mock papers 1

1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*.]

A ship P is moving with velocity (5**i** – 4**j**) km h^{-1} and a ship Q is moving with velocity (3**i** + 7**j**) km h^{-1}. Find the direction that ship Q appears to be moving in, to an observer on ship P, giving your answer as a bearing.

(5)

Question 1 continued

(Total 5 marks)

Q1

Turn over

2. Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is m kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(\mathbf{i} - 3\mathbf{j})$ m s^{-1}. Find the speed of B immediately after the collision.

(5)

Question 2 continued

(Total 5 marks)

Q2

Turn over

3. At time $t = 0$, a particle of mass m is projected vertically downwards with speed U from a point above the ground. At time t the speed of the particle is v and the magnitude of the air resistance is modelled as being mkv, where k is a constant.

Given that $U < \dfrac{g}{2k}$, find, in terms of k, U and g, the time taken for the particle to double its speed.

(8)

Question 3 continued

Q3

(Total 8 marks)

Turn over

4.

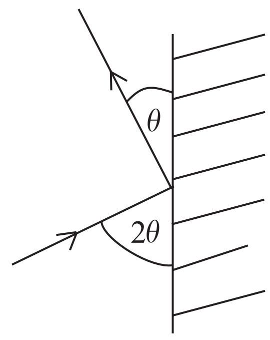

Figure 1

A small smooth ball B, moving on a horizontal plane, collides with a fixed vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is 2θ, where $0^{o} < \theta < 45^{o}$. Immediately after the collision the angle between the direction of motion of B and the wall is θ, as shown in Figure 1. Given that the coefficient of restitution between B and the wall is $\frac{3}{8}$, find the value of $\tan\theta$.

(8)

Question 4 continued

Q4

(Total 8 marks)

Turn over

5. A light elastic spring has natural length l and modulus of elasticity mg. One end of the spring is fixed to a point O on a rough horizontal table. The other end is attached to a particle P of mass m which is at rest on the table with $OP = l$. At time $t = 0$ the particle is projected with speed $\sqrt{(gl)}$ along the table in the direction OP. At time t the displacement of P from its initial position is x and its speed is v. The motion of P is subject to air resistance of magnitude $2mv\omega$, where $\omega = \sqrt{\frac{g}{l}}$. The coefficient of friction between P and the table is 0.5.

(a) Show that, until P first comes to rest,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\omega\frac{\mathrm{d}x}{\mathrm{d}t} + \omega^2 x = -0.5g.$$

(6)

(b) Find x in terms of t, l and ω.

(6)

(c) Hence find, in terms of ω, the time taken for P to first come to instantaneous rest.

(3)

Question 5 continued

Turn over

Question 5 continued

Question 5 continued

(Total 15 marks)

Q5

6.

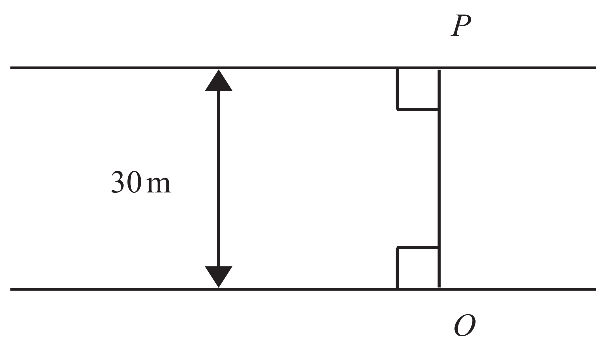

Figure 2

A river is 30 m wide and flows between two straight parallel banks. At each point of the river, the direction of flow is parallel to the banks. At time $t = 0$, a boat leaves a point O on one bank and moves in a straight line across the river to a point P on the opposite bank. Its path OP is perpendicular to both banks and $OP = 30$ m, as shown in Figure 2. The speed of flow of the river, r m s^{-1}, at a point on OP which is at a distance x m from O, is modelled as

$$r = \frac{1}{10}x, \quad 0 \leqslant x \leqslant 30.$$

The speed of the boat relative to the water is constant at 5 m s^{-1}. At time t seconds the boat is at a distance x m from O and is moving with speed v m s^{-1} in the direction OP.

(a) Show that

$$100v^2 = 2500 - x^2.$$

(3)

(b) Hence show that

$$\frac{d^2x}{dt^2} + \frac{x}{100} = 0.$$

(4)

(c) Find the total time taken for the boat to cross the river from O to P.

(9)

Question 6 continued

Question 6 continued

Question 6 continued

Q6

(Total 16 marks)

Turn over

7.

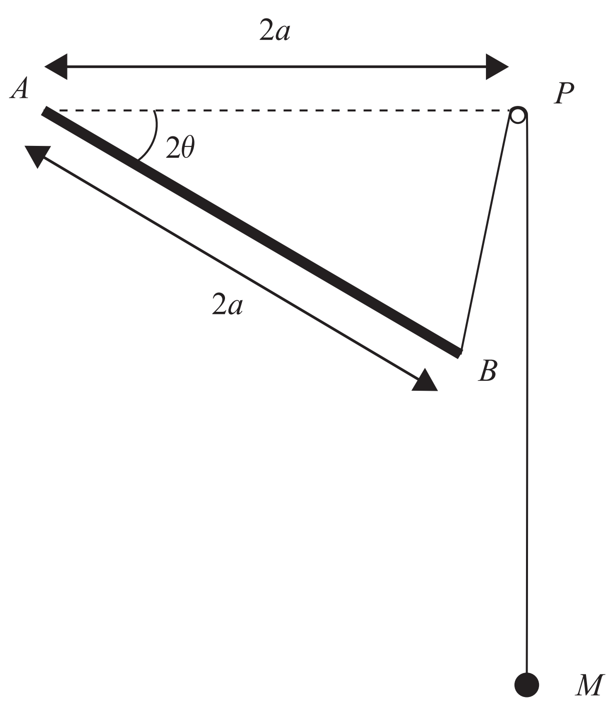

Figure 3

A uniform rod AB, of length $2a$ and mass kM where k is a constant, is free to rotate in a vertical plane about the fixed point A. One end of a light inextensible string of length $6a$ is attached to the end B of the rod and passes over a small smooth pulley which is fixed at the point P. The line AP is horizontal and of length $2a$. The other end of the string is attached to a particle of mass M which hangs vertically below the point P, as shown in Figure 3. The angle PAB is 2θ , where $0^\circ \leqslant \theta \leqslant 180^\circ$.

(a) Show that the potential energy of the system is

$$Mga(4\sin\theta - k\sin2\theta) + \text{constant.}$$

(5)

The system has a position of equilibrium when $\cos\theta = \frac{3}{4}$.

(b) Find the value of k.

(5)

(c) Hence find the value of $\cos\theta$ at the other position of equilibrium.

(3)

(d) Determine the stability of each of the two positions of equilibrium.

(5)

Question 7 continued

Turn over

Question 7 continued

Question 7 continued

Turn over

Question 7 continued

Q7

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

END

1.

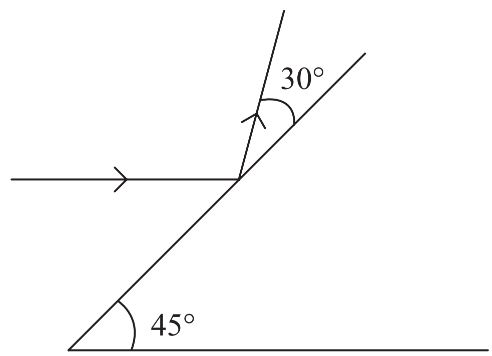

Figure 1

A fixed smooth plane is inclined to the horizontal at an angle of 45°. A particle P is moving horizontally and strikes the plane. Immediately before the impact, P is moving in a vertical plane containing a line of greatest slope of the inclined plane. Immediately after the impact, P is moving in a direction which makes an angle of 30° with the inclined plane, as shown in Figure 1.

Find the fraction of the kinetic energy of P which is lost in the impact.

(6)

Question 1 continued

Q1

(Total 6 marks)

2. At time $t = 0$, a particle P of mass m is projected vertically upwards with speed $\sqrt{\frac{g}{k}}$, where k is a constant. At time t the speed of P is v. The particle P moves against air resistance whose magnitude is modelled as being mkv^2 when the speed of P is v. Find, in terms of k, the distance travelled by P until its speed first becomes half of its initial speed.

(9)

Question 2 continued

Question 2 continued

Question 2 continued

Q2

(Total 9 marks)

3. At noon a motorboat P is 2 km north-west of another motorboat Q. The motorboat P is moving due south at 20 m s^{-1}. The motorboat Q is pursuing motorboat P at a speed of 12 m s^{-1} and sets a course in order to get as close to motorboat P as possible.

(a) Find the course set by Q, giving your answer as a bearing to the nearest degree.
(4)

(b) Find the shortest distance between P and Q.
(3)

(c) Find the distance travelled by Q from its position at noon to the point of closest approach.
(5)

Question 3 continued

Question 3 continued

Question 3 continued

Q3

(Total 12 marks)

4.

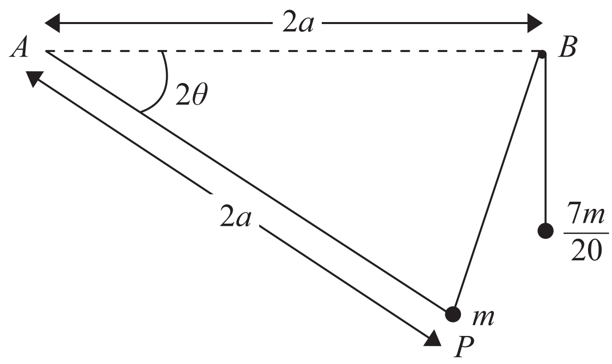

Figure 2

A light inextensible string of length $2a$ has one end attached to a fixed point A. The other end of the string is attached to a particle P of mass m. A second light inextensible string of length L, where $L > \frac{12a}{5}$, has one of its ends attached to P and passes over a small smooth peg fixed at a point B. The line AB is horizontal and $AB = 2a$. The other end of the second string is attached to a particle of mass $\frac{7}{20}m$, which hangs vertically below B, as shown in Figure 2.

(a) Show that the potential energy of the system, when the angle $PAB = 2\theta$, is

$$\frac{1}{5}mga(7\sin\theta - 10\sin 2\theta) + \text{constant.}$$

(4)

(b) Show that there is only one value of $\cos\theta$ for which the system is in equilibrium and find this value.

(8)

(c) Determine the stability of the position of equilibrium.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total 16 marks)

Q4

5. Two small smooth spheres A and B, of mass 2 kg and 1 kg respectively, are moving on a smooth horizontal plane when they collide. Immediately before the collision the velocity of A is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $-2\mathbf{i}$ m s^{-1}. Immediately after the collision the velocity of A is $\mathbf{j}$ m s^{-1}.

(a) Show that the velocity of B immediately after the collision is $2\mathbf{j}$ m s^{-1}.

(3)

(b) Find the impulse of B on A in the collision, giving your answer as a vector, and hence show that the line of centres is parallel to $\mathbf{i} + \mathbf{j}$.

(4)

(c) Find the coefficient of restitution between A and B.

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total 13 marks)

Q5

6. A light elastic spring AB has natural length $2a$ and modulus of elasticity $2mn^2a$, where n is a constant. A particle P of mass m is attached to the end A of the spring. At time $t = 0$, the spring, with P attached, lies at rest and unstretched on a smooth horizontal plane. The other end B of the spring is then pulled along the plane in the direction AB with constant acceleration f. At time t the extension of the spring is x.

(a) Show that $$\frac{d^2x}{dt^2} + n^2x = f.$$
(6)

(b) Find x in terms of n, f and t.
(8)

Hence find

(c) the maximum extension of the spring,
(3)

(d) the speed of P when the spring first reaches its maximum extension.
(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total 19 marks)

Q6

TOTAL FOR PAPER: 75 MARKS

END

mock papers 3

1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*]

A man cycles at a constant speed u m s^{-1} on level ground and finds that when his velocity is $u\mathbf{j}$ m s^{-1} the velocity of the wind appears to be $v(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where v is a positive constant.

When the man cycles with velocity $\frac{1}{5}u(-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}, the velocity of the wind appears to be $w\mathbf{i}$ m s^{-1}, where w is a positive constant.

Find, in terms of u, the true velocity of the wind.

(7)

Question 1 continued

(Total 7 marks)

Q1

2. Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5. Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i} + \mathbf{j})$ m s^{-1}. Given that when the spheres collide the line joining their centres is parallel to $\mathbf{i}$,

(a) find

(i) $\mathbf{u}_1$,

(ii) $\mathbf{u}_2$.

(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to $\mathbf{j}$. Given that the coefficient of restitution between T and the wall is also 0.5, find

(b) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,

(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.

(3)

Question 2 continued

Turn over

Question 2 continued

Question 2 continued

(Total 14 marks)

Q2

Turn over

3. At 12 noon, ship A is 8 km due west of ship B. Ship A is moving due north at a constant speed of 10 km h^{-1}. Ship B is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to A as possible.

(a) Find the bearing on which ship B moves.

(4)

(b) Find the shortest distance between the two ships.

(3)

(c) Find the time when the two ships are closest.

(3)

Question 3 continued

Turn over

Question 3 continued

Question 3 continued

(Total 10 marks)

Q3

Turn over

4. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U. The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(a) Show that the particle reaches its greatest height above the point of projection at time $\frac{k}{g}\tan^{-1}\left(\frac{U}{k}\right)$.

(6)

(b) Find the greatest height above the point of projection attained by the particle.

(6)

Question 4 continued

Turn over

Question 4 continued

Question 4 continued

(Total 12 marks)

Q4

Turn over

5.

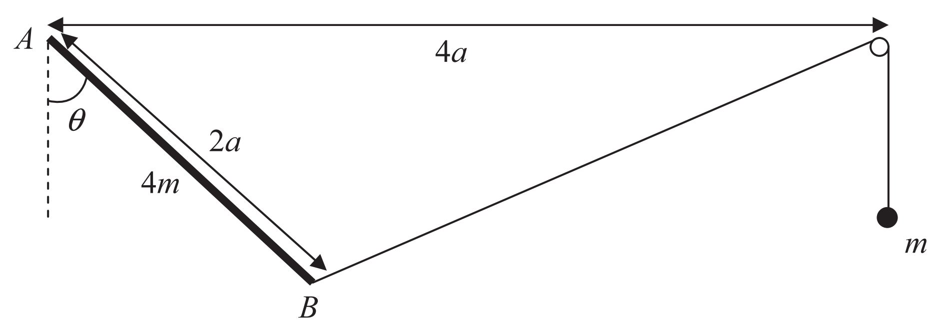

Figure 1

The end A of a uniform rod AB, of length $2a$ and mass $4m$, is smoothly hinged to a fixed point. The end B is attached to one end of a light inextensible string which passes over a small smooth pulley, fixed at the same level as A. The distance from A to the pulley is $4a$. The other end of the string carries a particle of mass m which hangs freely, vertically below the pulley, with the string taut. The angle between the rod and the downward vertical is θ, where $0 < \theta < \frac{\pi}{2}$, as shown in Figure 1.

(a) Show that the potential energy of the system is

$$2mga(\sqrt{(5 - 4\sin\theta)} - 2\cos\theta) + \text{constant} .$$

(5)

(b) Hence, or otherwise, show that any value of θ which corresponds to a position of equilibrium of the system satisfies the equation

$$4\sin^3\theta - 6\sin^2\theta + 1 = 0 .$$

(5)

(c) Given that $\theta = \frac{\pi}{6}$ corresponds to a position of equilibrium, determine its stability.

(5)

Question 5 continued

Turn over

Question 5 continued

Question 5 continued

(Total 15 marks)

Q5

Turn over

6. Two points A and B lie on a smooth horizontal table with $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $2mg$, is attached to A. The other end of the spring is attached to a particle P of mass m. Another light elastic spring, of natural length a and modulus of elasticity mg, has one end attached to B and the other end attached to P. The particle P is on the table at rest and in equilibrium.

(a) Show that $AP = \dfrac{5a}{3}$.

(4)

The particle P is now moved along the table from its equilibrium position through a distance $0.5a$ towards B and released from rest at time $t = 0$. At time t, P is moving with speed v and has displacement x from its equilibrium position. There is a resistance to motion of magnitude $4m\omega v$ where $\omega = \sqrt{\left(\dfrac{g}{a}\right)}$.

(b) Show that $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\omega\dfrac{\mathrm{d}x}{\mathrm{d}t} + 3\omega^2 x = 0$.

(5)

(c) Find the velocity, $\dfrac{\mathrm{d}x}{\mathrm{d}t}$, of P in terms of a, ω and t.

(8)

Question 6 continued

Turn over

Question 6 continued

Question 6 continued

Q6

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END