, the force, F, decreases, and as the acceleration is changing in the first 25s, so the force is also changing, so the graph of the force is not a straight line.

9. A supertanker of mass $4.0 \times 10^8 kg$, cruising at an initial speed of $4.5m/s$, takes one hour to come to rest.

(a) Assuming that the force slowing the tanker down is constant, calculate

(i) The deceleration of the tanker,

Solution:

The force slowing the tanker down is constant, so the tanker decelerates uniformly. Therefore, deceleration of the tanker is given by

$$a = \frac{0-4.5}{t} = \frac{-4.5}{1\times 60\times 60} = 1.25\times 10^{-3} m/s^2$$

(ii) The distance travelled by the tanker while slowing to a stop.

Solution:

The average speed is given by

$$\bar{v} = \frac{0+4.5}{2} = 2.25 m/s$$

So the distance traveled: $s = \bar{v}t = 2.25\times 1\times 60\times 60 = 8100m$

(b) Sketch, using the axes below, a distance-time graph representing the motion of the tanker until it stops.

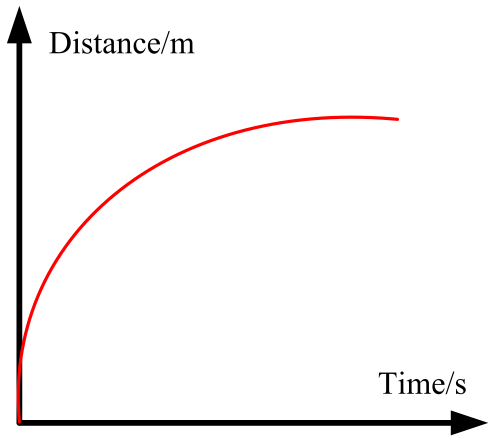


Fig. 9.1 Distance—time graph

(c) Explain the shape of the graph you have sketched in part (b).

Solution:

Because the speed is decreasing, the gradient of the curve decreases in the distance-time graph.

10. (a) A cheetah accelerating uniformly from rest reaches a speed of $29m/s$ in 2.0 s and then maintains this speed for 15 s. Calculate

(i) Its acceleration,

Solution:

Using $a = \frac{v-u}{t} = \frac{29-0}{2} = 14.5m/s^2$

(ii) The distance it travels while accelerating,

Solution:

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 14.5 \times 2^2 = 29m$

(iii) The distance it travels while it is moving at constant speed.

Solution:

$s = vt = 29 \times 15 = 435m$

(b) The cheetah and an antelope are both at rest and 100 m apart. The cheetah starts to chase the antelope. The antelope takes 0.50 s to react. It then accelerates uniformly for 2.0 s to a speed of $25m/s$ and then maintains this speed. **Fig. 10.1** shows the speed-time graph for the cheetah.

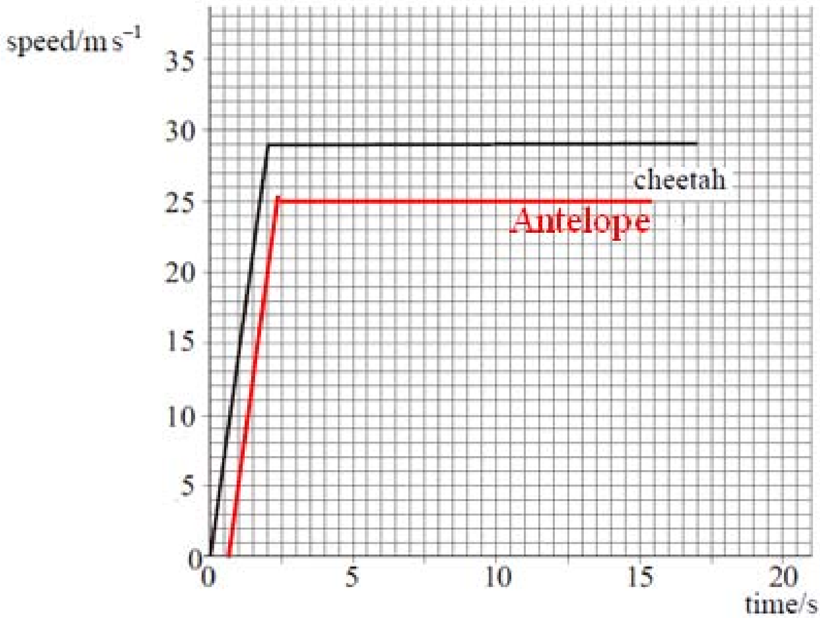


Fig. 10.1 speed—time graph for cheetah and antelope

(i) Using the same axes plot the speed-time graph for the antelope during the chase.

Solution:

The antelope takes 0.50 s to react and accelerates uniformly for 2.0 s to a speed of 25 m/s. thus we can get the speed-time graph beginning with 0.50 s.

(ii) Calculate the distance covered by the antelope in the 17 s after the cheetah started to run.

Solution:

The antelope accelerates from rest, and reaches to a speed of 25 m/s in 2 s. then maintains this speed. Thus the distance is given by

$$s = \frac{v+u}{2} \times 2 + 25 \times (17 - 2 - 0.5) = 12.5 \times 2 + 25 \times 14.5 = 387.5m$$

(iii) How far apart are the cheetah and the antelope after 17 s?

Solution:

From (a), the distance of cheetah is $s_1 = 435 + 29 = 464m$

And at the beginning, they are 100 m apart. Thus

$\Delta s = s + 100 - s_1 = 387.5 + 100 - 464 = 23.5m$

11. Figure 11.1 shows a distance-time graphs for two runners, A and B, in a 100 m race.

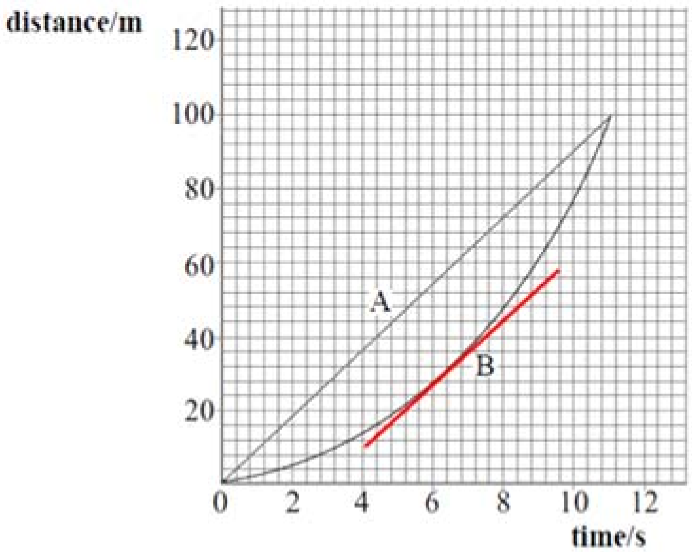


Fig. 11.1 distance—time graph for two runners

(a) Explain how the graph shows that athlete B accelerates throughout the race.

Solution:

The gradient is changing (increasing)

(b) Estimate the maximum distance between the athletes.

Solution:

When B's speed is the same as A's, it has the maximum distance between the athletes. From the graph is the gradient of B curve is the same that of A.

From the graph, the maximum distance is 25 m.

(c) Calculate the speed of athlete A during the race.

Solution:

For A, it has a distance in time 11 s, thus

$speed = \frac{distance}{time} = \frac{100m}{11s} = 9.1m/s$

(d) The acceleration of athlete B is uniform for the duration of the race.

(i) State what is meant by uniform acceleration.
(ii) Calculate the acceleration of athlete B.
Solution:
(i) The acceleration keeps the same or the velocity increases uniformly with time.
(ii) For B, its initial velocity is u = 0 m/s, distance s = 100 m, time taken t = 11 s.
Thus, using $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$, gives

$$a = \frac{2s}{t^2} = \frac{2 \times 100}{11^2} = 1.7 m/s^2$$

12.An aircraft accelerates horizontally from rest and takes off when its speed is 82 m s^{-1}. The mass of the aircraft is $5.6 \times 10^4 kg$ and its engines provide a constant thrust of $1.9 \times 10^5 N$.
(a) Calculate
(i) The initial acceleration of the aircraft,
Solution:
(i) Initially, the resultant force $F = 1.9 \times 10^5 N$, from Newton's second law: $F = ma$, we can get that

$$a = \frac{F}{m} = \frac{1.9 \times 10^5 N}{5.6 \times 10^4 kg} = 3.4 m/s^{-2}$$

(ii) The minimum length of runway required, assuming the acceleration is constant.
Solution: let the minimum length of the runway required L. thus
$v^2 - u^2 = 2aL$
Therefore

$$L = \frac{v^2 - u^2}{2a} = \frac{82^2 - 0}{2 \times 3.4} = 989m$$

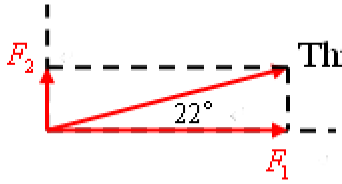
(b) In practice, the acceleration is unlikely to be constant. State a reason for this and explain what effect this will have on the minimum length of runway required.
Solution:
In practice, the air resistance increases with speed, hence the runway will be longer.

(c) After taking off, the aircraft climbs at an angle of 22° to the ground. The thrust from the engines remains at $1.9\times10^5 N$. Calculate

(i) The horizontal component of the thrust,

(ii) The vertical component of the thrust.

Solution:



The thrust $T = 1.9\times10^5 N$

The horizontal component of the thrust is given by

$F_1 = T\cos 22^0 = 1.76\times10^5 N$

The vertical component of the thrust is given by

$F_2 = T\sin 22^0 = 0.71\times10^5 N$

13. Figure 13.1 shows how the velocity, v, of a car varies with time, t.

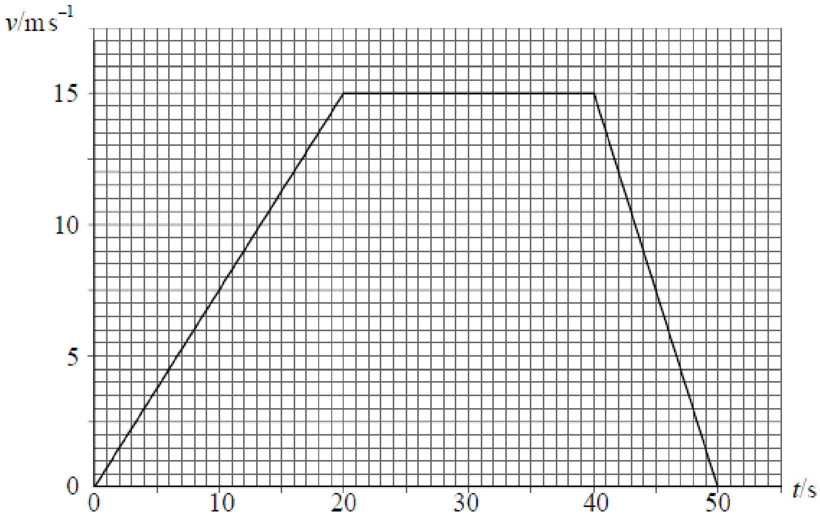


Fig. 13.1 velocity—time graph

(a) Describe the motion of the car for the 50 s period.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

0—20 s: the car uniformly accelerates to a velocity of 15 m/s.

20—40 s: the car moves with constant velocity 15 m/s.

40—50 s: the car uniformly decelerates from a velocity of 15 m/s to 0 m/s.

(b) The mass of the car is 1200 kg. Calculate for the first 20 s of motion, (b)
(i) the change in momentum of the car,
(b) (ii) the rate of change of momentum,
(b) (iii) the distance travelled.

Solution: for the first 20 s of motion

(i) At t = o s, the initial velocity is u = 0 m/s; at t = 20 s, the final velocity is v = 15 m/s. thus the change in momentum of the car is given by

Therefore,

$$\Delta p = mv - mu = (1200kg)\times 15m/s - 0 = 1.8\times 10^4 kg\cdot m/s$$

(ii)

$$\text{The rate of change of momentum} = \frac{change\ \ in\ \ momentum}{time\ taken} = \frac{1.8\times 10^4 kg\cdot m/s}{20} = 0.9\times 10^3 kg\cdot m/s^2$$

(iii) The area under a velocity—time graph measures the displacement traveled.

Thus the area for the first 20 s is given by

$$A = \frac{1}{2}\times 20\times 15 = 150$$

Therefore the distance traveled is 150 m.

14. A car is travelling on a level road at a speed of 15.0 m s^{-1} towards a set of traffic lights when the lights turn red. The driver applies the brakes 0.5 s after seeing the lights turn red and stops the car at the traffic lights. **Table 14.1** shows how the speed of the car changes from when the traffic lights turn red.

Table 14.1

Time/s	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Speed/ms^{-1}	15.0	15.0	12.5	10.0	7.5	5.0	2.5	0.0

(a) Draw a graph of speed on the y-axis against time on the x-axis on the grid provided (**Fig. 14.1**).

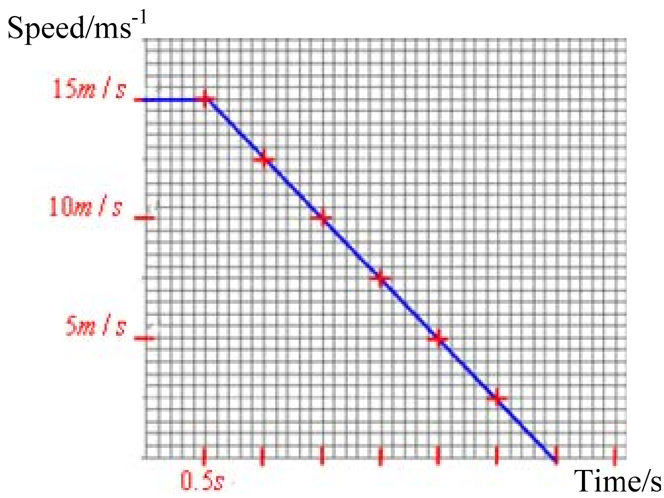


Fig. 14.1 speed—time graph

(b) (i) State and explain what feature of the graph shows that the car's deceleration was uniform.

Solution:

Deceleration is uniform because the graph is a decreasing straight line. And the gradient of the line represents the deceleration.

(b) (ii) Use your graph to calculate the distance the car travelled after the lights turned red to when it stopped.

Solution:

Distance traveled = area under the line (0s to 3.5s).

$$\text{Area } = \frac{1}{2} \times (0.5 + 3.5) \times 15 = 30$$

Therefore, distance traveled = 30 m.

15. Galileo used an inclined plane, similar to the one shown in **Fig. 15.1**, to investigate the motion of falling objects.

(a) Explain why using an inclined plane rather than free fall would produce data which is valid when investigating the motion of a falling object.

Solution:

Freefall is too quick; Galileo had no accurate method to time freefall.

(b) In a demonstration of Galileo's investigation, the number of swings of a pendulum was used to time a trolley after it was released from rest. A block was positioned to mark the distance that the trolley had travelled after a chosen whole number of swings.

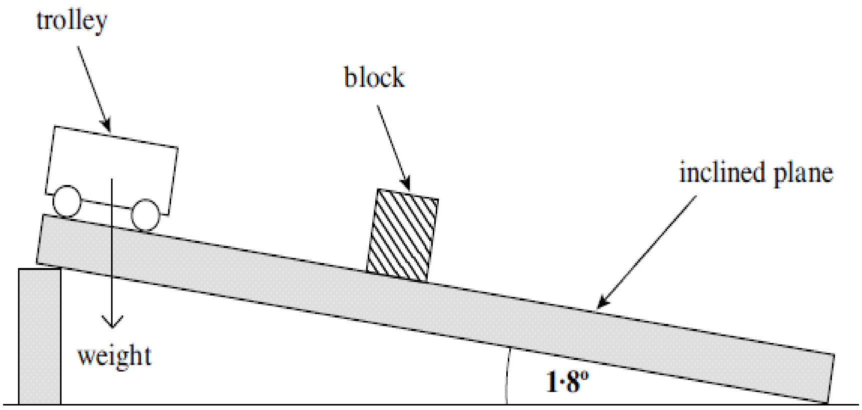


Fig. 15.1

The mass of the trolley in **Fig. 15.1** is 0.20 kg and the slope is at an angle of 1.8° to the horizontal.

(b) (i) Show that the component of the weight acting along the slope is about 0.06 N.

Solution:

The component of weight acting along the slope is given by

$W_1 = W\sin 1.8^0 = 0.2\times 9.81\times 0.031 = 0.06N$

(b) (ii) Calculate the initial acceleration down the slope.

Solution:

The initial resultant force along the slope equals to W_1, thus

$$a = \frac{W_1}{m} = \frac{0.06}{0.2} = 0.3m/s^{-2}$$

(c) In this experiment, the following data was obtained. A graph of the data (**Fig. 15.2**) is shown below it.

Time/pendulum swings	Distance travelled/m
1	0.29
2	1.22
3	2.70
4	4.85

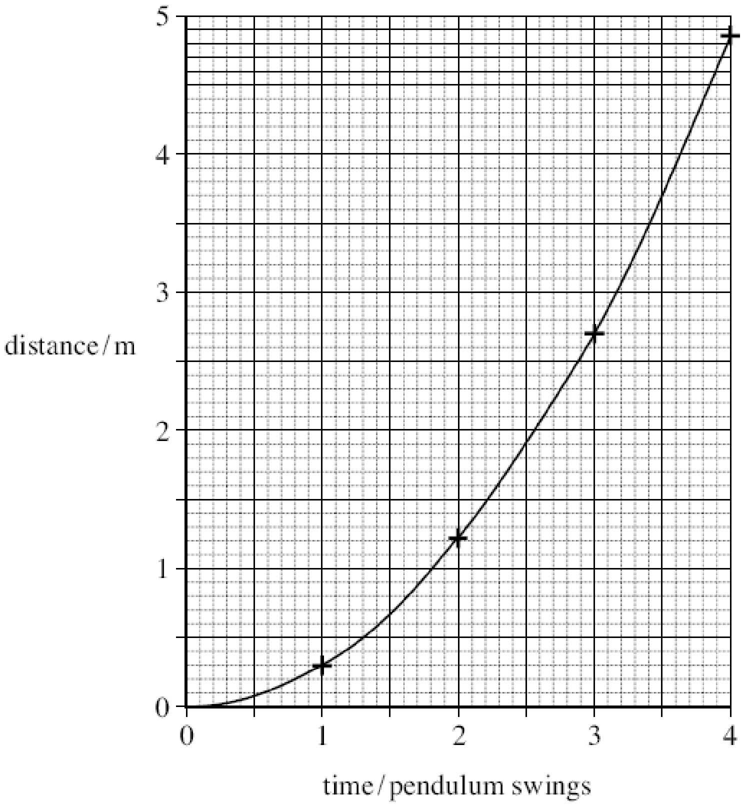


Fig. 15.2 distance—time graph

(c) From **Fig. 15.2**, state what you would conclude about the motion of the trolley?

Give a reason for your answer.

Solution:

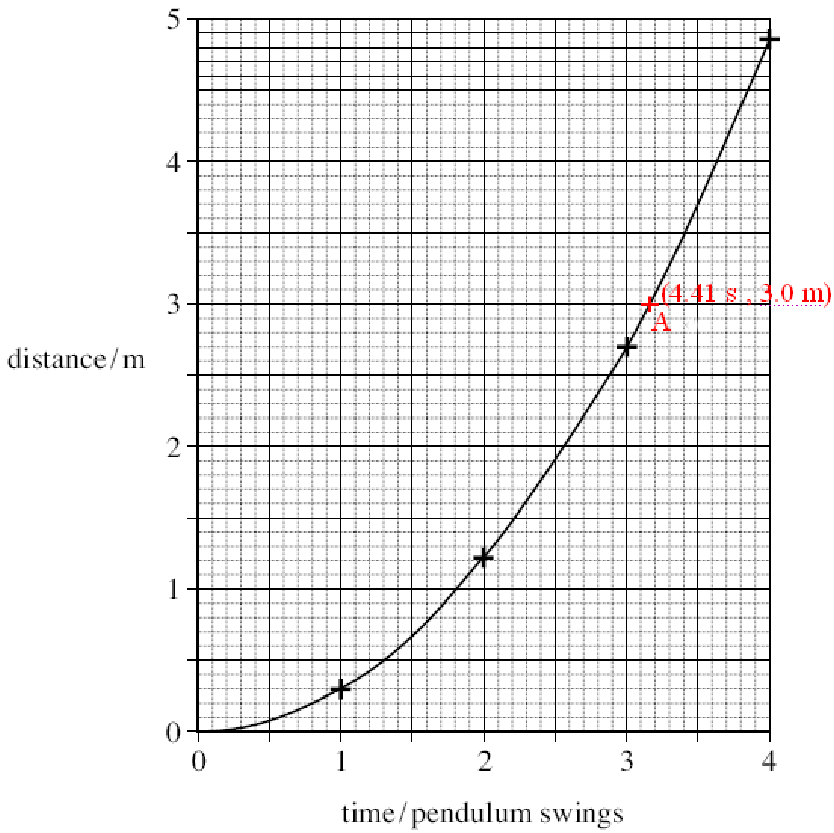
The gradient of the curve increases as time increasing. Thus the speed of the trolley is increasing.

(d) Each complete pendulum swing had a period of 1.4 s. Use the distance-time graph above to find the speed of the trolley after it had travelled 3.0 m.

Solution:

From **Fig. 15.2**, the time taken for traveling 3.0 m is given by

$$t = 3 \times 1.4 + 1.5 \times \frac{1.4}{10} = 4.41s$$



And initial speed $u = 0m/s$, thus

$s = \frac{u+v}{2} \times t = \frac{vt}{2}$, gives

Speed, v $= \frac{2s}{t} = \frac{2 \times 3.0m}{4.41s} = 1.36m/s$

16. An object is projected horizontally at a speed of $15ms^{-1}$ from the top of a tall tower of height $35.0m$. Calculate:

a. how long it takes to fall to the ground,

b. How far it travels horizontally.

c. Its speed just before it hits the ground.

Solution:

Strategy: horizontal projection, data: $u_x = 15ms^{-1}$, $h = 35m$

a. choose downwards as '+' direction

And $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 35}{9.8}} = 2.67s$

b. $X = u_x t = 15 \times 2.67 = 40m$

c. To find the velocity of the particle at any time t = 2.67s, the two components v_x and v_y must be added vectorially.

$v_x = u_x = 15ms^{-1}$

$v_y = gt = 9.8 \times 2.67 = 26.2ms^{-1}$

So the resultant speed $v = \sqrt{v_x^2 + v_y^2} = 30.2ms^{-1}$

Its direction is $\theta = \arctan\frac{v_x}{v_y} = 29.8^\circ$ to the vertical.

17. The aeroplane shown in **Fig. 17.1** is traveling horizontally at 95ms^{-1}. It has to drop a crate of emergency supplies.
The air resistance acting on the crate may be neglected.

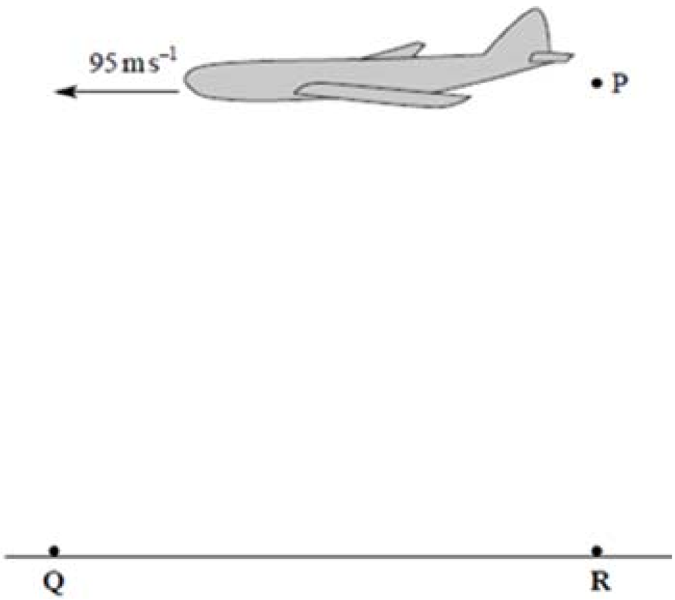


Fig. 17.1

(a) (i) The crate is released from the aircraft at point P and lands at point Q. Sketch the path followed by the crate between P and Q as seen from the ground.

Note: the path is a parabola from P to Q.

(ii) Explain why the horizontal component of the crate's velocity remains constant while it is moving through the air.

Because there is no horizontal force, by the Newton's second law, F = ma, there is no acceleration. So the crate's velocity remains constant while it is moving through the air.

(b) (i) To avoid damage to the crate, the maximum vertical component of the crate's velocity on landing should be 32ms^{-1}. Show that the maximum height from which the crate can be dropped is approximately 52 m.

Strategy: the initial vertically velocity, $u=0$, final vertically velocity, $v=32ms^{-1}$, so by the equation, $v^2-u^2=2gH$

So the height, $H=\dfrac{v^2-u^2}{2g}=\dfrac{32^2}{2\times 9.8}\approx 52m$

(ii) Calculate the time taken for the crate to reach the ground if the crate is dropped from a height of 52 m.

Strategy: from the equation, $v=u+gt$

$t=\dfrac{v}{g}=\dfrac{32}{9.8}=3.3s$

(iii) If R is a point on the ground directly below P, calculate the horizontal distance QR.

Strategy: the horizontal displacement x is given by $x=u_x t$, where $u_x=95ms^{-1}$ $t=3.3s$

So QR = 95×3.3 = 313.5 m

(c) In practice air resistance is not negligible. State and explain the effect this has on the maximum height from which the crate can be dropped.

Strategy: considering the air resistance, then the resultant force = weight of the crate minus the air resistance, so the resultant force decreases, then the acceleration decreases. So the maximum height from which the crate can be dropped increases.

18. A dart is thrown horizontally at a speed of 8.0 m s^{-1} towards the centre of a dartboard that is 2.0 m away. At the same instant that the dart is released, the support holding the dartboard fails and the dartboard falls freely, vertically downwards. The dart hits the dartboard in the centre before they both reach the ground.

(a) State and explain the motion of the dart and the dartboard, while the dart is in flight.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

There is no air resistance. The acceleration of both the dart and the dartboard is always equal to g and is always downwards because the force of gravity acts downwards. The acceleration therefore only affects the vertical motion of the object. Thus the dartboard accelerates vertically downwards at the same rate as the dart.

The horizontal velocity of the dart is constant because the acceleration of the object does not have a horizontal component, which results in a parabolic path.

(b) Calculate

(i) The time taken for the dart to hit the dartboard,

(ii) The vertical component of the dart's velocity just before it strikes the dartboard,

(iii) The magnitude and direction of the resultant velocity of the dart as it strikes the dartboard.

Solution:

(i) The horizontal distance $s = 2m$ and the horizontal speed $u = 8m/s$, thus

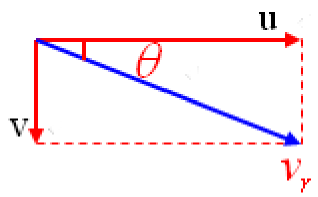
$$t = \frac{s}{u} = \frac{2}{8} = 0.25s$$

(ii) The dart accelerates downwards at an acceleration $g = 9.8m/s^2$, thus when t = 0.25 s.

$$v = u + at = 0 + gt = 9.8 \times 0.25 = 2.45m/s$$

(iii) The magnitude of the resultant velocity of the dart is given by

$$v_r = \sqrt{u^2 + v^2} = \sqrt{8^2 + (2.45)^2} = 8.4m/s$$



Thus

$$\theta = \tan^{-1}(\frac{2.45}{8}) = 17^0$$

19. Fig. 19.1 shows the path of a ball thrown horizontally from the top of a tower of height 24 m which is surrounded by level ground.

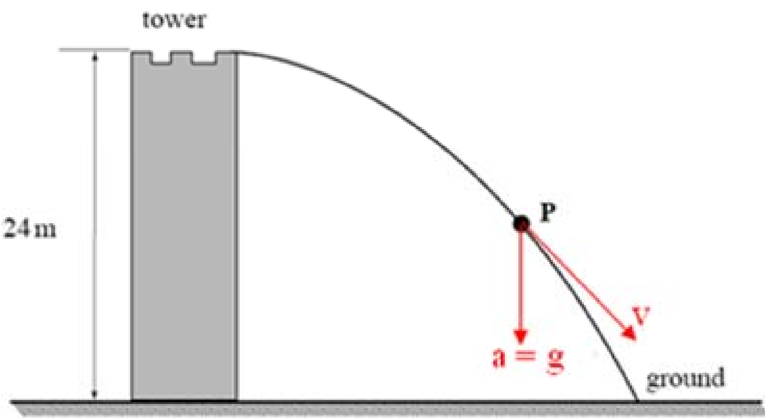


Fig. 19.1

(a) Using two labelled arrows, show on Figure 3 the direction of the velocity, v, and the acceleration, a, of the ball when it is at point P.

Solution:

The velocity vector is tangential to path.

The acceleration vector is vertically downwards.

Note:

A projectile is any object acted upon only by the force of gravity. And

(i) The acceleration of the object is always equal to g and is always downwards because the force of gravity acts downwards. The acceleration therefore only affects the vertical motion of the object.

(ii) The horizontal velocity of the objects is constant because the acceleration of the object does not have a horizontal component.

(iii) The motions in the horizontal and vertical directions are independent of each other. And can be treated separately.

(b) (i) Calculate the time taken from when the ball is thrown to when it first hits the ground. Assume air resistance is negligible.

Solution:

Vertically, $H = \frac{1}{2}gt^2$, gives

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 24}{9.8}} = 2.21s$$

(b) (ii) The ball hits the ground 27 m from the base of the tower. Calculate the speed at which the ball is thrown.

Solution:

Horizontally, the speed is constant, and $s = vt$, gives

$$v = \frac{s}{t} = \frac{27}{2.21} = 12.2m/s$$

20. A digital camera was used to obtain a sequence of images of a tennis ball being struck by a tennis racket. The camera was set to take an image every 5.0 ms. The successive positions of the racket and ball are shown in **Fig. 20.1**.

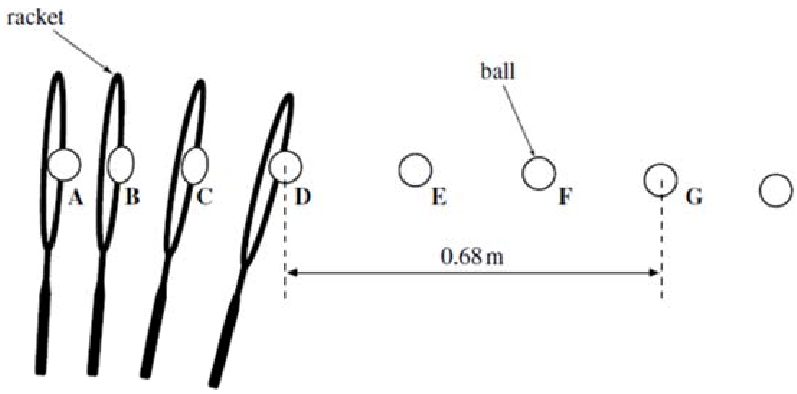


Fig. 20.1

(a) The ball has a horizontal velocity of zero at A and reaches a constant horizontal velocity at D as it leaves the racket. The ball travels a horizontal distance of 0.68m between D and G.

(a) (i) Show that the horizontal velocity of the ball between positions D and G in **Fig. 20.1** is about 45ms^{-1}.

Solution:

The camera was set to take an image every 5.0 ms, thus the time interval between D and G is $3 \times 5.0 = 15ms = 15 \times 10^{-3} s = 0.015s$.

The horizontal velocity is given by

$$v = \frac{s}{t} = \frac{0.68m}{0.015s} = 45m/s$$

(a) (ii) Calculate the horizontal acceleration of the ball between A and D.

Solution:

The time interval between A and D is $t = 3 \times 5.0 = 15ms = 15 \times 10^{-3} s = 0.015s$.

And $v_A = 0m/s$, $v_D = 45m/s$

Thus the acceleration is given by

$$a = \frac{v_D - v_A}{t} = \frac{45 - 0}{0.015} = 3000m/s^2$$

(b) At D, the ball was projected horizontally from a height of 2.3 m above level ground.

(b) (i) Show that the ball would fall to the ground in about 0.7 s.

Solution:

Assuming the ball falls freely. Thus vertically:

$$h = \frac{1}{2}gt^2$$

Gives

$$t_{fall} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2.3}{9.81}} = 0.7s$$

(b) (ii) Calculate the horizontal distance that the ball will travel after it leaves the racket before hitting the ground. Assume that only gravity acts on the ball as it falls.

Solution:

The horizontal distance is given by

$$s = v_D t_{fall} = (45m/s) \times 0.7s = 31.5m$$

(b) (iii) Explain why, in practice, the ball will not travel this far before hitting the ground.

Solution:

In practice, there is air resistance causing horizontal deceleration.

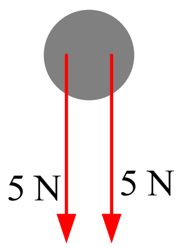
Chapter 3 Forces and Newton's laws of motion

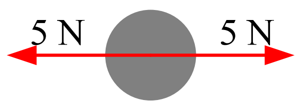
3.1 Force definition

Force is a vector; the SI unit is the **Newton (N)**.

If two or more forces act on something, their combined effect is called the **resultant force.**

Two simple examples are shown below:





Newton definition:

1 Newton (N), as the amount of force that will give an object of mass 1 kg an

acceleration of $1\,ms^{-2}$.

3.2 Weight and g

On Earth, everything feels the downward force of gravity. This gravitational force is called weight. As for other forces, its SI unit is the Newton (N).

Near the Earth's surface, the gravitational force on each kg is about 10 N; the gravitational field strength is 10 N kg $^{-1}$. This is represented by the symbol g.

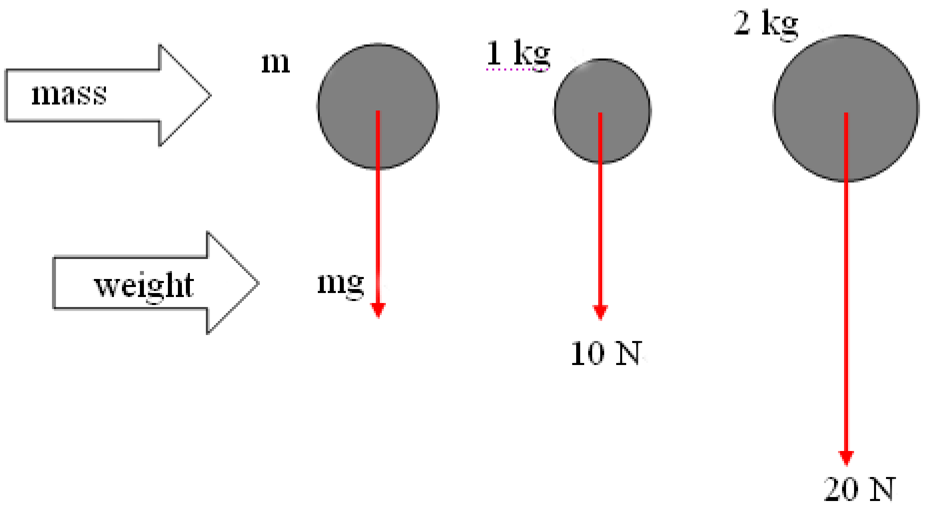
So weight = mass × gravitational field strength

In symbol

$W = mg$

For example, in the diagram below, all the masses are falling freely (gravity is the only force acting). From $F = ma$, it follows that all the masses have the same downward acceleration, g. this is the acceleration of free fall.

$$acceleration = \frac{weight}{mass} = 10\,ms^{-2} = g$$



Note: you can think of g:

Either as a gravitational field strength of 10 N kg $^{-1}$

Or as an acceleration of free fall of 10 m s $^{-2}$

In more accurate calculations, the value of g is normally taken to be 9.81, rather than 10.

3.3 Newton's first law of motion

If there is no resultant force acting:

① A stationary object will stay at rest,

② A moving object will maintain a constant velocity (a steady speed in a straight line).

From Newton's first law, it follows that if an object is at rest or moving at constant velocity, then the forces on it must be balanced.

Note: the more mass an object has, the more it resists any change in motion (because more force is needed for any given acceleration). Newton called this resistance to change in motion inertia.

Momentum:

The product of an object's mass m and velocity v is called its momentum:

$momentum = mv$

Momentum is measured in $kg \cdot ms^{-1}$. It is a vector.

3.4 Newton's second law

The rate of change of momentum of an object is proportional to the resultant force acting.

This can be written in the following form:

$$resultant\ force = \frac{change\ in\ momentum}{time\ taken}$$

In symbol:

$$F = \frac{mv - mu}{t} \cdots\cdots \quad ①$$

Where v is final velocity, u is initial velocity of an object.

Equation ① can be rewritten $F = \dfrac{m(v-u)}{t}$

And acceleration, $a = \dfrac{v-u}{t}$. So

$$F = ma \cdots\cdots \quad ②$$

Note:

1. Equation ① and ② are therefore different versions of the same principle.
2. F = ma cannot be used for a particle traveling at very high speeds because its mass increases.
3. When using equations ① and ②, remember that F is the resultant force acting. For example, for the figure below, the resultant force is $26 - 20 = 6N$ to the right. The acceleration a can be worked out as follows:

$$a = \frac{F}{m} = \frac{6}{2} = 3\,ms^{-2}$$

Impulse:

As $F = \frac{m(v-u)}{t}$ can be rewritten $Ft = mv - mu$

In words force × time = change in momentum.

The quantity 'force × time' is called an impulse.

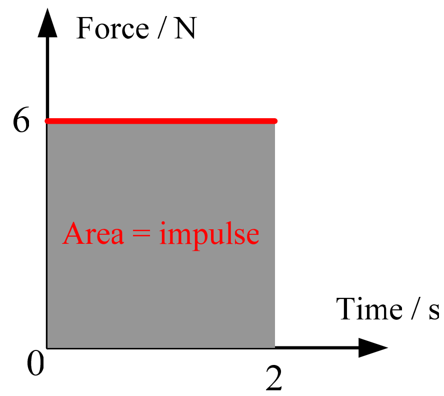
A given impulse always produces the same change in momentum, irrespective of the mass. For example, if a resultant force of 6 N acts for 2s, the impulse delivered is 6×2=12Ns.

This will produce a momentum change of $12\,kgms^{-1}$

So a 4 kg mass will gain $3ms^{-1}$ of velocity

Or a 2 kg mass will gain $6ms^{-1}$ of velocity, and so on.

The graph below is for a uniform force of 6 N. in 2s, the impulse delivered is 12 Ns. numerically, and this is equal to the area of the graph between the 0 and 2 s points.



3.5 Newton's third law of motion

If A is exerting a force on B, then B is exerting an equal but opposite force on A.

The law is sometimes expressed as follows:

To every action, there is an equal but opposite reaction.

Note:

·It does not matter which force you call the action and which the reaction. One cannot exist without the other.

·the action and reaction do not cancel each other out because they are acting on different objects.

3.6 Balanced forces

When forces act on a point object, the object is in equilibrium means the resultant force is zero (the object keeps at rest or moving at constant speed). In other words, when a point object keeps at rest or moving at constant speed, means it is in equilibrium and the resultant forces on it is zero.

Conditions for equilibrium for two or three coplanar forces acting at a point:

(i) When two forces act on a point object, the object is in equilibrium (at rest or moving at constant velocity) only if the two forces are equal and opposite of each other. The resultant of the two forces is therefore zero. The two forces are said to be balanced.

(ii) When three forces act on a point object, the object is in equilibrium (at rest or moving at constant velocity) only if the resultant of any two of the forces is equal and opposite to the third force.

- resolve each force along the same parallel and perpendicular lines
- balance the components along each line.

3.7 Centre of gravity and Determination of Centre of Gravity (c.g.) of irregular lamina using the plumb line method

(i) An object may be made to balance at a particular point. When it is balanced at this point, the object does not turn and so all the weight on one side of the pivot is balanced by the weight on the other side. Supporting the object at the pivot means that the only force which has to be applied at the pivot is one to stop the object falling—that is, a force equal to the weight of the object. Although all parts of the object have weight, the whole weight of the object appears to act at his balance point. This point is called the centre of gravity of the object.

Center of Gravity: The point on the object that no turning effect produced by the force of the gravity.

Note: for a uniform body such as a ruler, the centre of gravity is at the geometrical centre.

(ii) Determination of Centre of Gravity (c.g.) of irregular lamina using the plumb line method:

Suppose we have to find the c.g. of an irregularly shaped lamina of cardboard (**Fig. 3.1**).

Make a hole A in the lamina and hang it so that it can swing freely on a nail clamped in a stand. It will come to rest with its c.g. vertically below A. to locate the vertical line through A tie a plumb line to the nail, the figure below, and mark its position AB on the lamina. The c.g. lies on AB.

Hang the lamina from another position C and mark the plumb line position CD. The c.g. lies on CD and must be at the point of intersection of AB and CD. Check this by hanging the lamina from a third hole. Also try balancing it at its c.g. on the rip of your fore-finger.

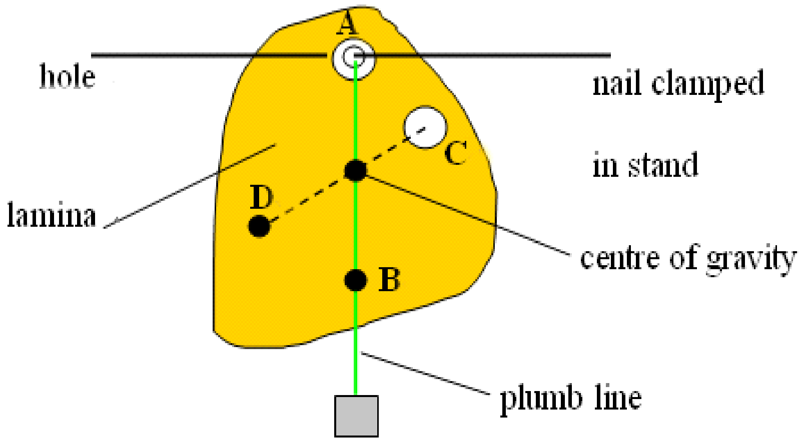


Fig. 3.1 c.g. of an irregularly shaped lamina of cardboard

3.8 11 Worked examples

1. A rocket engine ejects 100kg of exhaust gas per second at a velocity (relative to the rocket) of 200m/s (**Fig. 1.1**). What is the forward thrust (force) on the rocket?

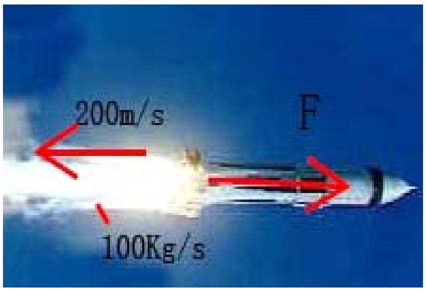


Fig. 1.1

By Newton's third law, the forward force on the rocket is equal to the backward force pushing out the exhaust gas. By Newton's second law, this force F is equal to the momentum gained per second by the gas, so it can be calculated using equation $F=\frac{m(v-u)}{t}$ with the following values:

$m=100kg \quad t=1s \quad u=0 \quad v=200ms^{-1}$

So, $F=\frac{m(v-u)}{t}=\frac{100\times(200-0)}{1}=20000N$.

2. A block of mass 2kg is pushed along a table with a constant velocity by a force of 5N. When the push is increased to 9N, what is

a. the resultant force,

b. the acceleration?

Solution: when the block moves with constant velocity the forces acting on it are balanced. The force of friction opposing its motion must therefore be 5N.

a. When the push is increased to 9N the resultant force F on the block is (9-5) N=4N, (since the frictional force is still 5N).

b. The acceleration a is obtained from $F=ma$ where F=4N and m=2kg.

So $a=\frac{F}{m}=\frac{4N}{2kg}=\frac{4kgms^{-2}}{2kg}=2ms^{-2}$

3. A car of mass 1200kg traveling at 72km/h is brought to rest in 4s. Find

a. the average deceleration,

b. the average braking force,

c. The distance moved during the deceleration.

Solution:

a. The deceleration is found from $v=u+at$ where $v=0$.

$u=72km/h=\frac{72\times1000}{60\times60}=20ms^{-1}$

And $t=4s$

Hence $0=20+a\times4$

So $a=-5ms^{-2}$

The deceleration is $5ms^{-2}$

b. The average braking force F is given by $F=ma$, where $m=1200kg$ and $a=-5ms^{-2}$. Therefore

$F=1200\times(-5)=-6000N$

'—' represents the direction of the braking force is opposite to the motion of

the car.
So the braking force is 6000N.
c. To find the distance moved, we used

$$s = \frac{u+v}{2}t = \frac{20+0}{2} \times 4 = 40m$$

4. (a). what resultant force produces an acceleration of $5ms^{-2}$ in a car of mass 1000kg.
(b). what acceleration is produced in a mass of 2kg by a resultant force of 30N.

Solution:

a. use $F = ma = 1000 \times 5 = 5000N$

b. $F = ma \Rightarrow a = \frac{F}{m} = \frac{30}{2} = 15ms^{-2}$

5. A rocket has a mass of 500kg.
a. What is its weight on earth where g=10N/kg.
b. At lift-off the rocket engine exerts an upward force of 25000N. What is the resultant force on the rocket? What is its initial acceleration?

Solution:

a. weight=mass × gravitational field strength
weight $= 500 \times 10 = 5000N$

b. resultant force = upward force - weight=25000 – 5000 = 20000N

So initial acceleration, $a = \frac{resultant\ force}{mass} = \frac{20000}{500} = 40ms^{-2}$

6. An athlete trains by dragging a heavy load across a rough horizontal surface (**Fig. 6.1**).

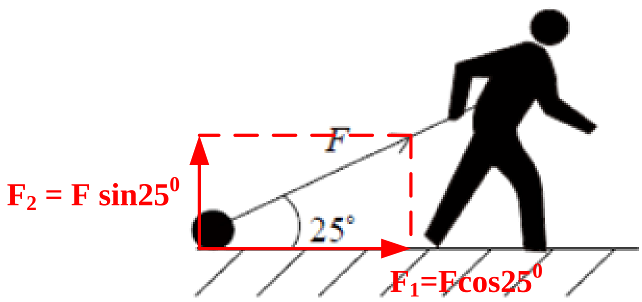


Fig. 6.1

The athlete exerts a force of magnitude F on the load at an angle of 25^0 to the horizontal.

(a) Once the load is moving at a steady speed, the average horizontal frictional force acting on the load is 470 N.

Calculate the average value of F that will enable the load to move at constant speed.

Solution:

The load is moving at constant speed, from Newton's first law, the resultant force is equal to zero. Thus

$F_1 = F\cos 25^0$ = frictional force = f = 470 N

The average value of **F** is given by

$$F = \frac{470N}{\cos 25^0} = 519N$$

(b) The load is moved a horizontal distance of 2.5 km in 1.2 hours.

Calculate

(i) The work done on the load by the force F.

Solution:

Work done=force × distance moved in direction of force.

$$W = F_1 \times S = (F\cos 25^0 N)\times(2.5\times10^3 m) = 470\times2.5\times10^3 = 1175kJ$$

(ii) The minimum average power required to move the load.

Solution:

$$power = \frac{work\ \ done}{time\ taken} = \frac{1175\times10^3 J}{1.2\times60\times60s} = 272W$$

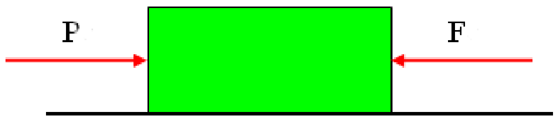
(c) The athlete pulls the load uphill at the same speed as in part (a).

Explain, in terms of energy changes, why the minimum average power required is greater than in (b)(ii).

Solution:

When the load is pulled uphill, some of the work need to be done to increase the gravitational potential energy.

7. For the figure below, if P is a force of 20N and the object moves with constant velocity. What is the value of the opposing force F?



Solution:

By the Newton's first law of motion, the object is moving with constant velocity, its resultant force is zero, that is $P - F = 0$

So $F = P = 20N$

8. An object resting on a horizontal surface (**Fig. 8.1**), the resultant force is zero.

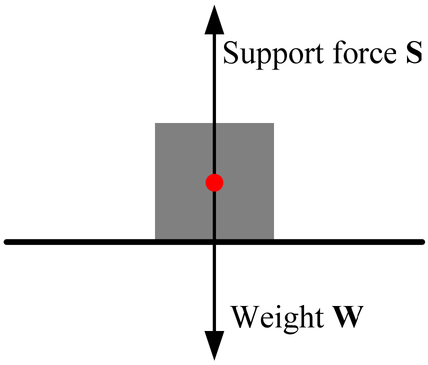


Fig. 8.1 An object resting on a horizontal surface

Then $W = S$

9. An object of weight $\boldsymbol{W}$ = 5N is moving along a rough slope that is at an angle of $\theta = 30°$ to the horizontal with a constant speed, the object is acted by a frictional force $\boldsymbol{F}$ and a support force $\boldsymbol{S}$, as shown in **Fig 9.1:**

Calculate the frictional force $\boldsymbol{F}$ and the support force $\boldsymbol{S}$.

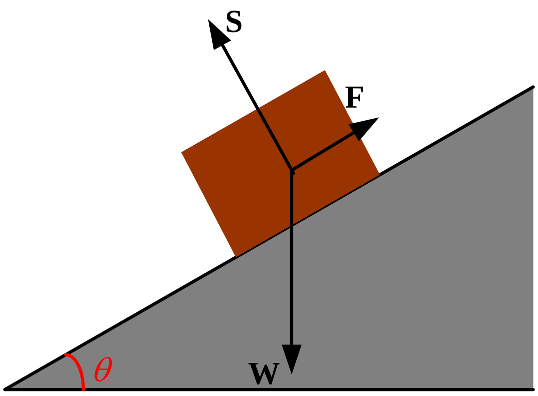


Fig. 9.1

Strategy:

The object moving down the slope with a constant speed means it keeping in equilibrium, that is the resultant of the three forces W, F, S is zero. Therefore resolve the forces along the slope and vertically to the slope (**Fig. 9.2**).

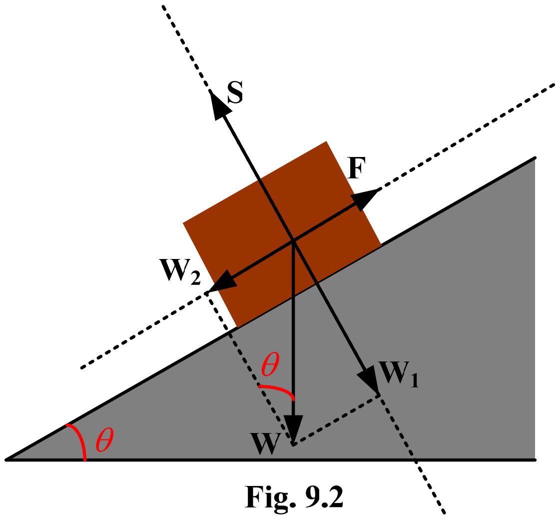


Fig. 9.2

From the figure, by the equilibrium condition,

$\boldsymbol{W_2 = F}$ $\boldsymbol{W_1 = S}$

And $W_2 = W\sin\theta = 5\sin 30^o = 2.5N$

$W_1 = W\cos\theta = 5\cos 30^o = 4.3N$

Frictional force $\boldsymbol{F = W_2} = 2.5\text{N}$

Support force $\boldsymbol{S = W_1} = 4.3\text{N}$

10. Figure 10.1 shows a stationary metal block hanging from the middle of a stretched wire which is suspended from a horizontal beam. The tension in each half of the wire is 15 N.

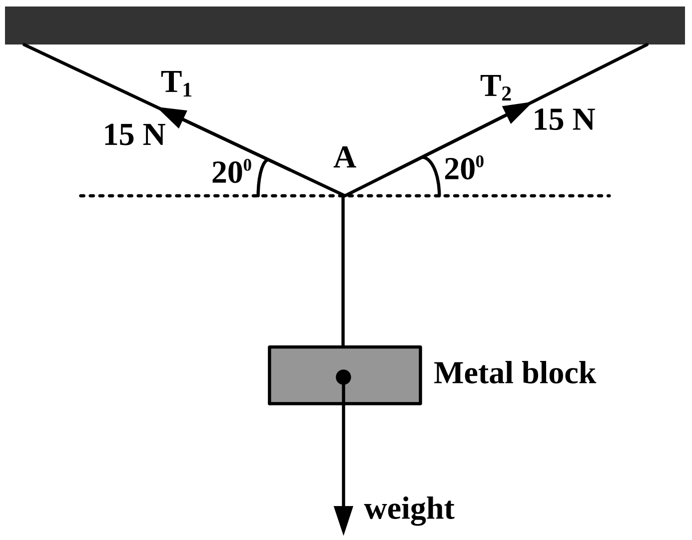


Fig. 10.1

(a) Calculate for the wire at A,

(i) The resultant horizontal component of the tension forces,

The resultant horizontal component of the tension forces is equal to

$T_1 \cos 20 - T_2 \cos 20 = 0$

(ii) The resultant vertical component of the tension forces.

The resultant vertical component of the tension forces,

$T = T_1 \sin 20 + T_2 \sin 20 = 10.3\,N$

(b) (i) State the weight of the metal block.

Strategy: the metal block is at a stationary state,

So weight of the metal block, $W = T = 10.3\,N$

(ii) Explain how you arrived at your answer, with reference to an appropriate law of motion.

Strategy: From Newton's first law, it follows that if an object is at rest or moving at constant velocity, then the forces on it must be balanced.

11. Figure 11.1 shows a sledge moving down a slope at constant velocity. The angle of the slope is 22°.

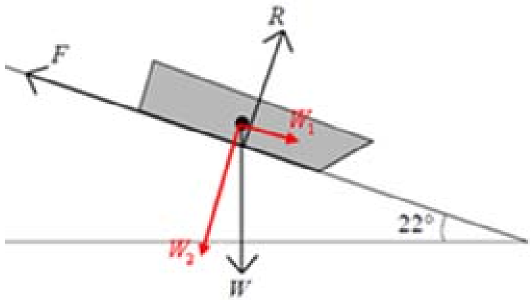


Fig. 11.1

The three forces acting on the sledge are weight, W, friction, F, and the normal reaction force, R, of the ground on the sledge.

(a) With reference to an appropriate law of motion, explain why the sledge is moving at constant velocity.

Solution:

Because the sledge is moving at constant velocity, the resultant force must be zero.

(b) The mass of the sledge is 4.5 kg. Calculate the component of W,

(b) (i) parallel to the slope,

(b) (ii) perpendicular to the slope,

Solution:

(i) parallel to the slope:

$$W_1 = W\sin 22^0 = mg\sin 22^0 = (4.5kg)\cdot(9.81N/kg)\sin 22^0 = 16.5N$$

(ii) Perpendicular to the slope

$$W_2 = W\cos 22^0 = mg\sin 22^0 = (4.5kg)\cdot(9.81N/kg)\cos 22^0 = 41N$$

(c) State the values of F and R.

Solution: The sledge is in equilibrium (moving with constant velocity), thus the resultant force is zero.

Therefore

$$F = W_1 = 16.5N$$

$$R = W_2 = 41N$$

Chapter 4 Energy and Power

4.1 Work

Work is done whenever a force makes something move. It is calculated like this:

Work done=force × distance moved in direction of force.

In symbol: $W = FS$

Work is a scalar quantity. In SI, its unit is **joule**, denoted by **J**. 1 J equals the work done by a force of 1 N when it moves a body through 1 m in its direction.

$1\,J = 1\,N \times 1\,m = 1\,N \cdot m$

1.1 Force and displacement

Imagine an object is acted by a constant force, F, at an angle θ to the direction in which the object moves, and the object moves to a distance S (**Fig. 4.1**).

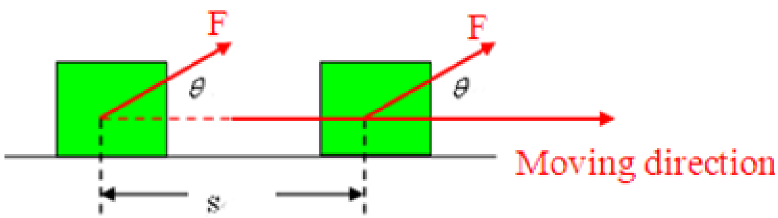


Fig. 4.1

The force has a component $F\cos\theta$ in the direction of motion of the object and a component $F\sin\theta$ at right angles to the direction of motion. So the work done on the object, W, is equal to the component of force in the direction of motion × the distance moved.

$$W = Fs\cos\theta$$

Note: if $\theta = 90^\circ$ (which means that the force is perpendicular to the direction of motion) then, because $\cos 90^\circ = 0$, the work done is zero.

1.2 Force—distance graphs

If a constant force F acts on an object and makes it move a distance s in the direction of the force, the work done on the object $W = Fs$. **Fig. 4.2** shows a graph of force against distance in this situation. The area under the line is a rectangle of height representing the force and of base length representing the distance moved. Therefore the area under the line represents the work done.

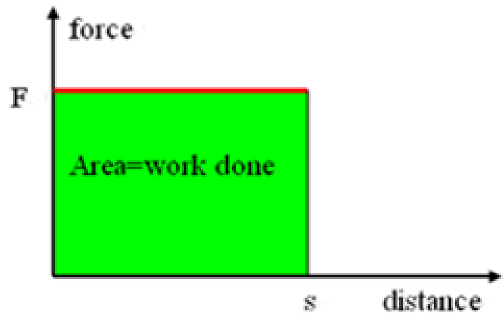


Fig. 4.2 force—distance graph

So the area under the line of a force—distance graph represents the total work done.

4.2 Power

The ratio of the work to the time taken to do the work is called **power**. Power is denoted by P, then

$$P = \frac{W}{t}$$

The SI unit for power is **watt**, denoted by **W**. 1 W = 1 J/s. A watt is a small unit. In technology we usually use kilowatt (kW) as the unit of power. 1 kW = 1000W.

Note: the process of doing work is the process of energy transforming from one state to another. The work done is equal to the energy transformed. Hence, work is the measure of the energy transformed.

Thus

Power can be calculated:

$$power = \frac{energy\ transferred}{time\ taken} \text{ or } power = \frac{work\ done}{time\ taken}$$

2.1 Power and velocity

Power can be calculated from force and velocity. If the force and the displacement are in the same direction, then $W = Fs$。 Substitute it into the equation of power, we get $P = \frac{W}{t} = \frac{Fs}{t}$. As $\frac{s}{t} = v$, then

$$P = F v$$

That means that the work done by force F equals the product of force F and the velocity v of the motion of the body. When the body is in variable motion, v in the above equation denotes the average velocity during time t, and P denotes the **average power** of force F during time t. if the time t is small enough, v in the above equation means the **instantaneous velocity** at a moment and P is the instantaneous power at that moment.

4.3 energy

Things have energy if they can do work. The SI unit of energy is also the joule (J).

3.1 Kinetic energy

This is energy which something has because it is moving.

3.2 Potential energy

This is energy which something has because of its position, shape, or state. A stone about to fall from a cliff has gravitational potential energy. A stretched spring has elastic potential energy. Foods and fuels have chemical potential energy. Charge from a battery has electrical potential energy. Particles from the nucleus of an atom have nuclear potential energy.

3.3 Internal energy

Matter is made up of tiny particles which are in random motion they have kinetic energy because of their motion, and potential energy because of the forces of attraction trying to pull them together. An object's internal energy is the total kinetic and potential energy of its particles.

3.4 Heat (thermal energy)

This is the energy transferred from one object to another because of a temperature difference. Usually, when heat is transferred, one object loses internal energy, and the other gains it.

3.5 Radiant energy

This is often in the form of waves. Sound and light are examples.

3.6 Mechanical energy

(i) Kinetic energy and gravitational and elastic potential energy are sometimes known as mechanical energy. They are the forms of energy most associated with machines and motion.

(ii) Gravitational potential energy is sometimes just called potential energy (or PE), even though there are other forms of potential energy as described above.

3.7 Energy changes

According to the law of conservation of energy:

Energy cannot be made or destroyed, but it can be changed from one form to another.

Whenever there is an energy change, work is done—although this may not always be obvious. For example, when a car's brakes are applied, the car slows down and the brakes heat up, so kinetic energy is being changed into internal energy. Work is done because tiny forces are making the particles of the brake materials move faster.

And an energy change is sometimes called an energy transformation. Whenever it takes place:

Work done = energy transformed

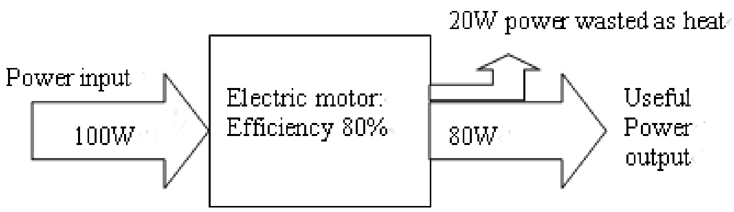
So, for each 1J of energy transformed, 1J of work is done.

4.4 Efficiency

Energy changers such as motors waste some of the energy supplied to them. Their efficiency is calculated like this:

$$Efficiency = \frac{useful\ \ energy\ output}{energy\ \ input} = \frac{useful\ \ power\ output}{power\ \ input}$$

For example, if an electric motor's power input is 100W, and its useful power output is 80W, then its efficiency is 0.8. This can be expressed as 80%.



4.5 Kinetic energy Theorem of kinetic energy

In **Fig. 4.3**, an object of mass m is accelerated from velocity u to ***v*** by a resultant force ***F***. while gaining this velocity, its displacement is ***s*** and its acceleration is ***a***.

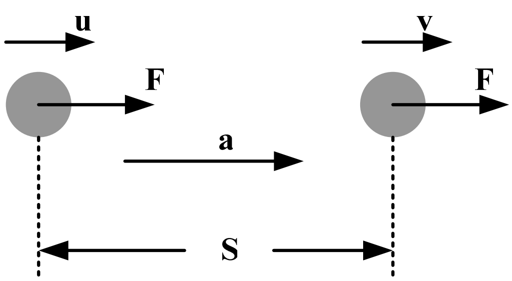


Fig. 4.3

From the law of conservation of energy, the KE gained by the object is equal to the work done on it, Fs.

By the equation $v^2 - u^2 = 2as$, rewritten as $as = \frac{1}{2}v^2 - \frac{1}{2}u^2$

So $mas = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

But $mas = Fs$ (Because $F = ma$)

So $Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Evidently the work done by force F equals the change of the physical quantity $\frac{1}{2}mv^2$. In physics the quantity $\frac{1}{2}mv^2$ is used to represent the kinetic energy of a body. Kinetic energy is denoted by E_k, i.e.

$$E_k = \frac{1}{2}mv^2$$

Kinetic energy is a scalar. It has the same unit as work, in SI it is joule (J) too.

$1 kg \cdot m^2 / s^2 = 1 N \cdot m = 1 J$

For example, if a 2kg tone has a speed of $10 ms^{-1}$, its $KE = \frac{1}{2} \times 2 \times 10^2 = 100J$

Theorem of kinetic energy: the work done by the resultant force on a body is equal to the change in kinetic energy of the body. This conclusion is called the **theorem of kinetic energy**.

The equation can be written as

$$W = E_{k2} - E_{k1}$$

Where E_{k2} represents the final kinetic energy $\frac{1}{2}mv^2$ and E_{k1} represents the initial kinetic energy $\frac{1}{2}mu^2$.

4.6 Gravitational potential energy

Suppose a body of mass m falls from point A of height h_1 to point B of height h_2 (**Fig. 4.4**). The work done by the gravitational force is

$$W_G = mg\Delta h = mgh_1 - mgh_2$$

We see that W_G equals the change of mgh. In physics mgh represents the gravitational potential energy of a body. If gravitational potential energy is denoted by E_p, we have

$$E_p = mgh$$

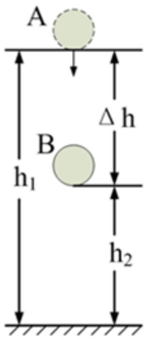


Fig. 4.4

The gravitational potential energy of a body equals the product of its weight and its height. Gravitational potential energy is a scalar. It has the same unit as work and in SI it is joule. $1kg \cdot m \cdot s^{-2} \cdot m = 1N \cdot m = 1J$

We can rewrite the equation $W_G = mgh_1 - mgh_2$ as $W_G = E_{P1} - E_{P2}$, where $E_{P1} = mgh_1$, representing the gravitational potential energy at the initial position and $E_{P2} = mgh_2$, representing the gravitational potential energy at the final position.

4.7 The law of conservation of mechanical energy

When only the gravitational force does work, the kinetic and the gravitational potential energy can be converted from one to another and the total mechanical energy remains conserved.

This conclusion is called the **law of conservation of mechanical energy.**

The equation can be written as:

$$E_{k2} + E_{p2} = E_{k1} + E_{p1}$$

4.8 17 Worked examples

1. On a fairground ride, the track descends by a vertical drop of 55m over a distance of 120 along the track. A train of mass 2500kg on the track reaches a speed of 30ms^{-1} at the bottom of the descent after being at rest at the top. Calculate (a) the loss of potential energy of the train, (b) its gain of kinetic energy, (c) the average frictional force on the train during the descent.

Solution:

(a) Loss of potential energy, $PE = mg\Delta h = 2500 \times 9.8 \times 55 = 1.35 \times 10^6 J$

(b) Its gain of Kinetic energy, $KE = \frac{1}{2}mv^2 = 0.5 \times 2500 \times 30^2 = 1.13 \times 10^6 J$

(c) Work done overcome friction = $mg\Delta h - \frac{1}{2}mv^2$

$$= 1.35 \times 10^6 - 1.13 \times 10^6 = 0.22 \times 10^6 \, J$$

Because the work done to overcome friction = friction force × distance moved along track.

$$the\ frictional\ force = \frac{work\ done\ to\ overcome\ friction}{distance\ moved}$$

$$= \frac{0.22 \times 10^6}{120} = 1830N$$

2. An aircraft powered by engines that exert a force of 40 kN is in level flight at a constant velocity of $80ms^{-1}$. Calculate the output power of the engine at this speed.

Solution:

$$power = force \times velocity = 40000 \times 80 = 3.2 \times 10^6 W$$

3. A 60W electric motor raises a weight of 20N through a height of 2.5m in 8.0s. Calculate:

a. the electrical energy supplied to the motor.

b. the useful energy transferred by the motor,

c. the efficiency of the motor.

Solution:

a.

Electrical energy transferred = power of the electric motor × time taken

So the electrical energy supplied to the motor, $E = Pt = 60 \times 8 = 480J$

a. The useful energy transferred by the motor is the potential energy of the weight gained.

$$PE = mgh = 20 \times 2.5 = 50J$$

c. $Efficiency = \dfrac{useful\ energy\ output}{energy\ input} = \dfrac{useful\ power\ output}{power\ input}$

$$= \frac{50J}{480J} \times 100\% = 10.4\%$$

4. A child tows a toy by means of a string as shown in **Fig. 4.1**.

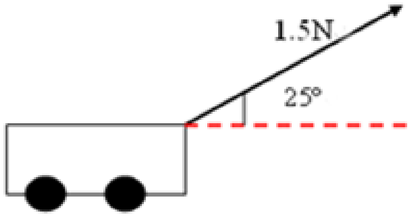


Fig. 4.1

The tension in the string is 1.5N and the string makes an angle of $25°$ with the horizontal. Calculate the work done in moving the toy horizontally through a distance of 265 cm.

Solution:

Work done = horizontal component of tension × distance moved

$$= 1.5\cos 25^{\circ} \times \frac{265}{100} = 3.6J$$

5. A skydiver of mass 70 kg, jumps from a stationary balloon and reaches a speed of 45ms^{-1} after falling a distance of 150 m.

(a) Calculate the skydiver's

(i) Loss of gravitational potential energy,

Strategy: loss of gravitational potential energy, $\Delta E_p = mgh$

So $\Delta E_p = mgh = 70 \times 9.8 \times 150 = 1.03 \times 10^5\ J$

(ii) Gain in kinetic energy.

Strategy: the speed gain is 45ms^{-1}, so the kinetic energy gain, $\Delta E_k = \frac{1}{2}mv^2$

So $\Delta E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times 45^2 = 7.09 \times 10^4\ J$

(b) The difference between the loss of gravitational potential energy and the gain in kinetic energy is equal to the work done against air resistance. Use this fact to calculate

(i) The work done against air resistance,

Answers: the work done, $W = \Delta E_p - \Delta E_k$

So $W = \Delta E_p - \Delta E_k = 1.03 \times 10^5 - 7.09 \times 10^4 = 0.321 \times 10^5\ J$

(ii) The average force due to air resistance acting on the skydiver.

Strategy: Work done = force × distance moved in direction of force.

So $W = 0.321 \times 10^5\ J = F \times 150$

$F = 214\,N$

6. A packing case is being lifted vertically at a constant speed by a cable attached to a crane. The packing case has a mass of 640 kg.

(a) With reference to one of Newton's laws of motion, explain why the tension, T, in the cable must be equal to the weight of the packing case.

You may be awarded marks for the quality of written communication in your answer.

Answers: because the packing case is being lifted vertically at a constant speed, by the Newton's first law, the resultant force on the case must be zero, so the tension, T, must be equal to the weight of the case.

(b) The packing case is lifted through a vertical height of 8.0 m in 4.5 s. Calculate

(i) The work done on the packing case,

Strategy: Work done = force × distance moved in direction of force.

In symbol: $W = FS$

And the tension on the packing case, $T = mg = 640 \times 9.8 = 6272N$

So $W = TS = 6272 \times 8 = 5.02 \times 10^4 J$

(ii) The power output of the crane in this situation.

Strategy: $power = \dfrac{energy\ transferred}{time\ taken}$ or $power = \dfrac{work\ done}{time\ taken}$

So $P = \dfrac{W}{t} = \dfrac{5.02 \times 10^4}{4.5} = 1.1 \times 10^4 W$

7. A fairground ride ends with the car moving up a ramp at a slope of 30° to the horizontal as shown in **Fig. 7.1**.

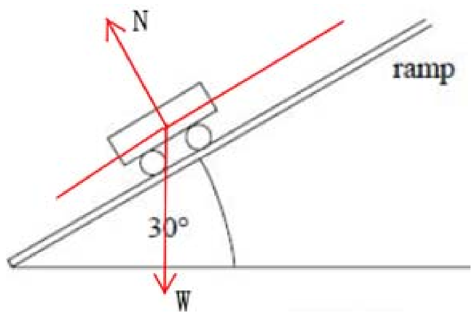


Fig. 7.1

(a) The car and its passengers have a total weight of 7.2×10^3 N. Show that the component of the weight parallel to the ramp is 3.6×10^3 N.

Strategy: from the figure, the component of the weight parallel to the ramp,

$W_p = W \sin 30^0 = 7.2 \times 10^3 \times 0.5 = 3.6 \times 10^3\ N$

(b) Calculate the deceleration of the car assuming the only force causing the car to decelerate is that calculated in part (a).

Strategy: mass of the car, $m = \frac{W}{g} = \frac{7.2 \times 10^3}{9.8} = 734.7kg$, and the only force cause the car decelerating is $W_p = 3.6 \times 10^3\ N$

So the deceleration of the car, $a = \frac{W_p}{m} = \frac{3.6 \times 10^3}{734.7} = 4.9ms^{-2}$

(c) The car enters at the bottom of the ramp at $18ms^{-1}$. Calculate the minimum length of the ramp for the car to stop before it reaches the end. The length of the car should be neglected.

Strategy: initial speed, $u = 18ms^{-1}$, final speed, $v = 0ms^{-1}$,

So $v^2 - u^2 = 2as \Rightarrow s = \frac{v^2 - u^2}{2a}$

So the minimum length of the ramp, $L = \frac{v^2 - u^2}{2a} = \frac{0 - 18^2}{2 \times (-4.9)} = 33.1m$

NOTE: $a = -4.9ms^{-2}$, because the car is decelerating.

(d) Explain why the stopping distance is, in practice, shorter than the value calculated in part (c).

Because in the process, the friction force must be considered, so the resultant force increases, the deceleration increases, and also some of the energy change to heat energy, so the stopping distance is shorter than the value calculated in part (c).

8. Figure 8.1 shows apparatus that can be used to investigate energy changes.

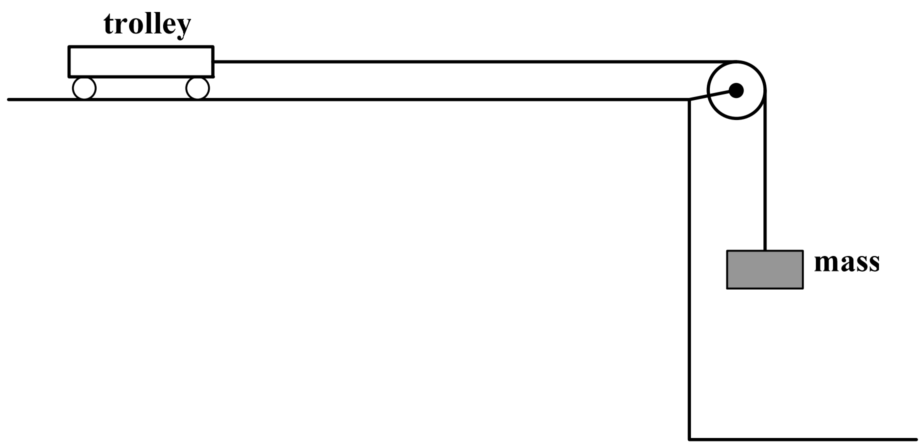


Fig. 8.1

The trolley and the mass are joined by an inextensible string. In an experiment to investigate energy changes, the trolley is initially held at rest, and is then released so that the mass falls vertically to the ground.

You may be awarded marks for the quality of written communication in your answer.

(a) (i) State the energy changes of the falling mass.

Answers: the gravitational potential energy change to kinetic energy.

(ii) Describe the energy changes that take place in this system.

Answers: The gravitational potential energy of the mass change to the kinetic energy of the trolley and the mass; and at the same time some of the energy change to thermal energy due to friction.

(b) State what measurements would need to be made to investigate the conservation of energy?

Strategy: the gravitational potential energy is equal to mgh, and the kinetic energy is equal to $\frac{1}{2}mv^2$,

So you need to know the masses of the trolley (M) and the falling mass (m), and also the height (h) the falling mass falls, and the speed (v) of the trolley and the falling mass.

(c) Describe how the measurements in part (b) would be used to investigate the conservation of energy.

Strategy:

(i) Determine the loss potential energy (mgh) of the falling mass when it falls from rest to the ground.

(ii) Calculate the speed of the trolley and the falling mass: v

(iii) Calculate the Kinetic energy of the trolley and the falling mass,

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}(M+m)v^2$$

(iv) Compare the loss of potential energy with the gain of the kinetic energy.

9. Figure 9.1 shows a skateboarder descending a ramp.

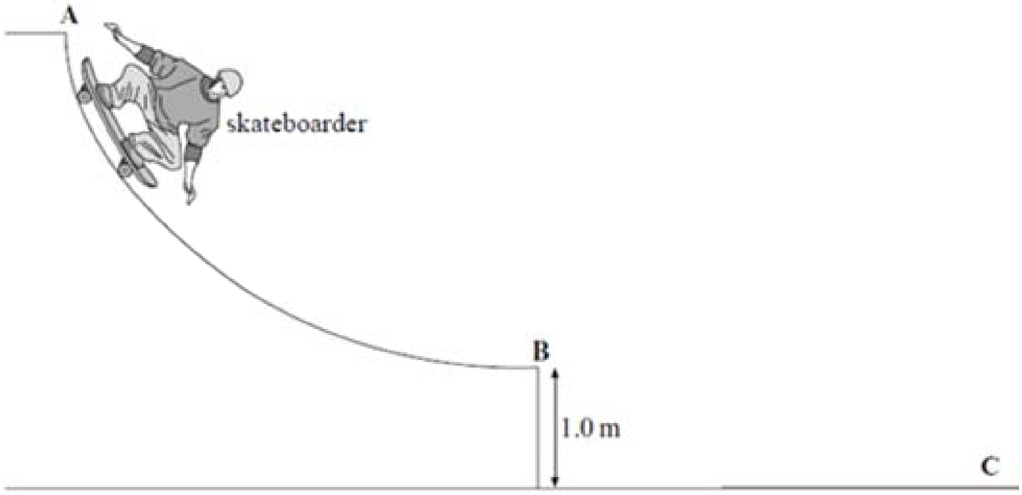


Fig. 9.1

The skateboarder starts from rest at the top of the ramp at A and leaves the ramp at B horizontally with a velocity *v*.

(a) State the energy changes that take place as the skateboarder moves from A to B.

Solution:

The gravitational potential energy changes to kinetic energy and heat energy.

(b) In going from A to B the skateboarder's centre of gravity descends a vertical height of 1.5 m. Calculate the horizontal velocity, v, stating an assumption that you make.

Solution:

Assuming the energy converts to thermal energy is negligible. Thus, loss of potential energy = gain in kinetic energy.

Therefore,

$\frac{1}{2}mv_h^{\ 2} = mg\Delta h$, gives

$$v_h = \sqrt{2g\Delta h} = \sqrt{2\times 9.81\times 1.5} = 5.42 m/s$$

(c) Explain why the acceleration decreases as the skateboarder moves from **A** to **B**.

Solution:

There is air resistance which increases with speed, thus when the skateboarder moves down the slope the resultant force decreases, causing the decreasing acceleration.

(d) After leaving the ramp at **B** the skateboarder lands on the ground at **C** 0.42 s later.

Calculate for the skateboarder

(i) The horizontal distance travelled between **B** and **C**,

Solution:

Horizontally, there is no force acting on the skateboarder. Thus, the skateboarder moves with constant speed horizontally ($v_h = 5.42 m/s$).

Therefore, the distance traveled is given by

$s = v_h t = 5.42 \times 0.42 = 2.28 m$

(ii) The vertical component of the velocity immediately before impact at C,

Solution:

Vertically, the acceleration is a = g = 9.81 m/s^2.

The vertical component of the velocity $v_v = at = gt = 9.81 \times 0.42 = 4.12 m/s$

(iii) The magnitude of the resultant velocity immediately before impact at C.

Solution:

Magnitude of the resultant velocity $v = \sqrt{v_h^2 + v_v^2} = \sqrt{5.42^2 + 4.12^2} = 6.4 m/s$

10. Fig. 10.1 shows a ship being pulled along by cables attached to two tugs.

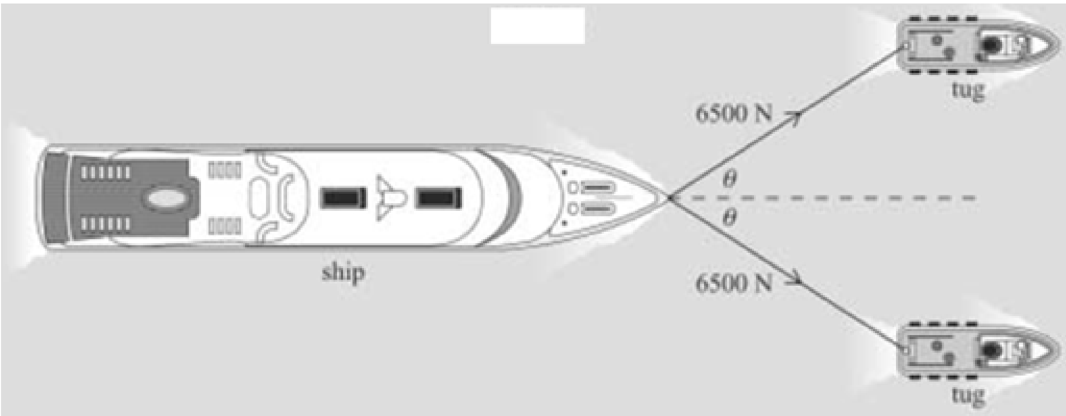


Fig. 10.1

(a) The tension in each cable is 6500 N and the ship is moving at a constant speed of $1.5m/s$. When θ is equal to 35°, calculate

(i) The resultant force exerted on the ship by the cables,

Solution:

The resultant force exerted on the ship by the cables is given by

$F = 6500\cos\theta + 6500\cos\theta = 2\times 6500\cos 35 = 10649N$

(ii) The work done by the tension in the cables in one minute.

Solution:

In one minute, the distance traveled is given by

$s = vt = 1.5\times 1\times 60 = 90m$

The worked done is

$W = F\cdot s = 10649\times 90 = 9.58\times 10^5 J$

(b) Explain why the work done on the ship does not result in a gain in its kinetic energy.

Solution:

There is frictional force on the ship, the total work done on this force. Thus it does not result in a gain in its kinetic energy.

(c) State and explain the initial effect on the ship if the angle θ is reduced while the tension in the cables remains constant.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

The resultant force on the ship by the cables increases Thus, the ship initially accelerates. And the ship eventually reaches new higher constant speed.

11. An athlete performs an experiment to measure the power developed as he runs up a flight of stairs. The athlete makes the assumption that the work done in climbing the stairs is equal to the gain in potential energy.

(i) State the measurements that would be needed to find the power developed by the athlete.

Solution:

(1) Find the students' weight

(2) Measure the vertical height of stairs.

(3) Time taken for the student to run up the stairs.

(ii) Show how the measurements would be used to calculate the power developed as the athlete runs up the stairs.

Solution:

Using $E_p = mgh$, we can get the work done by the athlete: $W = E_p = mgh$.

Then the power is given by

$$P = \frac{W}{t}$$

(iii) Explain why the power calculated by the athlete is likely to be less than the power actually developed.

Solution:

(1) Not all the work done goes to E_p.

(2) Do not considering the kinetic energy.

12. A pile driver is used to drive cylindrical poles, called piles, into the ground so that they form the foundations of a building. **Fig. 12.1** shows a possible arrangement for a pile driver. The hammer is held above the pile and then released so that it falls freely under gravity, until it strikes the top of the pile.

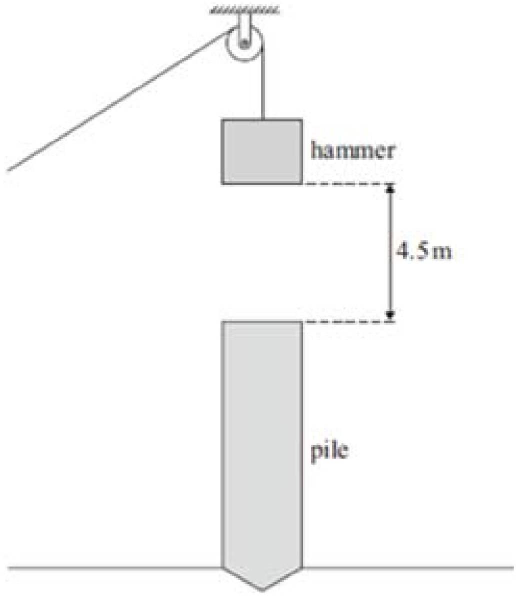


Fig. 12.1

(a) State the main energy changes that take place as the hammer is falling.

Solution:

The hammer falls freely under gravity, which means ignoring the air resistance. Thus the main energy changes: the potential energy converts to kinetic energy.

(b) The hammer has a mass of 250 kg and falls 4.5 m before striking the pile. After impact the hammer and pile move downwards together.
Calculate
(b) (i) the speed of the hammer just before impact,
(b) (ii) the momentum of the hammer just before the impact,
(b) (iii) the speed of the hammer and pile immediately after impact if the mass of the pile is 2000 kg.
Solutions:
(i) Let the speed of the hammer just before impact v.
When the hammer falls 4.5 m, loss of potential energy is given by

$$PE = mgH = (250kg) \times (9.81N/kg) \times 4.5m = 11036J$$

And the gain in kinetic energy is equal to the loss of potential, thus

$$\frac{1}{2}mv^2 = PE = 11036J$$

$$v = 9.4m/s$$

(ii) Let the mass of the hammer m = 250 kg.

$$p = mv = (250kg) \times 9.4m/s = 2350kg \cdot m/s$$

(iii) Let the speed of the hammer and the pile immediately after impact V, the total mass of hammer and the pile is M. by the Principle of conservation of linear momentum:

$$MV = mv = 2350kg \cdot m/s$$

$$V = \frac{2350kg \cdot m/s}{(2000+250)kg} = 1.0m/s$$

(c) After an impact the hammer and the pile move so that the pile sinks into the ground to a depth of 0.25 m.
Calculate
(c) (i) the loss of kinetic energy of the hammer and pile,
(c) (ii) the average frictional force the ground exerts on the pile while bringing it to rest.
Solution:
(i) The loss of kinetic energy is given by

$$\Delta E_k = \frac{1}{2}MV^2 - 0 = \frac{1}{2}MV^2 = \frac{1}{2} \times (2000kg + 250kg) \times 1^2 = 1125J$$

(ii) The work done by the average frictional force is equal to the loss of the kinetic energy. Thus

$$fs = 1125J$$

$$f = \frac{1125J}{0.25m} = 4500N$$

(d) The process is repeated several times and each time the hammer is raised 4.5 m above the pile. Suggest why the extra depth of penetration is likely to decrease with each impact.

Solution:

The resistive force from the ground will increase as the pile gets deeper in the ground.

13. A cyclist pedals downhill on a road, as shown in **Fig. 13. 1**, from rest at the top of the hill and reaches a horizontal section of the road at a speed of 16 m s^{-1}. The total mass of the cyclist and the cycle is 68 kg.

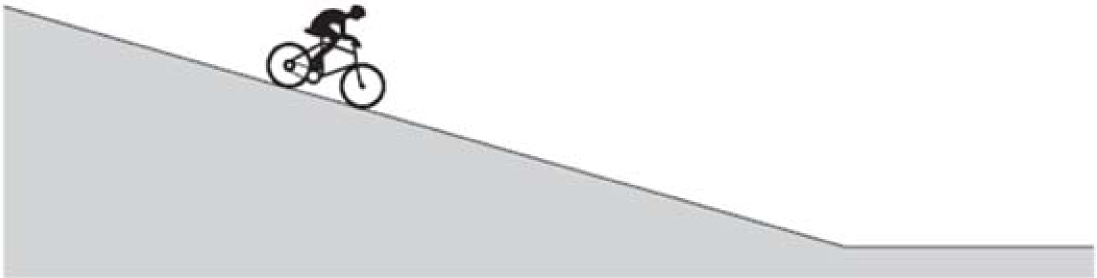


Fig. 13.1

(a) (i) Calculate the total kinetic energy of the cyclist and the cycle on reaching the horizontal section of the road.

Solution:

The total kinetic energy is given by

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\times 68\times 16^2 = 8704J$$

(a) (ii) The height difference between the top of the hill and the horizontal section of road is 12 m.

Calculate the loss of gravitational potential energy of the cyclist and the cycle.

Solution:

The loss of gravitational potential energy is given by

$$\Delta E_p = mg\Delta h = 68\times 9.8\times 12 = 7997J$$

(a) (iii) The work done by the cyclist when pedalling downhill is 2400 J.

Account for the difference between the loss of gravitational potential energy and the gain of kinetic energy of the cyclist and the cycle.

Solution:

Gain of kinetic energy is greater than the loss of potential energy, because the cyclist does wore, and some of the energy is wasted due to air resistance. Thus

Gain of kinetic energy = loss of potential energy + word done – energy 'lost'

Therefore, energy wasted = 7997 + 2400 – 8704 = 1693J

(b) The cyclist stops pedalling on reaching the horizontal section of the road and slows to a standstill 160 m further along this section of the road. Assume the deceleration is uniform.

(b) (i) calculate the time taken by the cyclist to travel this distance.

Solution:

On the horizontal section of the road, the initial velocity $u = 16m/s$, final velocity v = 0 m/s, distance traveled s = 160 m. thus

$v^2 - u^2 = 2as$, and $a = \dfrac{v-u}{t}$, therefore

$0 - 16^2 = 2 \times \dfrac{0-16}{t} \times 160$, gives t= 20 s

(b) (ii) Calculate the average horizontal force on the cyclist and the cycle during this time.

Solution:

By Newton's second law, $F = ma$, and $a = \dfrac{v-u}{t} = \dfrac{0-16}{20} = -0.8m/s^2$

Therefore $F = ma = 68 \times (-0.8) = -54.4N$

14. It has been predicted that in the future large offshore wind turbines may have a power output ten times that of the largest ones currently in use. These turbines could have a blade length of 100 m or more. A turbine such as this is shown in **Fig. 14.1**.

Fig. 14.1

(a) At a wind speed of $11 ms^{-1}$ the volume of air passing through the blades each second is $3.5 \times 10^5 m^3$.

(a) (i) Show that the mass of air that would pass through the blades each second is about 4×10^5 kg.

The density of air is 1.2 kg m^{-3}

Solution;

$m = \rho V = 1.2 \times 3.5 \times 10^5 = 4 \times 10^5 kg$

(a) (ii) Calculate the kinetic energy of the air that would enter the turbine each second.

Solution: in one second

The kinetic energy is given by

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 10^5 \times 11^2 = 2.42 \times 10^7 J$$

(a) (iii) It has been predicted that the turbine would produce an electrical power output of 10 MW in these wind conditions. Calculate the percentage efficiency of the turbine in converting this kinetic energy into electrical energy.

Solution;

The efficiency is calculated like this:

$$Efficiency = \frac{useful\ energy\ output}{energy\ input} = \frac{useful\ power\ output}{power\ input}$$

Thus

$$Efficiency = \frac{useful\ energy\ output}{energy\ input} = \frac{(10 \times 10^6 W) \times 1s}{2.42 \times 10^7 J} \times 100\% = 41.3\%$$

(b) State one advantage and one disadvantage of wind power in comparison to fossil fuel.

Solution:

Advantages: the wind causes no air pollution

Disadvantages: it can cause visual pollution (noise). Or there may be danger to birds.

15. Fig. 15.1 shows a gymnast trampolining.

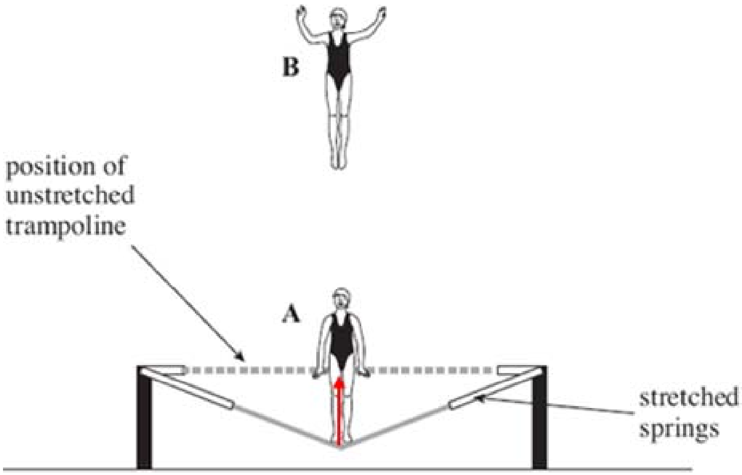


Fig. 15.1

In travelling from her lowest position at A to her highest position at B, her centre of mass rises 4.2 m vertically. Her mass is 55 kg.

(a) Calculate the increase in her gravitational potential energy when she ascends from position A to position B.

Solution:

The gravitational potential energy is given by

$E_p = mg\Delta h = 55 \times 9.81 \times 4.2 = 2266J$

(b) The gymnast descends from position B and regains contact with the trampoline when it is in its unstretched position. At this position, her centre of mass is 3.2 m below its position at B.

(b) (i) Calculate her kinetic energy at the instant she touches the unstretched trampoline.

Solution:

In this process, her potential energy change into kinetic energy, thus

$E_p = mg\Delta h = E_k = \frac{1}{2}mv^2$

Therefore,

$E_k = E_p = mg\Delta h = 55 \times 9.81 \times 3.2 = 1727\,J$

(b) (ii) Calculate her vertical speed at the same instant.

Solution:

$E_k = \frac{1}{2}mv^2 = 1727\,J$

Gives

$v = \sqrt{\frac{2 \times 1727}{55}} = 7.92\,m/s$

(c) Draw an arrow on **Fig. 15.2** to show the force exerted on the gymnast by the trampoline when she is in position A.

Solution:

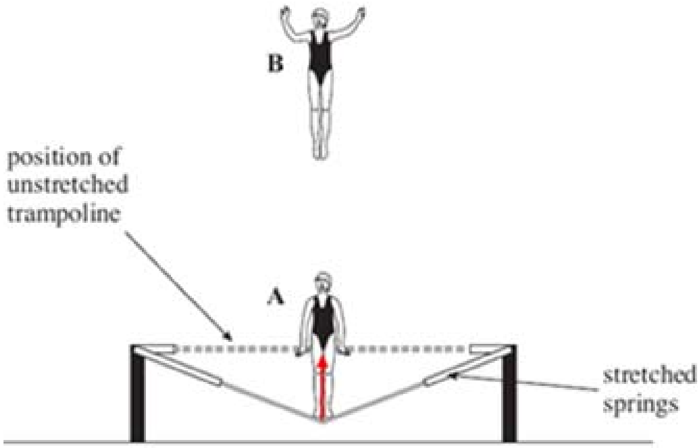


Fig. 15.2

(d) As she accelerates upwards again from position A, she is in contact with the trampoline for a further 0.26 s. Calculate the average acceleration she would experience while she is in contact with the trampoline, if she is to reach the same height as before.

Solution:

Because she is to reach the same height as before, the energy is conserved. Thus, the initial velocity $u = 0m/s$; the final velocity $v = 7.92m/s$, upwards. Therefore,

Acceleration, $a = \frac{v-u}{t} = \frac{7.92-0}{0.26} = 30m/s^2$

(e) On her next jump the gymnast decides to reach a height above position B. Describe and explain, in terms of energy and work, the transformations that occur as she ascends from her lowest position A until she reaches her new position above B.

The quality of your written communication will be assessed in this question.

Solution:

(i) The elastic potential energy is transformed to kinetic energy.

(ii) The kinetic energy is transformed into gravitational potential energy.

(iii) The gymnast must do work to increase height.

16 (a) Define a vector quantity and give one example.

Strategy and solution:

Scalar: quantity has direction only.

Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

Examples of vectors: force, acceleration, displacement, velocity, momentum…

(b) **Fig. 16.1** shows a force F at an angle of 30^0 to the horizontal direction.

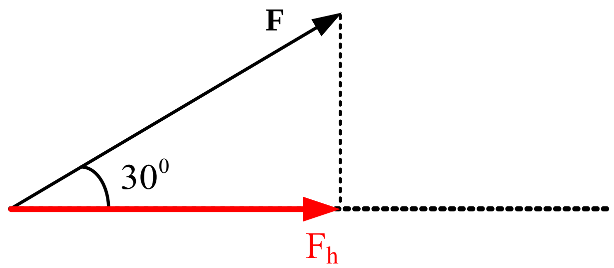


Fig. 16.1

(i) The horizontal component of the force F is 7.0 N. Calculate the magnitude of the force F.

Strategy and solution:

Form **Fig. 16.1**, $F_h = F\cos 30^0 = 7.0N$, thus, $F = \dfrac{7.0N}{\cos 30^0} = 8.10N$

(ii) The force F moves an object in the horizontal direction. In a time of 4.2 s, the object moves a horizontal distance of 5.0 m. calculate

1 the work done by the force

Strategy and solution:

Force and displacement

Imagine an object is acted by a constant force, F, at an angle θ to the direction in which the object moves, and the object moves to a distance S

(**Fig. 16.2**).

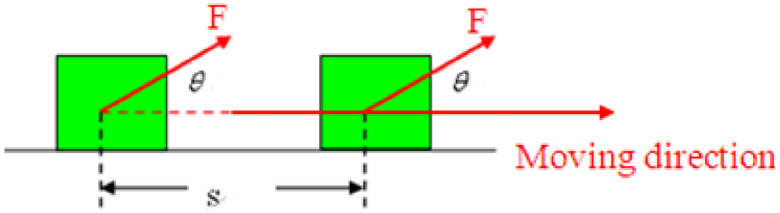


Fig. 16.2

The force has a component $F\cos\theta$ in the direction of motion of the object and a component $F\sin\theta$ at right angles to the direction of motion. So the work done on the object, W, is equal to the component of force in the direction of motion × the distance moved.

$W = Fs\cos\theta$

Note: if $\theta = 90^\circ$ (which means that the force is perpendicular to the direction of motion) then, because $\cos 90^\circ = 0$, the work done is zero.

Therefore,

$$W = Fs\cos\theta = 8.10N \times 5m \times \cos 30^0 = 35.1Nm$$

2 the rate of work done by the force

Strategy and solution:

The rate of work done by the force is Power. Thus

$$Power = \frac{work\ \ done}{time\ taken} = \frac{35J}{4.2s} = 8.33W$$

(c) **Fig. 16.3** shows the forces acting on a stage light of weight 120 N held stationary by two separate cables.

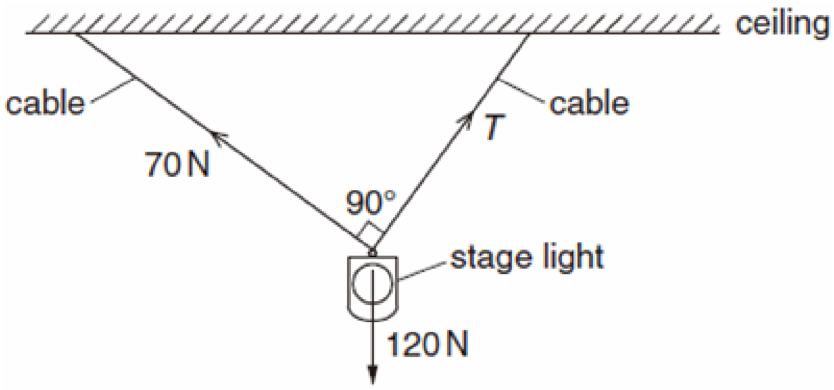


Fig. 16.3

The angle between the two cables is 90^0. One cable has tension 70 N and the other has tension T.

(i) State the magnitude and the direction of the resultant of the tensions in the two cables.

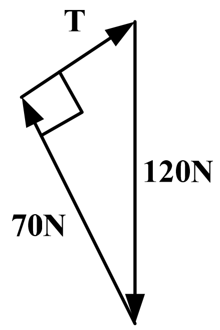
Strategy and solution:

The stage light is in stationary, thus the resultant force of 70N, T and 120N is zero. Therefore the resultant force of T and 70 N is equal but opposite to the 120N.

Therefore, the direction of the two cables is opposite to the direction of weight.

(ii) Sketch a labelled vector triangle for the forces acting on the stage light. Hence, determine the magnitude of the tension T.

Strategy and solution:



Thus,

$T^2 + 70^2 = 120^2$

T = 97.5N

17 (a) State the principle of conservation of energy.

Strategy and solution:

Energy can neither be created nor destroyed (but it can be transformed from one form to another). The total energy of a closed system remains constant.

(b) **Fig. 17.1** shows a glider on a horizontal frictionless track.

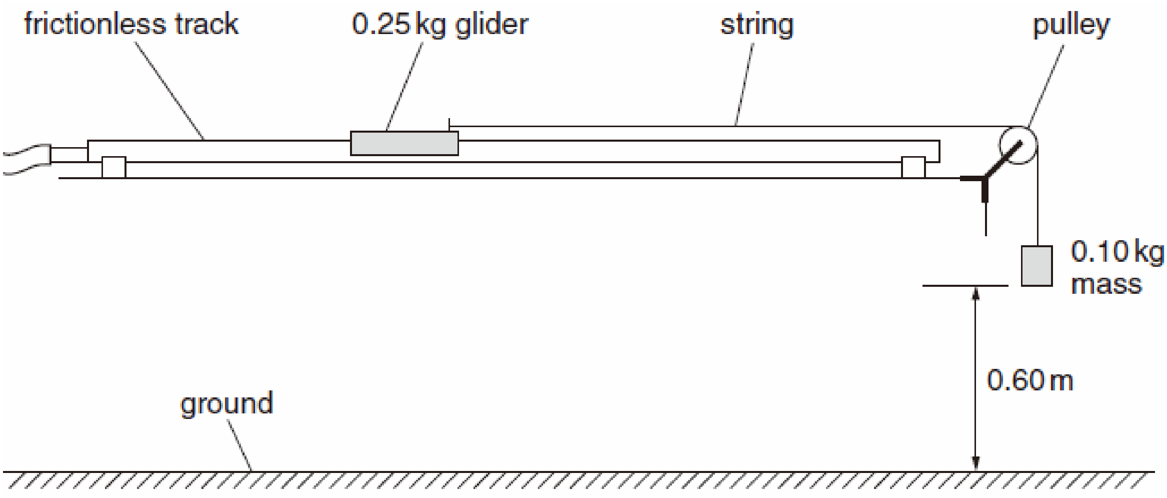


Fig. 17.1

The mass of the glider is 0.25 kg. One end of a string is fixed to the glider and the other end to a 0.10 kg mass. The 0.10 kg mass is held stationary at a height of 0.60m from the ground. The pulley is more than 0.60m away from the front of the glider. When the 0.10 kg mass is released, the glider has a constant acceleration of $2.8ms^{-2}$ towards the pulley. The 0.10 kg mass instantaneously comes to rest when it hits the ground.

(i) Calculate the loss in potential energy of the 0.10 kg mass as it falls through the distance of 0.60m

Strategy and solution:

$E_p = mg\Delta h = 0.10\times 9.81\times 0.6 = 0.59J$

(ii) The glider starts from rest. Show that the velocity of the glider after travelling a distance of 0.60m is about $1.8ms^{-1}$.

Strategy and solution:

The glider has a constant acceleration of $2.8ms^{-2}$ towards the pulley, thus, the 0.10kg mass also has a constant acceleration of $2.8ms^{-2}$.

Therefore, the final velocity of the mass after travelling a distance of 0.60m is given by

$v = \sqrt{2as} = \sqrt{2\times 2.8\times 0.6} = 1.83ms^{-1}$ ($v^2 = u^2 + 2as = 2as$ and initial velocity is zero)

(iii) Calculate the kinetic energy of the glider at this velocity of $1.8ms^{-1}$.

Strategy and solution: