**mock papers 1**

**1.**

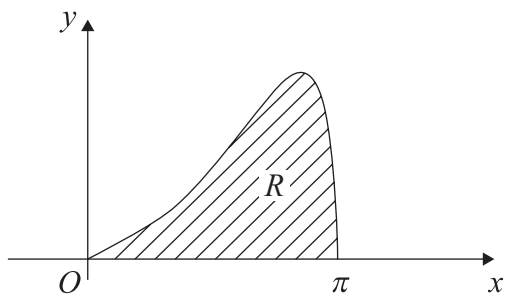


**Figure 1**

The curve shown in Figure 1 has equation $y = e^x\sqrt{(\sin x)}$ , $0 \leqslant x \leqslant \pi$ . The finite region $R$ bounded by the curve and the $x$-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of $y$ corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

| $x$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|
| $y$ | 0 | | | 8.87207 | 0 |

**(2)**

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region $R$. Give your answer to 4 decimal places.

**(4)**

**Question 1 continued**

**Q1**

**(Total 6 marks)**

**2.** (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of $x$, up to and including the term in $x^3$, giving each term as a simplified fraction.

**(5)**

(b) Use your expansion, with a suitable value of $x$, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

**(2)**

**Question 2 continued**

**Q2**

**(Total 7 marks)**

**3.**

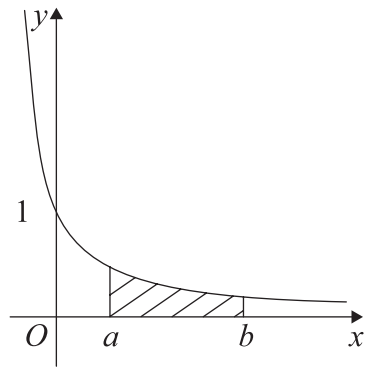


**Figure 2**

The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the $x$-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the $x$-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of $a$ and $b$.

**(5)**

**Question 3 continued**

**Q3**

**(Total 5 marks)**

**4.** (i) Find $\int \ln(\frac{x}{2})\,\mathrm{d}x$ .

**(4)**

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,\mathrm{d}x$.

**(5)**

**Question 4 continued**

**Q4**

**(Total 9 marks)**

**5.** A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

**(3)**

(b) Find the gradient of the curve at each of these points.

**(6)**

**Question 5 continued**

**Q5**

**(Total 9 marks)**

**6.** The points $A$ and $B$ have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line $l_1$ passes through the points $A$ and $B$.

(a) Find the vector $\overrightarrow{AB}$.

**(2)**

(b) Find a vector equation for the line $l_1$.

**(2)**

A second line $l_2$ passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line $l_1$ meets the line $l_2$ at the point $C$.

(c) Find the acute angle between $l_1$ and $l_2$.

**(3)**

(d) Find the position vector of the point $C$.

**(4)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total 11 marks)**

**Q6**

**7.**

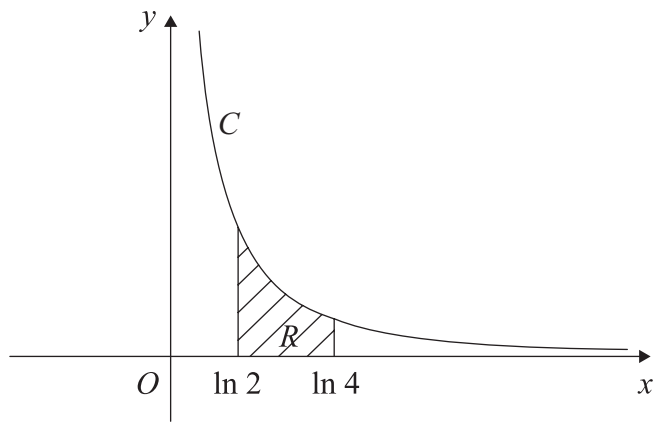


**Figure 3**

The curve $C$ has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region $R$ between the curve $C$ and the $x$-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of $R$ is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

**(4)**

(b) Hence find an exact value for this area.

**(6)**

(c) Find a cartesian equation of the curve $C$, in the form $y = \mathrm{f}(x)$.

**(4)**

(d) State the domain of values for $x$ for this curve.

**(1)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**Q7**

**(Total 15 marks)**

**8.** Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3\ s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 $cm^2$.

(a) Show that at time $t$ seconds, the height $h$ cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

**(3)**

When $h = 25$, water is leaking out of the hole at 400 $cm^3\ s^{-1}$.

(b) Show that $k = 0.02$

**(1)**

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$$

**(2)**

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$.

**(6)**

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

**(1)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

Q8

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

**mock papers 2**

**1.**

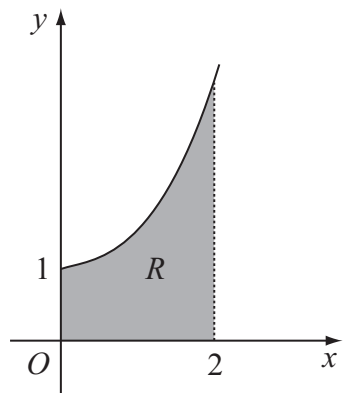


**Figure 1**

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, the $y$-axis and the line $x = 2$.

(a) Complete the table with the values of $y$ corresponding to $x = 0.8$ and $x = 1.6$.

| $x$ | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2 |
|---|---|---|---|---|---|---|
| $y$ | $e^0$ | $e^{0.08}$ | | $e^{0.72}$ | | $e^2$ |

**(1)**

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of $R$, giving your answer to 4 significant figures.

**(3)**

**Question 1 continued**

Q1

**(Total 4 marks)**

**2.** (a) Use integration by parts to find $\int x\,e^{x}\,dx$.

(b) Hence find $\int x^{2}\,e^{x}\,dx$.

**(3)**

**Question 2 continued**

Q2

**(Total 6 marks)**

**3.**

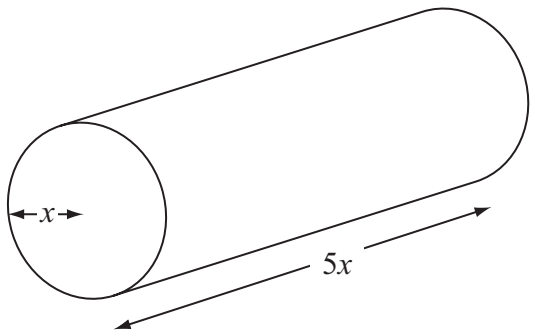


**Figure 2**

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After $t$ seconds the radius of the rod is $x$ cm and the length of the rod is $5x$ cm.
The cross-sectional area of the rod is increasing at the constant rate of 0.032 $cm^2\ s^{-1}$.

(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

**(4)**

(b) Find the rate of increase of the volume of the rod when $x = 2$.

**(4)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**Q3**

**(Total 8 marks)**

**4.** A curve has equation $3x^2 - y^2 + xy = 4$. The points $P$ and $Q$ lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at $P$ and at $Q$.

(a) Use implicit differentiation to show that $y - 2x = 0$ at $P$ and at $Q$.

**(6)**

(b) Find the coordinates of $P$ and $Q$.

**(3)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**Q4**

**(Total 9 marks)**

**5.** (a) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of $x$ up to and including the term in $x^2$. Simplify each term.

**(5)**

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of $x$.

**(4)**

**Question 5 continued**

**Q5**

**(Total 9 marks)**

**6.** With respect to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where $\lambda$ and $\mu$ are scalar parameters.

(a) Show that $l_1$ and $l_2$ meet and find the position vector of their point of intersection.

**(6)**

(b) Show that $l_1$ and $l_2$ are perpendicular to each other.

**(2)**

The point $A$ has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that $A$ lies on $l_1$.

**(1)**

The point $B$ is the image of $A$ after reflection in the line $l_2$.

(d) Find the position vector of $B$.

**(3)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total 12 marks)**

**Q6**

**7.** (a) Express $\dfrac{2}{4-y^2}$ in partial fractions.

**(3)**

(b) Hence obtain the solution of

$$2\cot x \frac{\mathrm{d}y}{\mathrm{d}x} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$.

**(8)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**Q7**

**(Total 11 marks)**

**8.**

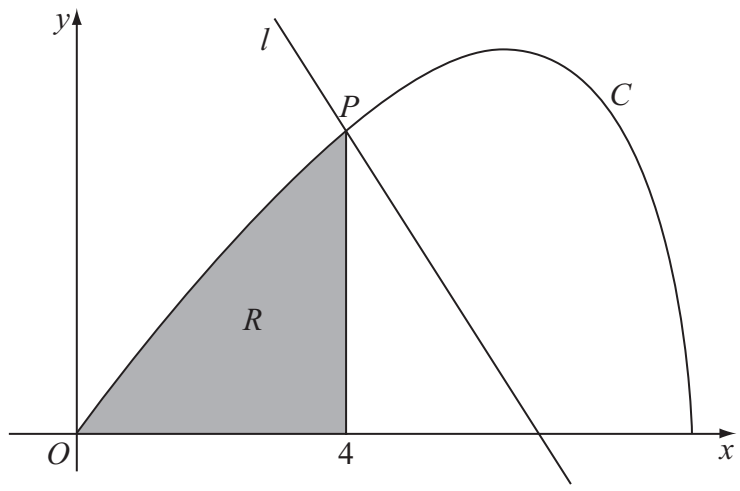


**Figure 3**

Figure 3 shows the curve $C$ with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point $P$ lies on $C$ and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of $t$ at the point $P$.

**(2)**

The line $l$ is a normal to $C$ at $P$.

(b) Show that an equation for $l$ is $y = -x\sqrt{3} + 6\sqrt{3}$.

**(6)**

The finite region $R$ is enclosed by the curve $C$, the $x$-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of $R$ is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t\,\mathrm{d}t$.

**(4)**

(d) Use this integral to find the area of $R$, giving your answer in the form $a + b\sqrt{3}$, where $a$ and $b$ are constants to be determined.

**(4)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

Q8

**(Total 16 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

**mock papers 3**

**1.** A curve $C$ has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(4)**

(b) Hence find the gradient of $C$ at the point where $y = 3$.

**(3)**

**Question 1 continued**

Q1

**(Total 7 marks)**

**2.**

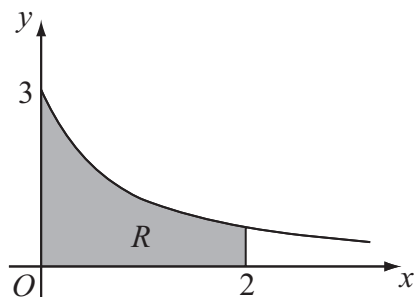


**Figure 1**

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region $R$ is bounded by the curve, the $x$-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of $R$.

**(4)**

The region $R$ is rotated 360° about the $x$-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

**(5)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**Q2**

**(Total 9 marks)**

**3.**

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}$$

Given that f($x$) can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of $B$ and $C$ and show that $A = 0$.

**(4)**

(b) Hence, or otherwise, find the series expansion of f($x$), in ascending powers of $x$, up to and including the term in $x^2$. Simplify each term.

**(6)**

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f(0.2). Give your answer to 2 significant figures.

**(4)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**Q3**

**(Total 14 marks)**

**4.** With respect to a fixed origin $O$ the lines $l_1$ and $l_2$ are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where $\lambda$ and $\mu$ are parameters and $p$ and $q$ are constants. Given that $l_1$ and $l_2$ are perpendicular,

(a) show that $q = -3$.

**(2)**

Given further that $l_1$ and $l_2$ intersect, find

(b) the value of $p$,

**(6)**

(c) the coordinates of the point of intersection.

**(2)**

The point $A$ lies on $l_1$ and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point $C$ lies on $l_2$.

Given that a circle, with centre $C$, cuts the line $l_1$ at the points $A$ and $B$,

(d) find the position vector of $B$.

**(3)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**Q4**

**(Total 13 marks)**

**5.**

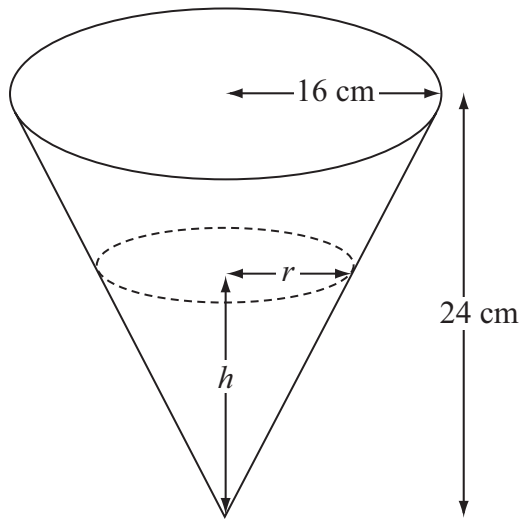


**Figure 2**

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is $h$ cm, the surface of the water has radius $r$ cm and the volume of water is $V$ cm$^3$.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

**(2)**

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$ .]

Water flows into the container at a rate of 8 cm$^3$ s$^{-1}$.

(b) Find, in terms of $\pi$, the rate of change of $h$ when $h = 12$.

**(5)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**Q5**

**(Total 7 marks)**

**6.** (a) Find $\int \tan^2 x \, dx$.

**(2)**

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$.

**(4)**

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where $k$ is a constant.

**(7)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**Q6**

**(Total 13 marks)**

**7.**

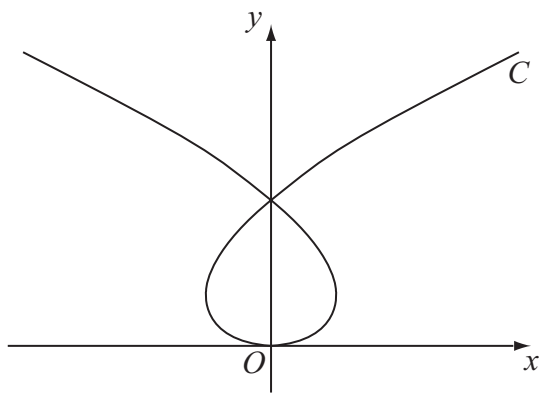


**Figure 3**

The curve $C$ shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where $t$ is a parameter. Given that the point $A$ has parameter $t = -1$,

(a) find the coordinates of $A$.

**(1)**

The line $l$ is the tangent to $C$ at $A$.

(b) Show that an equation for $l$ is $2x - 5y - 9 = 0$.

**(5)**

The line $l$ also intersects the curve at the point $B$.

(c) Find the coordinates of $B$.

**(6)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

Q7

**(Total 12 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

**mock papers 4**

**1.** $$f(x) = \frac{1}{\sqrt{(4+x)}}, \qquad |x| < 4$$

Find the binomial expansion of $f(x)$ in ascending powers of $x$, up to and including the term in $x^3$. Give each coefficient as a simplified fraction.

**(6)**

**Question 1 continued**

Q1

**(Total 6 marks)**

**2.**

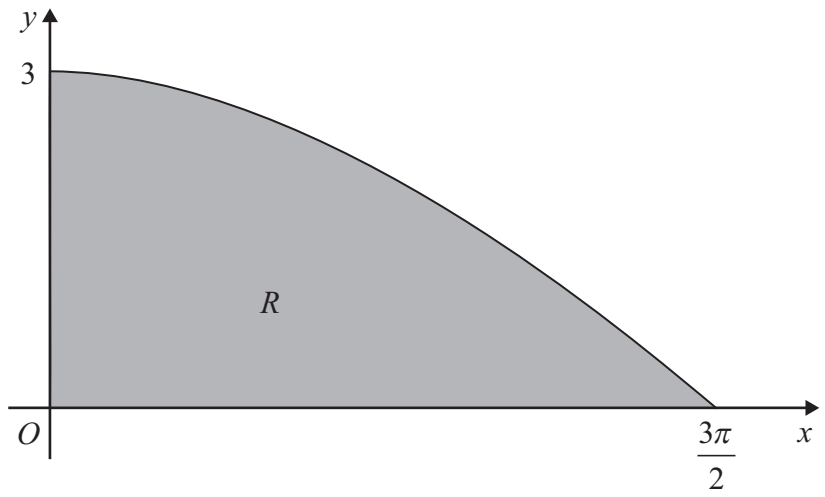


**Figure 1**

Figure 1 shows the finite region $R$ bounded by the $x$-axis, the $y$-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leqslant x \leqslant \frac{3\pi}{2}$.

The table shows corresponding values of $x$ and $y$ for $y = 3\cos\left(\frac{x}{3}\right)$.

| $x$ | 0 | $\frac{3\pi}{8}$ | $\frac{3\pi}{4}$ | $\frac{9\pi}{8}$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|---|
| $y$ | 3 | 2.77164 | 2.12132 | | 0 |

(a) Complete the table above giving the missing value of $y$ to 5 decimal places.

**(1)**

(b) Using the trapezium rule, with all the values of $y$ from the completed table, find an approximation for the area of $R$, giving your answer to 3 decimal places.

**(4)**

(c) Use integration to find the exact area of $R$.

**(3)**

**Question 2 continued**

**(Total 8 marks)**

**Q2**

**3.** $$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants $A$, $B$ and $C$.

**(4)**

(b) (i) Hence find $\int f(x)\,dx$.

**(3)**

(ii) Find $\int_0^2 f(x)\,dx$ in the form $\ln k$, where $k$ is a constant.

**(3)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**Q3**

**(Total 10 marks)**

**4.** The curve $C$ has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(5)**

The point $P$ on $C$ has coordinates (0, 1).

(b) Find the equation of the normal to $C$ at $P$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(4)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**Q4**

**(Total 9 marks)**

**5.**

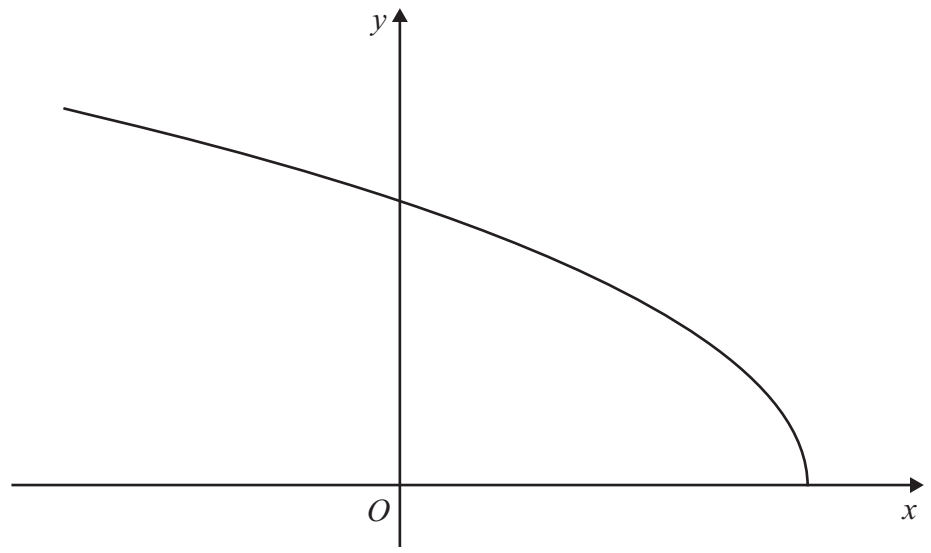


**Figure 2**

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

**(4)**

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant $k$.

**(4)**

(c) Write down the range of $\mathrm{f}(x)$.

**(2)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**Q5**

**(Total 10 marks)**

**6.** (a) Find $\int \sqrt{(5-x)}\,\mathrm{d}x$.

**(2)**

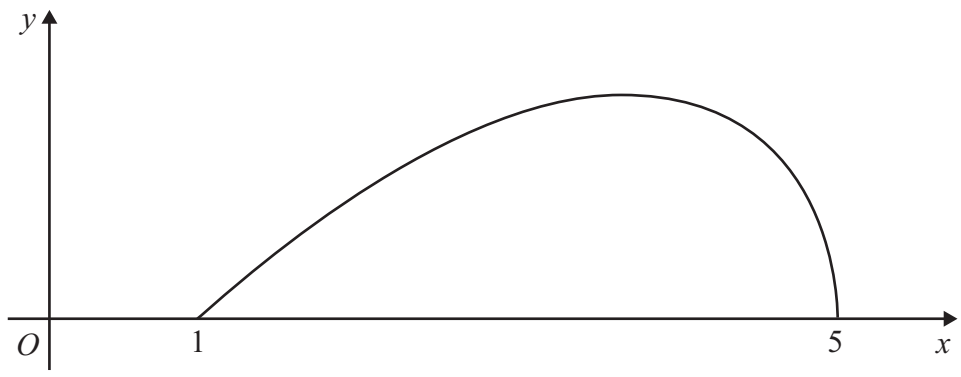


**Figure 3**

Figure 3 shows a sketch of the curve with equation

$$y = (x - 1)\sqrt{(5 - x)}, \qquad 1 \leqslant x \leqslant 5$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x$$

**(4)**

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\,\mathrm{d}x$.

**(2)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total 8 marks)**

**Q6**

**7.** Relative to a fixed origin $O$, the point $A$ has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point $B$ has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point $C$ has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line $l$ passes through the points $A$ and $B$.

(a) Find a vector equation for the line $l$.

**(3)**

(b) Find $\left|\overrightarrow{CB}\right|$.

**(2)**

(c) Find the size of the acute angle between the line segment $CB$ and the line $l$, giving your answer in degrees to 1 decimal place.

**(3)**

(d) Find the shortest distance from the point $C$ to the line $l$.

**(3)**

The point $X$ lies on $l$. Given that the vector $\overrightarrow{CX}$ is perpendicular to $l$,

(e) find the area of the triangle $CXB$, giving your answer to 3 significant figures.

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**Q7**

**(Total 14 marks)**

**8.** (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, \mathrm{d}\theta$.

**(2)**

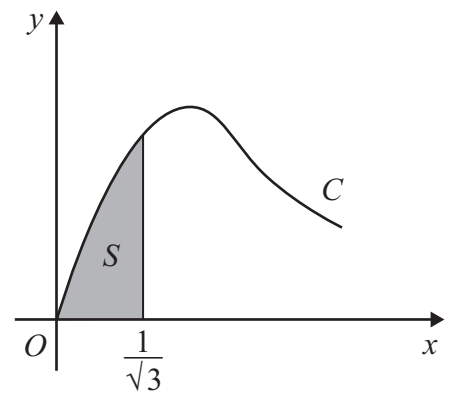


**Figure 4**

Figure 4 shows part of the curve $C$ with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The finite shaded region $S$ shown in Figure 4 is bounded by $C$, the line $x = \frac{1}{\sqrt{3}}$ and the $x$-axis. This shaded region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, \mathrm{d}\theta$$

where $k$ is a constant.

**(5)**

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where $p$ and $q$ are constants.

**(3)**

**Question 8 continued**

**Question 8 continued**

**Q8**

**(Total 10 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**