mock papers 1

1. A parcel of mass 2.5 kg is moving in a straight line on a smooth horizontal floor. Initially the parcel is moving with speed 8 m s^{-1}. The parcel is brought to rest in a distance of 20 m by a constant horizontal force of magnitude R newtons. Modelling the parcel as a particle, find

(a) the kinetic energy lost by the parcel in coming to rest,

(2)

(b) the value of R.

(3)

Question 1 continued

(Total 5 marks)

Q1

Turn over

2. At time t seconds ($t \geqslant 0$), a particle P has position vector $\mathbf{p}$ metres, with respect to a fixed origin O, where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(a) the velocity of P at time t seconds,

(2)

(b) the value of t when P is moving parallel to the vector $\mathbf{i}$.

(3)

When $t = 1$, the particle P receives an impulse of $(2\mathbf{i} - 6\mathbf{j})$ N s. Given that the mass of P is 0.5 kg,

(c) find the velocity of P immediately after the impulse.

(4)

Question 2 continued

Q2

(Total 9 marks)

Turn over

3. A car of mass 1000 kg is moving at a constant speed of 16 m s^{-1} up a straight road inclined at an angle θ to the horizontal. The rate of working of the engine of the car is 20 kW and the resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 550 N.

(a) Show that $\sin\theta = \frac{1}{14}$.

(5)

When the car is travelling up the road at 16 m s^{-1}, the engine is switched off. The car comes to rest, without braking, having moved a distance y metres from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 550 N.

(b) Find the value of y.

(4)

Question 3 continued

Q3

(Total 9 marks)

Turn over

4.

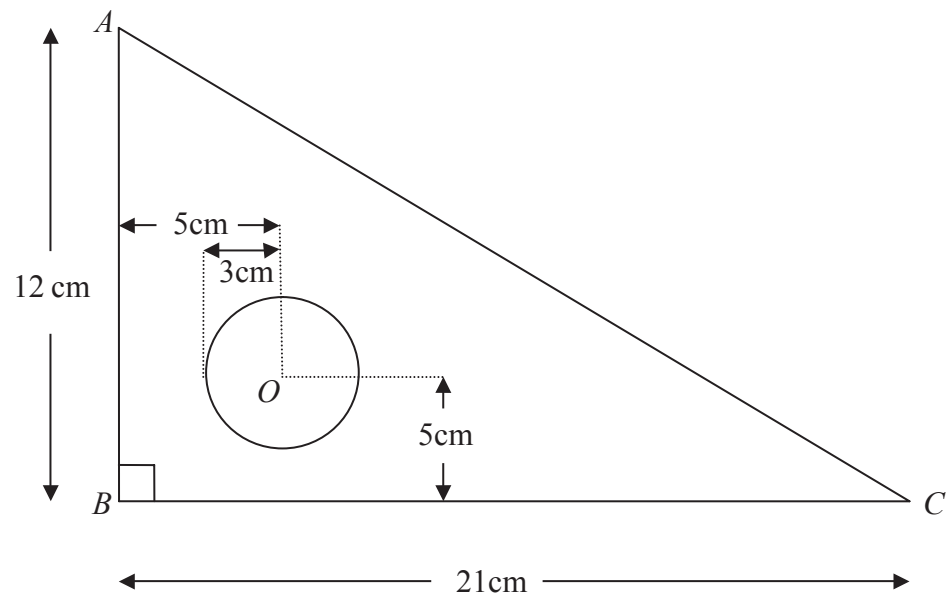

Figure 1

A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC, with $\angle ABC = 90°$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC, as shown in Figure 1.

(a) Find the distance of the centre of mass of S from

(i) AB,

(ii) BC.

(9)

The set square is freely suspended from C and hangs in equilibrium.

(b) Find, to the nearest degree, the angle between CB and the vertical.

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total 12 marks)

Q4

5.

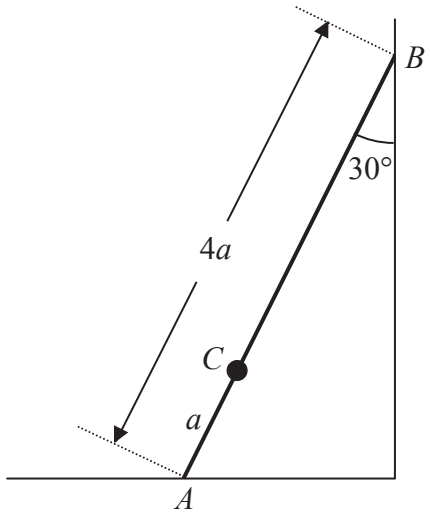

Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

Question 5 continued

Question 5 continued

Question 5 continued

(Total 10 marks)

Q5

Turn over

6.

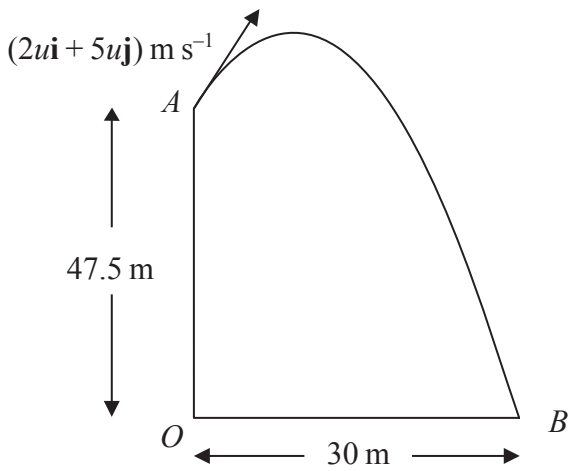

Figure 3

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle P is projected from the point A which has position vector $47.5\mathbf{j}$ metres with respect to a fixed origin O. The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity passing through the point B with position vector $30\mathbf{i}$ metres, as shown in Figure 3.

(a) Show that the time taken for P to move from A to B is 5 s.

(6)

(b) Find the value of u.

(2)

(c) Find the speed of P at B.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total 13 marks)

Q6

7. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$.

(5)

(b) Find the total kinetic energy lost in the collision.

(5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(c) Calculate the range of values of e for which there will be a second collision between P and Q.

(7)

Question 7 continued

Question 7 continued

Question 7 continued

Q7

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END

mock papers 2

1. A lorry of mass 2000 kg is moving down a straight road inclined at angle α to the horizontal, where $\sin\alpha = \frac{1}{25}$. The resistance to motion is modelled as a constant force of magnitude 1600 N. The lorry is moving at a constant speed of 14 m s^{-1}.

Find, in kW, the rate at which the lorry's engine is working.

(6)

Question 1 continued

Q1

(Total 6 marks)

Turn over

2. A particle A of mass $4m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass $3m$ moving with speed $2u$ in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is $4eu$.

(a) Show that $e = \frac{3}{4}$.

(5)

(b) Find the total kinetic energy lost in the collision.

(4)

Question 2 continued

Q2

(Total 9 marks)

3.

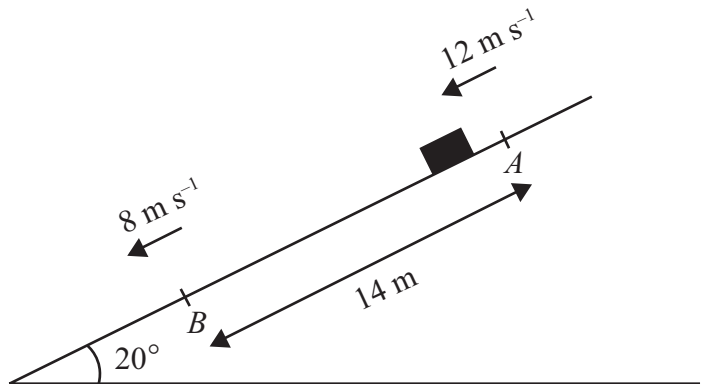

Figure 1

A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of 20° to the horizontal. The package slides down a line of greatest slope of the plane from a point A to a point B, where $AB = 14$ m. At A the package has speed 12 m s^{-1} and at B the package has speed 8 m s^{-1}, as shown in Figure 1. Find

(a) the total energy lost by the package in travelling from A to B,

(5)

(b) the coefficient of friction between the package and the ramp.

(5)

Question 3 continued

Q3

(Total 10 marks)

Turn over

4. A particle P of mass 0.5 kg is moving under the action of a single force **F** newtons. At time t seconds,

$$\mathbf{F} = (6t - 5)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j}.$$

The velocity of P at time t seconds is **v** m s^{-1}. When $t = 0$, $\mathbf{v} = \mathbf{i} - 4\mathbf{j}$.

(a) Find **v** at time t seconds.

(6)

When $t = 3$, the particle P receives an impulse $(-5\mathbf{i} + 12\mathbf{j})$ N s.

(b) Find the speed of P immediately after it receives the impulse.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total 12 marks)

Q4

5.

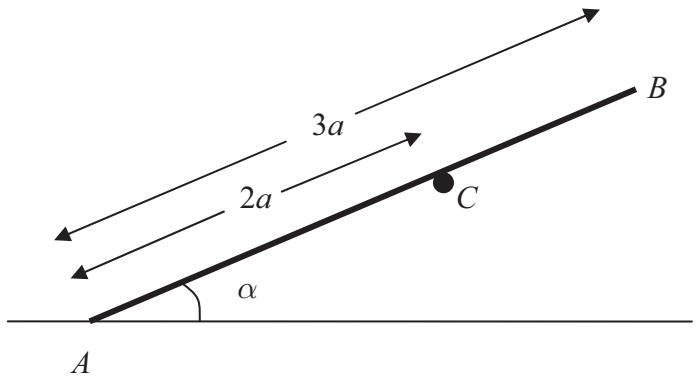

Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB. The rod has length $3a$ and weight W and rests on the peg at C, where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3\cos^2 \alpha)\, W$.

(6)

Given that the rod is in limiting equilibrium and that $\cos \alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total 11 marks)

Q5

6.

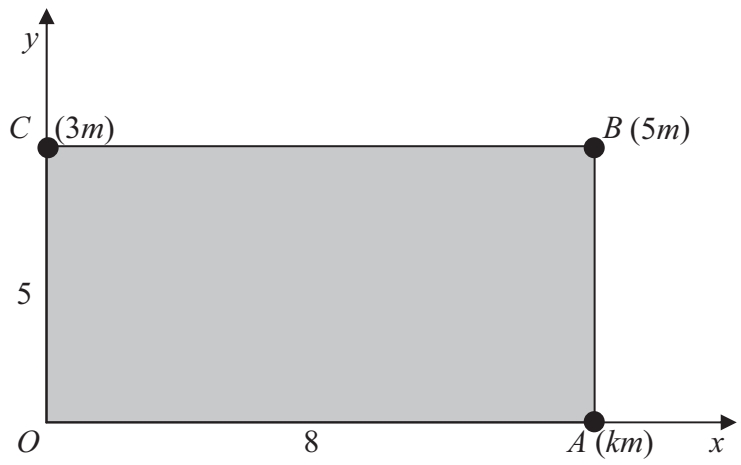

Figure 3

Figure 3 shows a rectangular lamina $OABC$. The coordinates of O, A, B and C are (0, 0), (8, 0), (8, 5) and (0, 5) respectively. Particles of mass km, $5m$ and $3m$ are attached to the lamina at A, B and C respectively.

The x-coordinate of the centre of mass of the three particles *without the lamina* is 6.4.

(a) Show that $k = 7$.

(4)

The lamina $OABC$ is uniform and has mass $12m$.

(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina.

(6)

The combined system is freely suspended from O and hangs at rest.

(c) Find the angle between OC and the horizontal.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total 13 marks)

Q6

7.

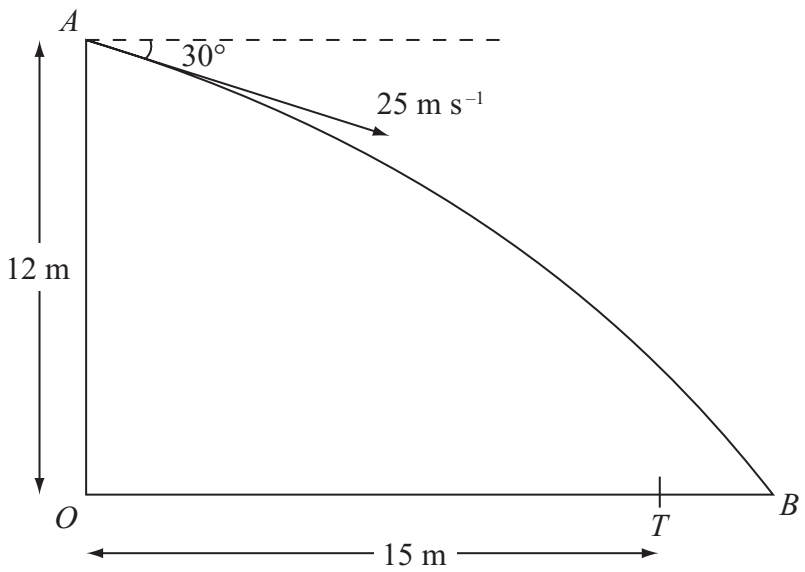

Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

(5)

Question 7 continued

Question 7 continued

Question 7 continued

Q7

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

mock papers 3

1. A car of mass 1500 kg is moving up a straight road, which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The resistance to the motion of the car from non-gravitational forces is constant and is modelled as a single constant force of magnitude 650 N. The car's engine is working at a rate of 30 kW.

Find the acceleration of the car at the instant when its speed is 15 m s^{-1}.

(5)

Question 1 continued

Q1

(Total 5 marks)

2.

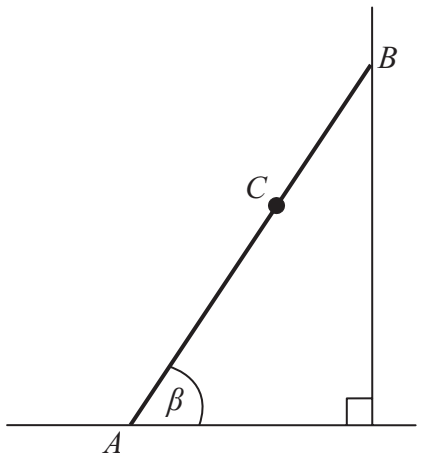

Figure 1

Figure 1 shows a ladder AB, of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(a) Find the magnitude of the frictional force of the ground on the ladder.

(3)

(b) Find, to the nearest degree, the value of β.

(6)

(c) State how you have used the modelling assumption that Reece is a particle.

(1)

Question 2 continued

Question 2 continued

Question 2 continued

(Total 10 marks)

Q2

Turn over

3. A block of mass 10 kg is pulled along a straight horizontal road by a constant horizontal force of magnitude 70 N in the direction of the road. The block moves in a straight line passing through two points A and B on the road, where $AB = 50$ m. The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is $\frac{4}{7}$.

(a) Calculate the work done against friction in moving the block from A to B.

(4)

The block passes through A with a speed of 2 m s^{-1}.

(b) Find the speed of the block at B.

(4)

Question 3 continued

Q3

(Total 8 marks)

Turn over

4. A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & 0 \leqslant t \leqslant 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

(a) $t = 6$,

(3)

(b) $t = 10$.

(5)

Question 4 continued

Q4

(Total 8 marks)

Turn over

5.

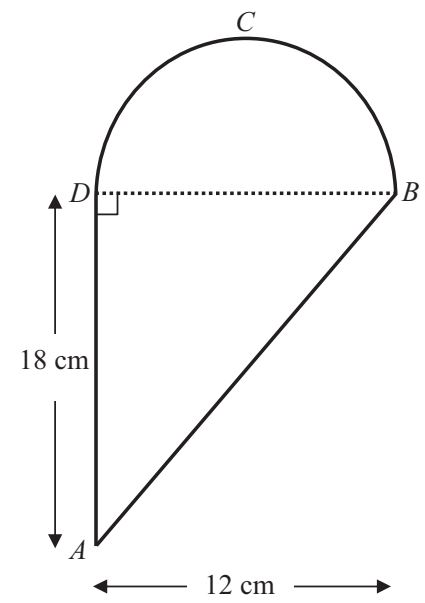

Figure 2

A uniform lamina $ABCD$ is made by joining a uniform triangular lamina ABD to a uniform semi-circular lamina DBC, of the same material, along the edge BD, as shown in Figure 2. Triangle ABD is right-angled at D and $AD = 18$ cm. The semi-circle has diameter BD and $BD = 12$ cm.

(a) Show that, to 3 significant figures, the distance of the centre of mass of the lamina $ABCD$ from AD is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius r, is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

(b) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina $ABCD$ from BD.

(4)

The lamina is freely suspended from B and hangs in equilibrium.

(c) Find, to the nearest degree, the angle which BD makes with the vertical.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

Q5

(Total 12 marks)

6.

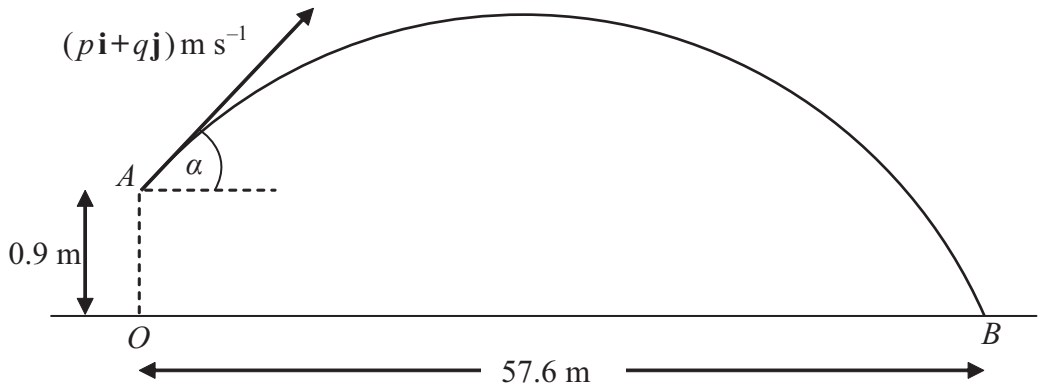

Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors **i** and **j** are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and $OB = 57.6$ m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(a) find the value of p,

(2)

(b) show that $q = 14.4$,

(3)

(c) find the initial speed of the cricket ball,

(2)

(d) find the exact value of $\tan \alpha$.

(1)

(e) Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(f) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)

Question 6 continued

Question 6 continued

Question 6 continued

(Total 15 marks)

Q6

7. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$.

(4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall.

(4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total 17 marks)

Q7

TOTAL FOR PAPER: 75 MARKS

END

mock papers 4

1. A particle of mass 0.25 kg is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ m s^{-1} when it receives the impulse $(5\mathbf{i} - 3\mathbf{j})$ N s.

Find the speed of the particle immediately after the impulse.

(5)

Question 1 continued

(Total 5 marks)

Q1

Turn over

2. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2 .$$

(a) Find the maximum value of v.

(4)

(b) Find the time taken for P to return to O.

(5)

Question 2 continued

Q2

(Total 9 marks)

3. A truck of mass of 300 kg moves along a straight horizontal road with a constant speed of 10 m s^{-1}. The resistance to motion of the truck has magnitude 120 N.

(a) Find the rate at which the engine of the truck is working.

(2)

On another occasion the truck moves at a constant speed up a hill inclined at θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The resistance to motion of the truck from non-gravitational forces remains of magnitude 120 N. The rate at which the engine works is the same as in part (a).

(b) Find the speed of the truck.

(4)

Question 3 continued

Q3

(Total 6 marks)

Turn over

4.

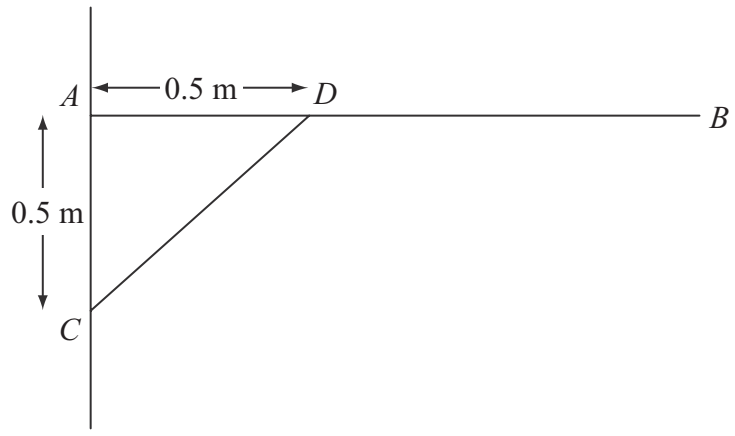

Figure 1

A uniform rod AB, of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at A. The rod is held in equilibrium in a horizontal position by a light strut CD as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end C of the strut is freely jointed to the wall at a point 0.5 m vertically below A. The end D is freely joined to the rod so that AD is 0.5 m.

(a) Find the thrust in CD.

(4)

(b) Find the magnitude and direction of the force exerted on the rod AB at A.

(7)

Question 4 continued

Question 4 continued

Question 4 continued

Q4

(Total 11 marks)

Turn over

5.

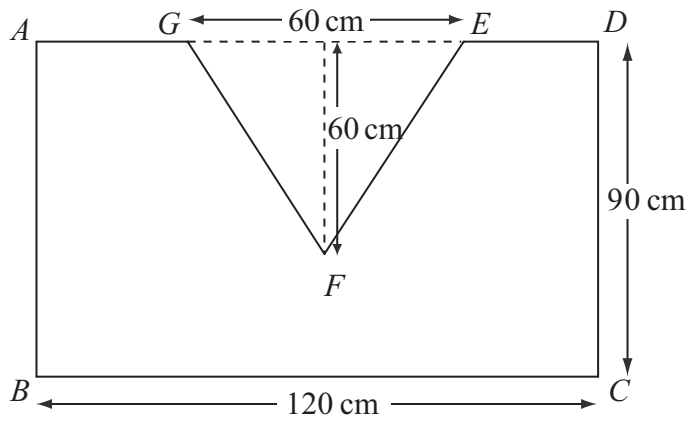

Figure 2

A shop sign *ABCDEFG* is modelled as a uniform lamina, as illustrated in Figure 2. *ABCD* is a rectangle with $BC = 120$ cm and $DC = 90$ cm. The shape *EFG* is an isosceles triangle with $EG = 60$ cm and height 60 cm. The mid-point of *AD* and the mid-point of *EG* coincide.

(a) Find the distance of the centre of mass of the sign from the side *AD*.

(5)

The sign is freely suspended from *A* and hangs at rest.

(b) Find the size of the angle between *AB* and the vertical.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

Q5

(Total 9 marks)

Turn over

6.

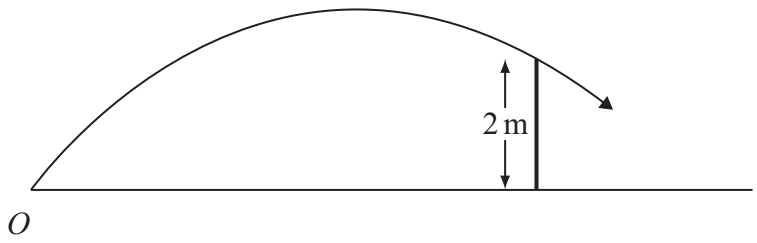

Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}.$$

(6)

Given that $\alpha = 45°$,

(b) find the speed of the ball as it passes over the fence.

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total 12 marks)

Q6

7.

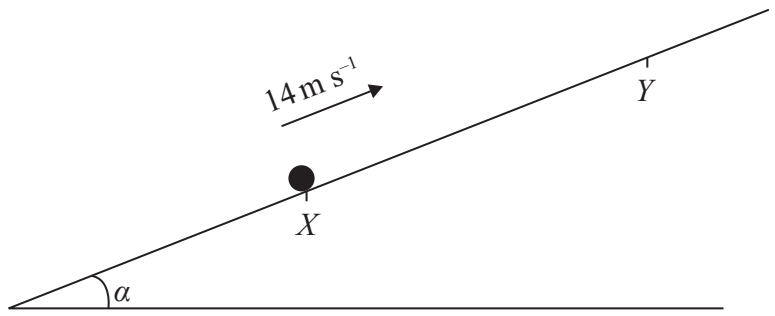

Figure 4

A particle P of mass 2 kg is projected up a rough plane with initial speed 14 m s^{-1}, from a point X on the plane, as shown in Figure 4. The particle moves up the plane along the line of greatest slope through X and comes to instantaneous rest at the point Y. The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{7}{24}$. The coefficient of friction between the particle and the plane is $\frac{1}{8}$.

(a) Use the work-energy principle to show that $XY = 25$ m.

(7)

After reaching Y, the particle P slides back down the plane.

(b) Find the speed of P as it passes through X.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

Q7

(Total 11 marks)

Turn over

8. Particles A, B and C of masses $4m$, $3m$ and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u m s^{-1} and v m s^{-1} respectively, and collide directly.

As a result of the collision, A is brought to rest and B rebounds with speed kv m s^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Show that $u = 3v$.

(6)

(b) Find the value of k.

(2)

Immediately after the collision between A and B, particle C is projected with speed $2v$ m s^{-1} towards B so that B and C collide directly.

(c) Show that there is no further collision between A and B.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END