1 A particle P is moving with simple harmonic motion in a straight line. The period is 6.1 s and the amplitude is 3 m. Calculate, in either order,

(i) the maximum speed of P, [3]

(ii) the distance of P from the centre of motion when P has speed $2.5\,\mathrm{m\,s^{-1}}$. [3]

2 A tennis ball of mass 0.057 kg has speed $10\,\mathrm{m\,s^{-1}}$. The ball receives an impulse of magnitude 0.6 N s which reduces the speed of the ball to $7\,\mathrm{m\,s^{-1}}$. Using an impulse-momentum triangle, or otherwise, find the angle the impulse makes with the original direction of motion of the ball. [7]

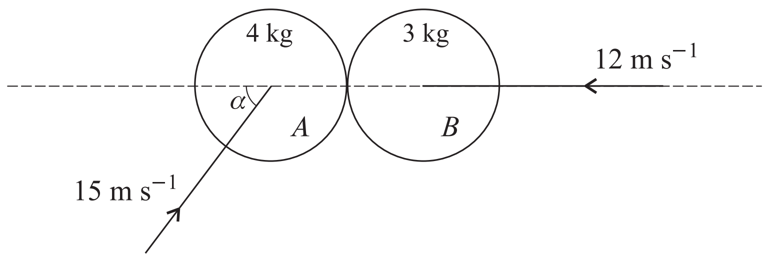
3 A particle P of mass 0.2 kg is projected horizontally with speed $u\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. P moves in a straight line and, at time t s after projection, P has speed $v\,\mathrm{m\,s^{-1}}$ and is x m from O. The only force acting on P has magnitude $0.4v^2$ N and is directed towards O.

(i) Show that $\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} = -2$. [2]

(ii) Hence show that $v = u\mathrm{e}^{-2x}$. [4]

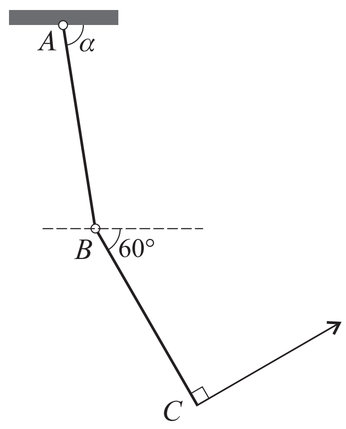
(iii) Find u, given that $x = 2$ when $t = 4$. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 3 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, A is moving with speed $15\,\mathrm{m\,s^{-1}}$ at an angle α to the line of centres, where $\sin\alpha = 0.8$, and B is moving along the line of centres with speed $12\,\mathrm{m\,s^{-1}}$ (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [10]

5

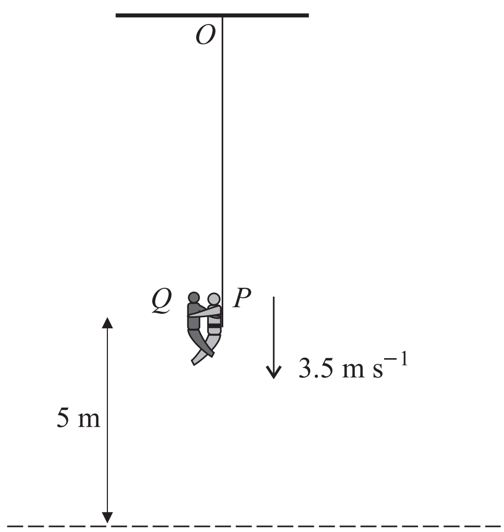


Two uniform rods AB and BC, each of length 1.4 m and weight 80 N, are freely jointed to each other at B, and AB is freely jointed to a fixed point at A. They are held in equilibrium with AB at an angle α to the horizontal, and BC at an angle of 60° to the horizontal, by a light string, perpendicular to BC, attached to C (see diagram).

(i) By taking moments about B for BC, calculate the tension in the string. Hence find the horizontal and vertical components of the force acting on BC at B. [7]

(ii) Find α. [4]

6



A circus performer P of mass 80 kg is suspended from a fixed point O by an elastic rope of natural length 5.25 m and modulus of elasticity 2058 N. P is in equilibrium at a point 5 m above a safety net. A second performer Q, also of mass 80 kg, falls freely under gravity from a point above P. P catches Q and together they begin to descend vertically with initial speed 3.5 m s^{-1} (see diagram). The performers are modelled as particles.

(i) Show that, when P is in equilibrium, OP = 7.25 m. [3]

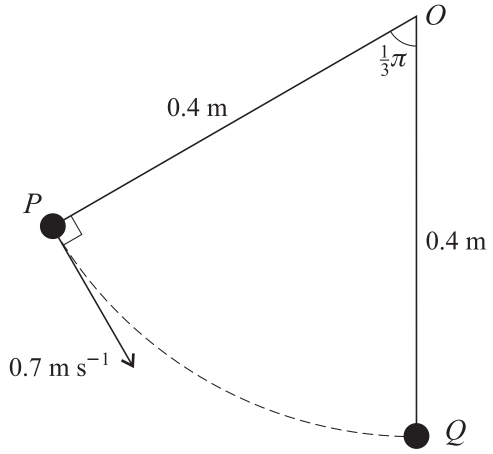
(ii) Verify that P and Q together just reach the safety net. [5]

(iii) At the lowest point of their motion P releases Q. Prove that P subsequently just reaches O. [3]

(iv) State two additional modelling assumptions made when answering this question. [2]

[Turn over

7



A particle P of mass 0.8 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. A particle Q is suspended from O by an identical string. With the string OP taut and inclined at $\frac{1}{3}\pi$ radians to the vertical, P is projected with speed $0.7\,\text{m}\,\text{s}^{-1}$ in a direction perpendicular to the string so as to strike Q directly (see diagram). The coefficient of restitution between P and Q is $\frac{1}{7}$.

(i) Calculate the tension in the string immediately after P is set in motion. [4]

(ii) Immediately after P and Q collide they have equal speeds and are moving in opposite directions. Show that Q starts to move with speed $0.15\,\text{m}\,\text{s}^{-1}$. [4]

(iii) Prove that before the second collision between P and Q, Q is moving with approximate simple harmonic motion. [5]

(iv) Hence find the time interval between the first and second collisions of P and Q. [2]

mock papers 2

1. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A small bead of mass 0.5 kg is threaded on a smooth horizontal wire. The bead is initially at rest at the point with position vector (**i** – 6**j**) m. A constant horizontal force **P** N then acts on the bead causing it to move along the wire. The bead passes through the point with position vector (7**i** – 14**j**) m with speed $2\sqrt{7}$ m s^{-1}.

Given that **P** is parallel to (6**i** + **j**), find **P**.

(6)

Question 1 continued

Q1

(Total 6 marks)

Turn over

2. The velocity **v** m s^{-1} of a particle P at time t seconds satisfies the vector differential equation

$$\frac{d\mathbf{v}}{dt} + 4\mathbf{v} = 0 \ .$$

The position vector of P at time t seconds is **r** metres.

Given that at $t = 0$, $\mathbf{r} = (\mathbf{i} - \mathbf{j})$ and $\mathbf{v} = (-8\mathbf{i} + 4\mathbf{j})$, find **r** at time t seconds.

(7)

Question 2 continued

Q2

(Total 7 marks)

Turn over

3. A system of forces consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on a rigid body.

$\mathbf{F}_1 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_1 = (\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

$\mathbf{F}_2 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_2 = (4\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ m.

Given that the system is equivalent to a single force **R** N, acting at the point with position vector $(5\mathbf{i} + \mathbf{j} - \mathbf{k})$ m, together with a couple **G** N m, find

(a) **R** ,

(2)

(b) the magnitude of **G**.

(9)

Question 3 continued

(Total 11 marks)

Q3

Turn over

4. At time $t = 0$ a rocket is launched from rest vertically upwards. The rocket propels itself upwards by expelling burnt fuel vertically downwards with constant speed U m s^{-1} relative to the rocket. The initial mass of the rocket is M_0 kg. At time t seconds, where $t < 2$, its mass is $M_0(1 - \frac{1}{2}t)$ kg, and it is moving upwards with speed v m s^{-1}.

(a) Show that

$$\frac{dv}{dt} = \frac{U}{(2 - t)} - 9.8$$

(7)

(b) Hence show that $U > 19.6$

(2)

(c) Find, in terms of U, the speed of the rocket one second after its launch.

(5)

Question 4 continued

Turn over

Question 4 continued

Question 4 continued

Q4

(Total 14 marks)

5.

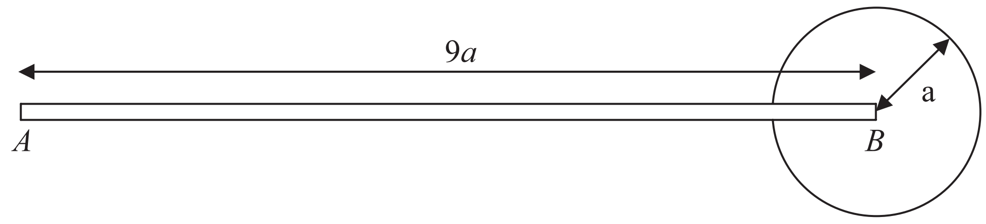


Figure 1

A pendulum P is modelled as a uniform rod AB, of length $9a$ and mass m, rigidly fixed to a uniform circular disc of radius a and mass $2m$. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in Figure 1. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through the end A and is perpendicular to the plane of the disc.

(a) Show that the moment of inertia of P about L is $190ma^2$.

(4)

The pendulum makes small oscillations about L.

(b) By writing down an equation of motion for P, find the approximate period of these small oscillations.

(7)

Question 5 continued

Turn over

Question 5 continued

Question 5 continued

(Total 11 marks)

Q5

6. A uniform solid right circular cylinder has mass M, height h and radius a. Find, using integration, its moment of inertia about a diameter of one of its circular ends.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius a, about a diameter is* $\frac{1}{4}ma^2$.]

(10)

Question 6 continued

Turn over

Question 6 continued

Question 6 continued

(Total 10 marks)

Q6

Turn over

7. A uniform square lamina *ABCD*, of mass $2m$ and side $3a\sqrt{2}$, is free to rotate in a vertical plane about a fixed smooth horizontal axis *L* which passes through *A* and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about *L* is $24ma^2$.

The lamina is at rest with *C* vertically above *A*. At time $t = 0$ the lamina is slightly displaced. At time *t* the lamina has rotated through an angle θ.

(a) Show that

$$2a\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = g(1-\cos\theta).$$

(4)

(b) Show that, at time *t*, the magnitude of the component of the force acting on the lamina at *A*, in a direction perpendicular to *AC*, is $\frac{1}{2}mg\sin\theta$.

(7)

When the lamina reaches the position with *C* vertically below *A*, it receives an impulse which acts at *C*, in the plane of the lamina and in a direction which is perpendicular to the line *AC*. As a result of this impulse the lamina is brought immediately to rest.

(c) Find the magnitude of the impulse.

(5)

Question 7 continued

Turn over

Question 7 continued

Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

1 A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.8 m and modulus of elasticity $1.35mg$ N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P is held at rest at a point on the surface 3 m from O. The particle is then released. Find

(i) the initial acceleration of P, [3]

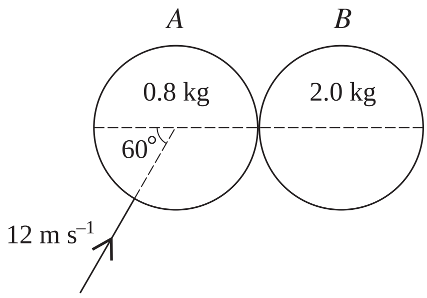
(ii) the speed of P at the instant the string becomes slack. [3]

2 A particle P of mass 0.2 kg is moving with speed $8\,\mathrm{m\,s^{-1}}$ when it hits a horizontal smooth surface. The direction of motion of P immediately before impact makes an angle of $27°$ with the surface. Given that the coefficient of restitution between the particle and the surface is 0.6, find

(i) the vertical component of the velocity of P immediately after impact, [3]

(ii) the magnitude of the impulse exerted on P. [3]

3



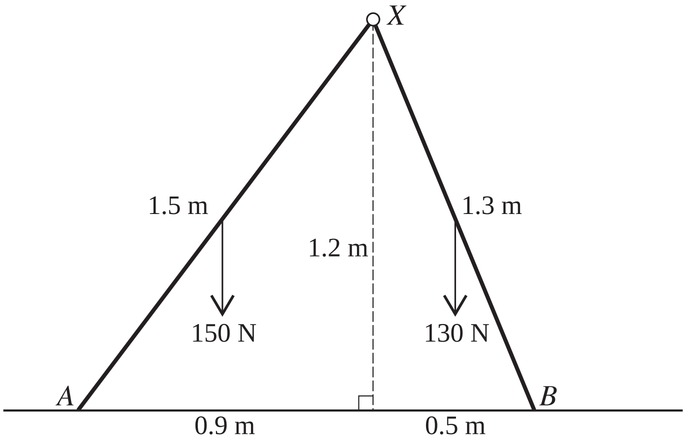
Two uniform smooth spheres A and B, of equal radius, have masses 0.8 kg and 2.0 kg respectively. The spheres are on a horizontal surface. A is moving with speed $12\,\mathrm{m\,s^{-1}}$ at $60°$ to the line of centres when it collides with B, which is stationary (see diagram). The coefficient of restitution between the spheres is 0.75. Find the speed and direction of motion of A immediately after the collision. [10]

4 A particle P of mass m kg is held at rest at a point O on a fixed plane inclined at an angle $\sin^{-1}\left(\frac{4}{7}\right)$ to the horizontal. P is released and moves down the plane. The total resistance acting on P is $0.2mv$ N, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at time t s after leaving O.

(i) Show that $5\frac{\mathrm{d}v}{\mathrm{d}t} = 28 - v$ and hence find an expression for v in terms of t. [8]

(ii) Find the acceleration of P when $t = 10$. [2]

5



Two uniform rods XA and XB are freely jointed at X. The lengths of the rods are 1.5 m and 1.3 m respectively, and their weights are 150 N and 130 N respectively. The rods are in equilibrium in a vertical plane with A and B in contact with a rough horizontal surface. A and B are at distances horizontally from X of 0.9 m and 0.5 m respectively, and X is 1.2 m above the surface (see diagram).

(i) The normal components of the contact forces acting on the rods at A and B are R_A N and R_B N respectively. Show that $R_A = 125$ and find R_B. [4]

(ii) Find the frictional components of the contact forces acting on the rods at A and B. [4]

(iii) Find the horizontal and vertical components of the force exerted on XA at X, stating their directions. [3]

6 A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal surface. A force of $(0.36 - 0.144x)$ N acts on P in the direction from O to P, where x m is the displacement of P from a point O on the surface at time t s.

(i) By using the substitution $x = y + 2.5$, or otherwise, show that P moves with simple harmonic motion of period 5.24 s, correct to 3 significant figures. [5]

The maximum value of x during the motion is 3.

(ii) Write down the amplitude of P's motion and find the two possible values of x for which P's speed is $0.48\,\text{m}\,\text{s}^{-1}$. [4]

(iii) On each of the first two occasions when P has speed $0.48\,\text{m}\,\text{s}^{-1}$, P is moving towards O. Find the time interval between

(a) these first two occasions,

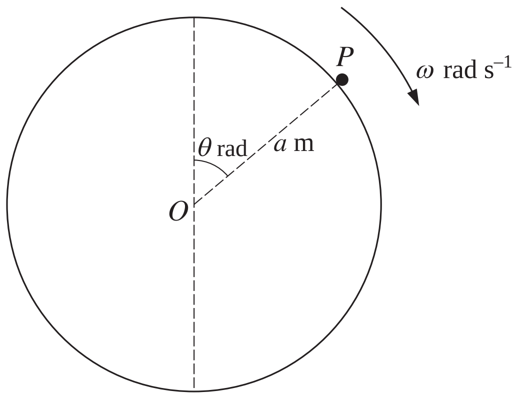
(b) the second and third occasions when P has speed $0.48\,\text{m}\,\text{s}^{-1}$.

[5]

[Question 7 is printed overleaf.]

[Turn over

7



A particle P of mass m kg is slightly disturbed from rest at the highest point on the surface of a smooth fixed sphere of radius a m and centre O. The particle starts to move downwards on the surface. While P remains on the surface OP makes an angle of θ radians with the upward vertical and has angular speed ω rad s^{-1} (see diagram). The sphere exerts a force of magnitude R N on P.

(i) Show that $a\omega^2 = 2g(1 - \cos\theta)$. [3]

(ii) Find an expression for R in terms of m, g and θ. [4]

At the instant that P loses contact with the surface of the sphere, find

(iii) the transverse component of the acceleration of P, [4]

(iv) the rate of change of R with respect to time t, in terms of m, g and a. [4]

mock papers 4

1. At time $t = 0$, a particle P of mass 3 kg is at rest at the point A with position vector $(\mathbf{j} - 3\mathbf{k})$ m. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ then act on the particle P and it passes through the point B with position vector $(8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ N and $\mathbf{F}_2 = (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$ N and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the *only* two forces acting on P, find the velocity of P as it passes through B, giving your answer as a vector.

(7)

Question 1 continued

Q1

(Total 7 marks)

Turn over

2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the vector differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 4\mathbf{r} = \mathrm{e}^{2t}\mathbf{j}\,.$$

When $t = 0$, P has position vector $(\mathbf{i} + \mathbf{j})$ m and velocity $2\mathbf{i}$ m s^{-1}.

Find an expression for $\mathbf{r}$ in terms of t.

(11)

Question 2 continued

Turn over

Question 2 continued

Question 2 continued

(Total 11 marks)

Q2

3. A spaceship is moving in a straight line in deep space and needs to increase its speed. This is done by ejecting fuel backwards from the spaceship at a constant speed c relative to the spaceship. When the speed of the spaceship is v, its mass is m.

(a) Show that, while the spaceship is ejecting fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}.$$

(5)

The initial mass of the spaceship is m_0 and at time t the mass of the spaceship is given by $m = m_0(1 - kt)$, where k is a positive constant.

(b) Find the acceleration of the spaceship at time t.

(4)

Question 3 continued

Turn over

Question 3 continued

Question 3 continued

Q3

(Total 9 marks)

Turn over

4.

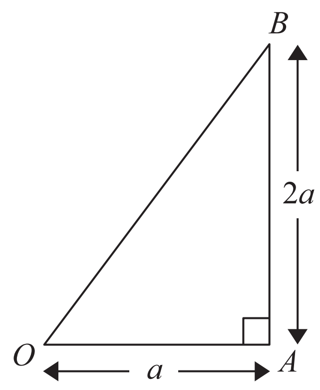


Figure 1

A uniform lamina of mass M is in the shape of a right-angled triangle OAB. The angle OAB is 90°, $OA = a$ and $AB = 2a$, as shown in Figure 1.

(a) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina OAB is free to rotate about a fixed smooth horizontal axis along the edge OA and hangs at rest with B vertically below A. The lamina is then given a horizontal impulse of magnitude J. The impulse is applied to the lamina at the point B, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(b) find an expression for J, in terms of M, a and g.

(7)

Question 4 continued

Turn over

Question 4 continued

Question 4 continued

(Total 13 marks)

Q4

5. Two forces $\mathbf{F}_1 = (2\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}_2 = (-2\mathbf{j} - \mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts at the point with position vector $\mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $\mathbf{r}_2 = (\mathbf{i} - 2\mathbf{j})$ m. A third force $\mathbf{F}_3$ acts on the body such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in equilibrium.

(a) Find the magnitude of $\mathbf{F}_3$.

(4)

(b) Find a vector equation of the line of action of $\mathbf{F}_3$.

(8)

The force $\mathbf{F}_3$ is replaced by a fourth force $\mathbf{F}_4$, acting through the origin O, such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ are equivalent to a couple.

(c) Find the magnitude of this couple.

(4)

Question 5 continued

Turn over

Question 5 continued

Question 5 continued

(Total 16 marks)

Q5

Turn over

6. A pendulum consists of a uniform rod AB, of length $4a$ and mass $2m$, whose end A is rigidly attached to the centre O of a uniform square lamina $PQRS$, of mass $4m$ and side a. The rod AB is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through B. The axis L is perpendicular to AB and parallel to the edge PQ of the square.

(a) Show that the moment of inertia of the pendulum about L is $75ma^2$.

(4)

The pendulum is released from rest when BA makes an angle α with the downward vertical through B, where $\tan \alpha = \frac{7}{24}$. When BA makes an angle θ with the downward vertical through B, the magnitude of the component, in the direction AB, of the force exerted by the axis L on the pendulum is X.

(b) Find an expression for X in terms of m, g and θ.

(9)

Using the approximation $\theta \approx \sin \theta$,

(c) find an estimate of the time for the pendulum to rotate through an angle α from its initial rest position.

(6)

Question 6 continued

Turn over

Question 6 continued

Question 6 continued

Turn over

Question 6 continued

(Total 19 marks)

Q6

TOTAL FOR PAPER: 75 MARKS

END