mock papers 1

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

Question 1 continued

(Total 4 marks)

Q1

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2} .$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

Question 2 continued

(Total 8 marks)

Q2

Turn over

3. $$\mathrm{f}(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $\mathrm{f}(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \; x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $\mathrm{f}(x) = 0$ correct to 3 decimal places.

(2)

Question 3 continued

Q3

(Total 7 marks)

4.

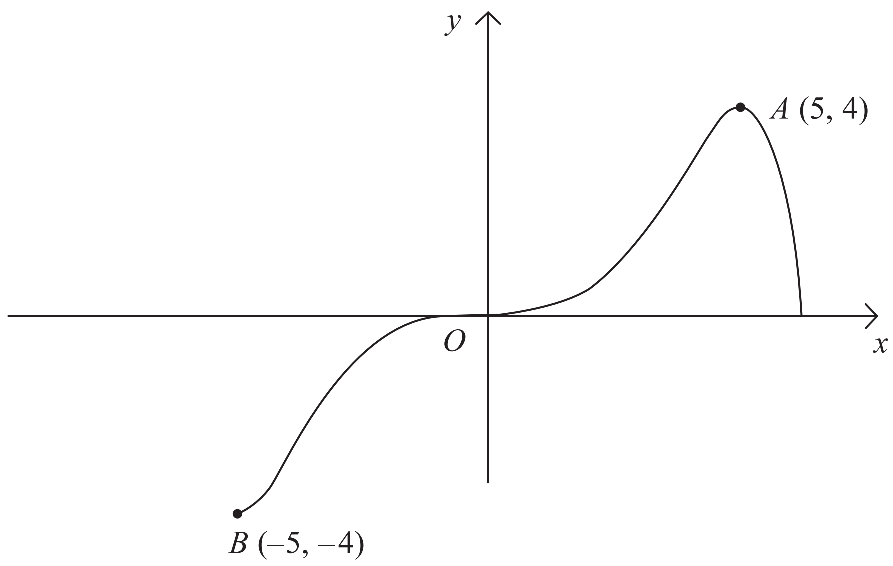

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}(|x|)$,

(3)

(c) $y = 2\mathrm{f}(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

Question 4 continued

Turn over

Question 4 continued

Question 4 continued

Q4

(Total 10 marks)

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \qquad t \geqslant 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when $t = 22\ 920$.

(2)

(d) In the space provided on page 13, sketch the graph of R against t .

(2)

Question 5 continued

(Total 9 marks)

Q5

Turn over

6. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

Question 6 continued

Turn over

Question 6 continued

Question 6 continued

(Total 11 marks)

Q6

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

Question 7 continued

Turn over

Question 7 continued

Question 7 continued

Q7

(Total 13 marks)

Turn over

8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$
$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

Question 8 continued

Turn over

Question 8 continued

(Total 13 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END

mock papers 2

1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

2. $$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leqslant x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

3.

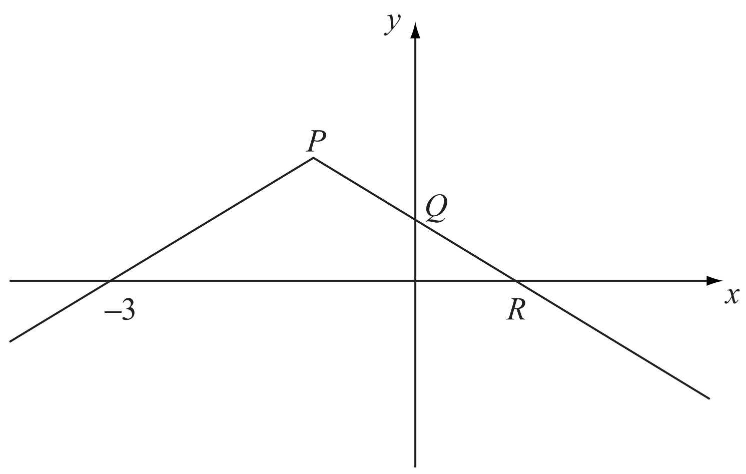

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.
The graph consists of two line segments that meet at the point *P*.
The graph cuts the *y*-axis at the point *Q* and the *x*-axis at the points (–3, 0) and *R*.
Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$,

(2)

(b) $y = \mathrm{f}(-x)$.

(2)

Given that $\mathrm{f}(x) = 2 - |x+1|$,

(c) find the coordinates of the points *P*, *Q* and *R*,

(3)

(d) solve $\mathrm{f}(x) = \frac{1}{2}x$.

(5)

4. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2 - 2x - 3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \dfrac{1}{x+1}, \quad x > 3.$

(4)

(b) Find the range of f.

(2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \dfrac{1}{8}$.

(3)

5. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leqslant \theta < 180°$, the equation

$$2\cot^2\theta - 9\,\text{cosec}\,\theta = 3,$$

giving your answers to 1 decimal place.

(6)

6. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x+2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}, \quad x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

7.

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with x_0=1.43, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

Question 7 continued

(Total 11 marks)

Q7

TOTAL FOR PAPER: 75 MARKS

END

mock papers 3

1. (a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

2.
$$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $f(x)$ as a single fraction in its simplest form.
(4)

(b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$
(3)

3.

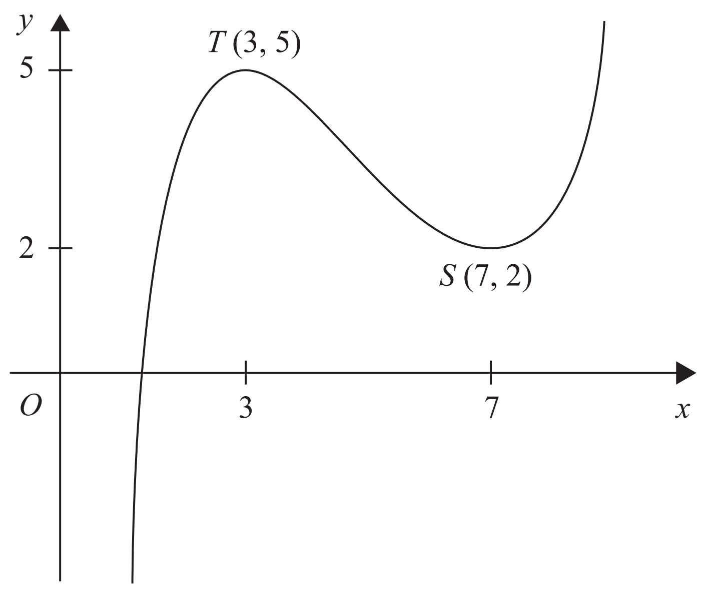

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x) - 4$, **(3)**

(b) $y = |\mathrm{f}(x)|$. **(3)**

Indicate on each diagram the coordinates of any turning points on your sketch.

4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$.

(6)

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3\theta - 6 \sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

7. $$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

8. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

mock papers 4

1.

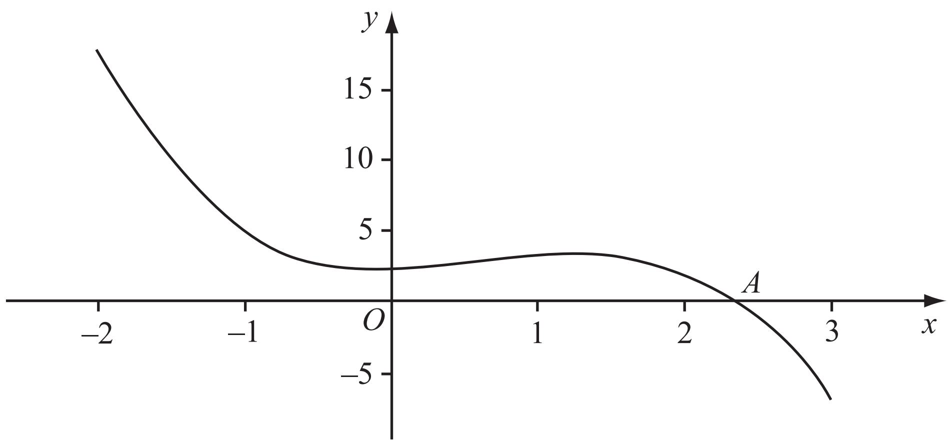

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

2. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

(6)

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \qquad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.
(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.
(2)

(c) Find $\frac{dP}{dt}$.
(2)

(d) Find P when $\frac{dP}{dt} = 50$.
(3)

4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y=\sqrt{(4x+1)}\,, \quad x>-\tfrac{1}{4},\ y>0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax+by+c=0$, where a, b and c are integers.

(6)

5.

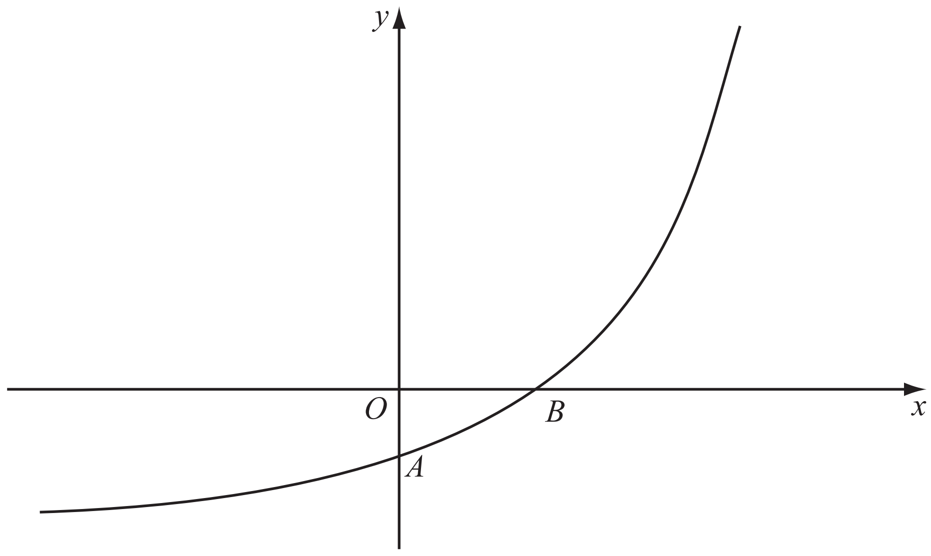

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$,

where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$, **(3)**

(b) $y = \mathrm{f}^{-1}(x)$. **(2)**

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f , **(1)**

(d) find $\mathrm{f}^{-1}(x)$, **(3)**

(e) write down the domain of f^{-1}. **(1)**

6. (a) Use the identity $\cos(A+B) = \cos A\cos B - \sin A\sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leqslant x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $g'(x) = 1$ **(4)**

8. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)

Question 8 continued

(Total 6 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END