**1.** Solve

(*a*) $5^x = 8$, giving your answers to 3 significant figures,

**(3)**

(*b*) $\log_2 (x + 1) - \log_2 x = \log_2 7$.

**(3)**

---

**2.** (*a*) Write down the first three terms, in ascending powers of $x$, of the binomial expansion of $(1 + px)^{12}$, where $p$ is a non-zero constant.

**(2)**

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of $x$ is $(-q)$ and the coefficient of $x^2$ is $11q$,

(*b*) find the value of $p$ and the value of $q$.

**(4)**

---

**3.** A river, running between parallel banks, is 20 m wide. The depth, $y$ metres, of the river measured at a point $x$ metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \qquad 0 \le x \le 20.$$

(*a*) Complete the table below, giving values of $y$ to 3 decimal places.

| $x$ | 0 | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|---|
| $y$ | 0 | | 2.771 | | | 0 |

**(2)**

(*b*) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

**(4)**

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 m $s^{-1}$,

(*c*) estimate, in $m^3$, the volume of water flowing per minute, giving your answer to 3 significant figures.

**(2)**

---

**4.** The function f is defined by

$$f: x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $f(x) = \dfrac{2}{x-1}, x > 1$.

**(4)**

(*b*) Find $f^{-1}(x)$.

**(3)**

The function g is defined by

$$g: x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*b*) Solve $fg(x) = \frac{1}{4}$.

**(3)**

**5.**

**Figure 1**

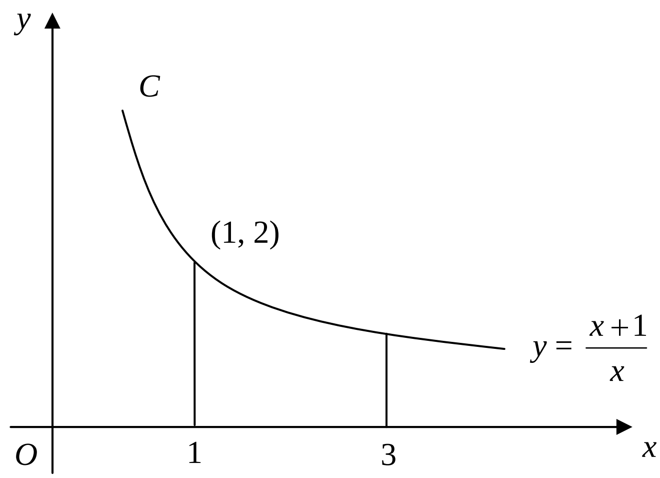


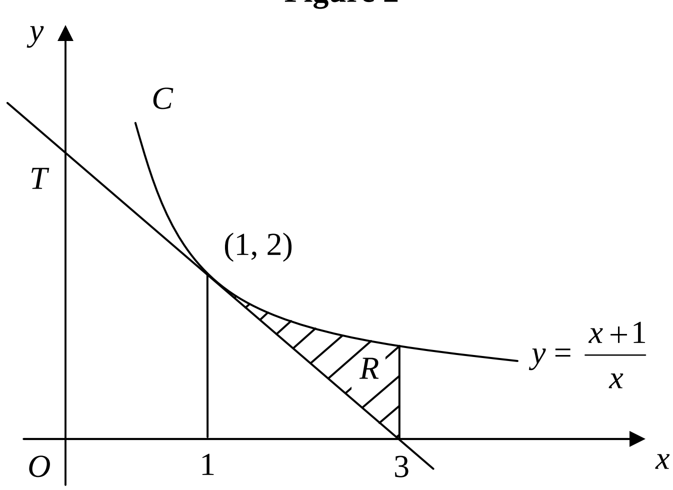

Figure 1 shows part of the curve $C$ with equation $y = \frac{x+1}{x}$, $x > 0$.

The finite region enclosed by $C$, the lines $x = 1$, $x = 3$ and the $x$-axis is rotated through 360° about the $x$-axis to generate a solid $S$.

(*a*) Using integration, find the exact volume of $S$.

**(7)**

**Figure 2**




The tangent $T$ to $C$ at the point (1, 2) meets the $x$-axis at the point (3, 0). The shaded region $R$ is bounded by $C$, the line $x = 3$ and $T$, as shown in Figure 2.

(*b*) Using your answer to part (*a*), find the exact volume generated by $R$ when it is rotated through 360° about the $x$-axis.

**(3)**

**6.** $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(*a*) Differentiate to find $f'(x)$.

**(3)**

The curve with equation $y = f(x)$ has a turning point at $P$. The $x$-coordinate of $P$ is $\alpha$.

(*b*) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$.

**(2)**

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for $\alpha$.

(*c*) Calculate the values of $x_1$, $x_2$, $x_3$ and $x_4$, giving your answers to 4 decimal places.

**(2)**

(*d*) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

**(2)**

**7.** **Figure 1**

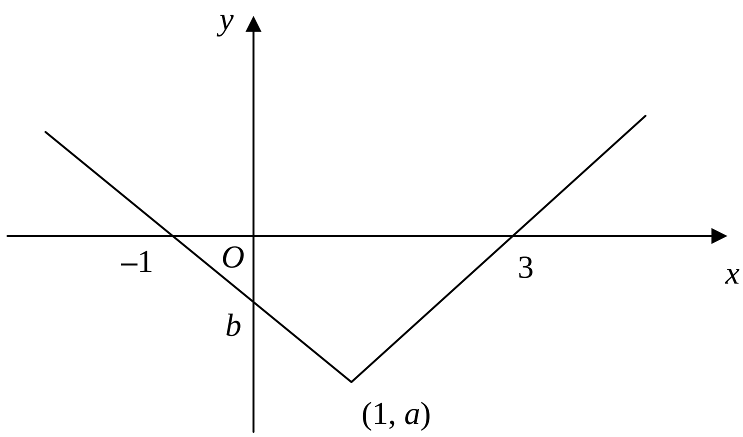



Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the $x$-axis at $(3, 0)$. The other line meets the $x$-axis at $(–1, 0)$ and the $y$-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = \mathrm{f}(x + 1)$,

**(2)**

(*b*) $y = \mathrm{f}(|x|)$.

**(3)**

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $\mathrm{f}(x) = |x – 1| – 2$, find

(*c*) the value of $a$ and the value of $b$,

**(2)**

(*d*) the value of $x$ for which $\mathrm{f}(x) = 5x$.

**(4)**

**8.** (*a*) Given that $2 \sin(\theta + 30)° = \cos(\theta + 60)°$, find the exact value of $\tan \theta°$.

**(5)**

(*b*) (i) Using the identity $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2 \sin^2 A.$$

**(2)**

(ii) Hence solve, for $0 \leq x < 2\pi$,

$$\cos 2x = \sin x,$$

giving your answers in terms of $\pi$.

**(5)**

(iii) Show that $\sin 2y \tan y + \cos 2y \equiv 1$, for $0 \leq y < \frac{1}{2}\pi$.

**(3)**

---

**TOTAL FOR PAPER: 75 MARKS**

**END**

**1.** Find the values of $x$ for which

$$5 \cosh x - 2 \sinh x = 11,$$

giving your answers as natural logarithms.

**(6)**

---

**2.** The point $S$, which lies on the positive $x$-axis, is a focus of the ellipse with equation $\frac{x^2}{4} + y^2 = 1$. Given that $S$ is also the focus of a parabola $P$, with vertex at the origin, find

($a$) a cartesian equation for $P$,

**(4)**

($b$) an equation for the directrix of $P$.

**(1)**

---

**3.** The radius of curvature of a curve $C$, at any point on $C$, is $e^{\sin \psi} \cos \psi$, where $\psi$ is the angle between the tangent to $C$ at $P$ and the positive axis, and $0 \leq \psi \leq \frac{\pi}{2}$.

Taking $s = 0$ at $\psi = 0$, find an intrinsic equation for $C$.

**(4)**

---

**4.** The curve $C$ has equation $y = \arctan x^2$, $0 \leq y < \frac{\pi}{2}$.

Find, in surd form, the value of the radius of curvature of $C$ at the point where $x = 1$.

**(6)**

---

**5.** The curve with equation

$$y = -x + \tanh 4x, \quad x \geq 0,$$

has a maximum turning point $A$.

($a$) Find, in exact logarithmic form, the $x$-coordinate of $A$.

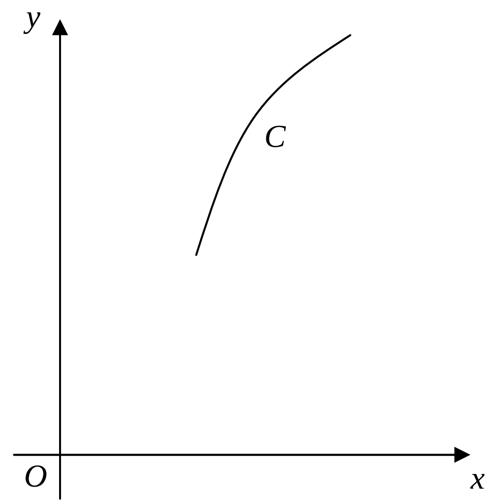
**(4)**

($b$) Show that the $y$-coordinate of $A$ is $\frac{1}{4}\{2\sqrt{3} - \ln(2 + \sqrt{3})\}$.

**(3)**

---

**6.** **Figure 1**




The curve $C$, shown in Figure 1, has parametric equations

$$x = t - \ln t,$$

$$y = 4\sqrt{t}, \qquad 1 \leq t \leq 4.$$

($a$) Show that the length of $C$ is $3 + \ln 4$.

**(7)**

The curve is rotated through $2\pi$ radians about the $x$-axis.

($b$) Find the exact area of the curved surface generated.

**(4)**

**7.** **Figure 2**

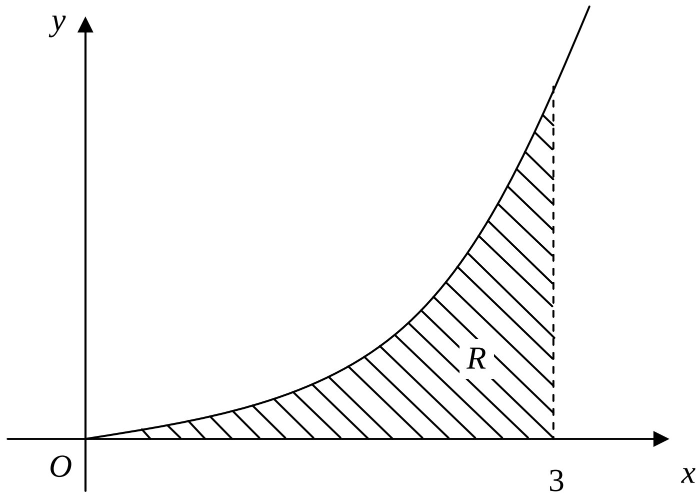



Figure 2 shows a sketch of part of the curve with equation

$$y = x^2 \text{ arsinh } x.$$

The region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the line $x = 3$.

Show that the area of $R$ is

$$9 \ln (3 + \sqrt{10}) - \tfrac{1}{9}(2 + 7\sqrt{10}).$$

**(10)**

---

**8.**

$$I_n = \int x^n \cosh x \; dx, \quad n \geq 0.$$

(*a*) Show that, for $n \geq 2$,

$$I_n = x^n \sinh x - nx^{n-1} \cosh x + n(n-1)I_{n-2}.$$

**(4)**

(*b*) Hence show that

$$I_4 = f(x) \sinh x + g(x) \cosh x + C,$$

where f($x$) and g($x$) are functions of $x$ to be found, and $C$ is an arbitrary constant.

**(5)**

(*c*) Find the exact value of $\int_0^1 x^4 \cosh x \; dx$, giving your answer in terms of e.

**(3)**

---

**9.** The ellipse $E$ has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $L$ has equation $y = mx + c$, where $m > 0$ and $c > 0$.

(*a*) Show that, if $L$ and $E$ have any points of intersection, the $x$-coordinates of these points are the roots of the equation

$$(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2(c^2 - b^2) = 0.$$

**(2)**

Hence, given that $L$ is a tangent to $E$,

(*b*) show that $c^2 = b^2 + a^2m^2$.

**(2)**

The tangent $L$ meets the negative $x$-axis at the point $A$ and the positive $y$-axis at the point $B$, and $O$ is the origin.

(*c*) Find, in terms of $m$, $a$ and $b$, the area of triangle $OAB$.

**(4)**

(*d*) Prove that, as $m$ varies, the minimum area of triangle $OAB$ is $ab$.

**(3)**

(*e*) Find, in terms of $a$, the $x$-coordinate of the point of contact of $L$ and $E$ when the area of triangle $OAB$ is a minimum.

**(3)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Leave blank

**1.** (a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

**(1)**

(b) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$.

**(5)**

Leave blank

**Question 1 continued**

Q1

**(Total 6 marks)**

Leave blank

**2.** Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\mathrm{i},$$

giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$.

**(6)**

Leave blank

**Question 2 continued**

Q2

**(Total 6 marks)**

Leave blank

**3.** Find the general solution of the differential equation

$$\sin x \frac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = \sin 2x \sin x,$$

giving your answer in the form $y = \mathrm{f}(x)$.

**(8)**

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**Question 3 continued**

**(Total 8 marks)**

Q3

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**4.**

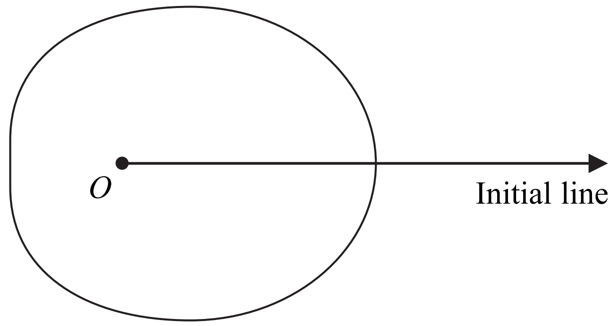


**Figure 1**

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leqslant \theta < 2\pi$$

The area enclosed by the curve is $\dfrac{107}{2}\pi$.

Find the value of $a$.

**(8)**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

Q4

**(Total 8 marks)**

Leave blank

**5.**

$$y = \sec^2 x$$

(a) Show that $\dfrac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$.

**(4)**

(b) Find a Taylor series expansion of $\sec^2 x$ in ascending powers of $\left(x - \dfrac{\pi}{4}\right)$, up to and including the term in $\left(x - \dfrac{\pi}{4}\right)^3$.

**(6)**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Q5

**(Total 10 marks)**

Leave blank

**6.** A transformation $T$ from the $z$-plane to the $w$-plane is given by

$$w = \frac{z}{z + \mathrm{i}}, \quad z \neq -\mathrm{i}$$

The circle with equation $|z| = 3$ is mapped by $T$ onto the curve $C$.

(a) Show that $C$ is a circle and find its centre and radius.

**(8)**

The region $|z| < 3$ in the $z$-plane is mapped by $T$ onto the region $R$ in the $w$-plane.

(b) Shade the region $R$ on an Argand diagram.

**(2)**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Q6

**(Total 10 marks)**

Leave blank

**7.** (a) Sketch the graph of $y = |x^2 - a^2|$, where $a > 1$, showing the coordinates of the points where the graph meets the axes.

**(2)**

(b) Solve $|x^2 - a^2| = a^2 - x$, $a > 1$.

**(6)**

(c) Find the set of values of $x$ for which $|x^2 - a^2| > a^2 - x$, $a > 1$.

**(4)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**(Total 12 marks)**

Q7

Leave blank

**8.**

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

(a) find $x$ in terms of $t$.

**(8)**

The solution to part (a) is used to represent the motion of a particle $P$ on the $x$-axis. At time $t$ seconds, where $t > 0$, $P$ is $x$ metres from the origin $O$.

(b) Show that the maximum distance between $O$ and $P$ is $\frac{2\sqrt{3}}{9}$ m and justify that this distance is a maximum.

**(7)**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

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**Question 8 continued**

Leave blank

**Question 8 continued**

Q8

**(Total 15 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

**mock papers 4**

Leave blank

**1.** (a) Express $\dfrac{3}{(3r-1)(3r+2)}$ in partial fractions.

**(2)**

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n}\frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)}$$

**(3)**

(c) Evaluate $\displaystyle\sum_{r=100}^{1000}\frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures.

**(2)**

Leave blank

**Question 1 continued**

Q1

**(Total 7 marks)**

Leave blank

**2.** The displacement $x$ metres of a particle at time $t$ seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = \frac{1}{2}$.

Find a Taylor series solution for $x$ in ascending powers of $t$, up to and including the term in $t^3$.

**(5)**

Leave blank

**Question 2 continued**

Q2

**(Total 5 marks)**

Leave blank

**3.** (a) Find the set of values of $x$ for which

$$x+4>\frac{2}{x+3}$$

**(6)**

(b) Deduce, or otherwise find, the values of $x$ for which

$$x+4>\frac{2}{|x+3|}$$

**(1)**

Leave blank

**Question 3 continued**

Q3

**(Total 7 marks)**

Leave blank

**4.** $$z = -8 + (8\sqrt{3})\text{i}$$

(a) Find the modulus of $z$ and the argument of $z$.

**(3)**

Using de Moivre's theorem,

(b) find $z^3$,

**(2)**

(c) find the values of $w$ such that $w^4 = z$, giving your answers in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$.

**(5)**

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**Question 4 continued**

Q4

**(Total 10 marks)**

Turn over

Leave blank

**5.**

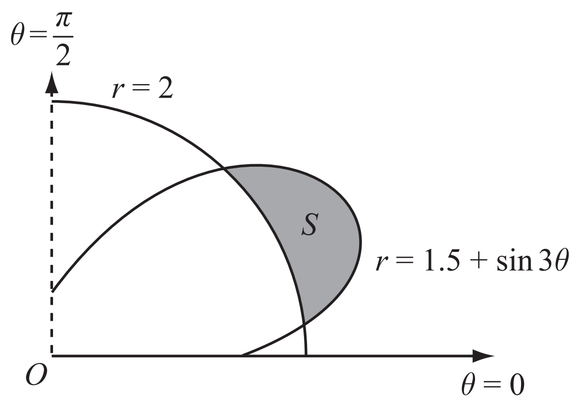


**Figure 1**

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2},$$

$$\text{and} \qquad r = 1.5 + \sin 3\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect.

**(3)**

The region $S$, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region $S$, giving your answer in the form $a\pi + b\sqrt{3}$, where $a$ and $b$ are simplified fractions.

**(7)**

Leave blank

**Question 5 continued**

**Turn over**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

**(Total 10 marks)**

Q5

Leave blank

**6.** A complex number $z$ is represented by the point $P$ in the Argand diagram.

(a) Given that $|z-6|=|z|$, sketch the locus of $P$.

**(2)**

(b) Find the complex numbers $z$ which satisfy both $|z-6|=|z|$ and $|z-3-4i|=5$.

**(3)**

The transformation $T$ from the $z$-plane to the $w$-plane is given by $w=\frac{30}{z}$.

(c) Show that $T$ maps $|z-6|=|z|$ onto a circle in the $w$-plane and give the cartesian equation of this circle.

**(5)**

Leave blank

**Question 6 continued**

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**Question 6 continued**

Leave blank

**Question 6 continued**

Q6

**(Total 10 marks)**

**Turn over**

Leave blank

**7.** (a) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{dy}{dx} - 4y\tan x = 2y^{\frac{1}{2}} \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 2z\tan x = 1 \qquad \text{(II)}$$

**(5)**

(b) Solve the differential equation (II) to find $z$ as a function of $x$.

**(6)**

(c) Hence obtain the general solution of the differential equation (I).

**(1)**

Leave blank

**Question 7 continued**

Turn over

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**(Total 12 marks)**

Q7

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**8.** (a) Find the value of $\lambda$ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x$$

**(4)**

(b) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x$$

**(3)**

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$.

**(5)**

(d) Sketch the curve with equation $y = f(x)$ for $0 \leqslant x \leqslant \pi$.

**(2)**

Leave blank

**Question 8 continued**

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**Question 8 continued**

Q8

**(Total 14 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**