1. Find the value of

(a) $25^{\frac{1}{2}}$

(1)

(b) $25^{-\frac{3}{2}}$

(2)

Q1

(Total 3 marks)

2. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$,

(3)

(b) $\int y \, dx$.

(4)

3. The points P and Q have coordinates (–1, 6) and (9, 0) respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

4. Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

(7)

5. A sequence $a_1, a_2, a_3, ...$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.
(1)

(b) Show that $a_3 = 25k + 18$.
(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.
(4)

6. Given that $\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^{p}+3x^{q}$,

(a) write down the value of p and the value of q.

(2)

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

7. $$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k+a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

8.

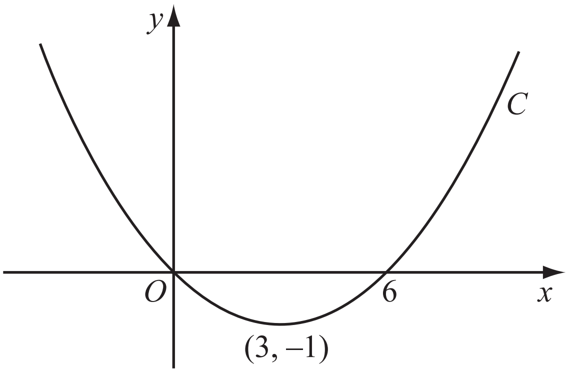


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$,

(3)

(b) $y = -\mathrm{f}(x)$,

(3)

(c) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

9. (a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \ldots\ldots + 100$$

(3)

(b) In the arithmetic series

$$k + 2k + 3k + \ldots\ldots + 100$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}$$

(4)

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), (4k + 4), (6k + 7), \ldots\ldots,$$

giving your answer in its simplest form.

(2)

10. The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate −5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B.

(3)

mock papers 2

1. Find $\int(3x^2 + 4x^5 - 7)\,\mathrm{d}x$.

(4)

Q1

(Total 4 marks)

2. (a) Write down the value of $16^{\frac{1}{4}}$.

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$.

(2)

Q2

(Total 3 marks)

Turn over

3. Simplify

$$\frac{5-\sqrt{3}}{2+\sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(4)

Q3

(Total 4 marks)

4. The point A (–6, 4) and the point B (8, –3) lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.
(3)

Q4

(Total 7 marks)

5. (a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

Question 5 continued

(Total 6 marks)

Q5

Turn over

6.

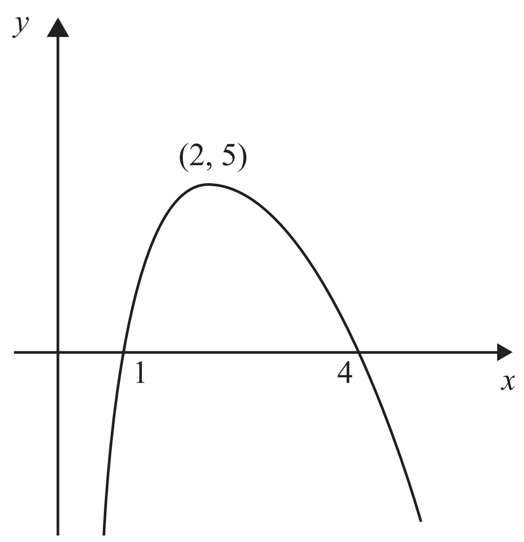


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5).
In separate diagrams sketch the curves with the following equations.
On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2f(x)$,

(3)

(b) $y = f(-x)$.

(3)

The maximum point on the curve with equation $y = f(x + a)$ is on the y-axis.

(c) Write down the value of the constant a.

(1)

Question 6 continued

(Total 7 marks)

Q6

7. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant $(p \neq 0)$.

(a) Find x_2 in terms of p.

(1)

(b) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(c) find the value of p,

(3)

(d) write down the value of x_{2008} .

(2)

Question 7 continued

Q7

(Total 8 marks)

8. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

Question 8 continued

Q8

(Total 7 marks)

Turn over

9. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $\mathrm{f}(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

Question 9 continued

Q9

(Total 10 marks)

10. The curve C has equation

$$y = (x+3)(x-1)^2 .$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

Question 10 continued

Turn over

Question 10 continued

Question 10 continued

Q10

(Total 12 marks)

11. The first term of an arithmetic sequence is 30 and the common difference is –1.5

(a) Find the value of the 25th term.
(2)

The rth term of the sequence is 0.

(b) Find the value of r.
(2)

The sum of the first n terms of the sequence is S_n.

(c) Find the largest positive value of S_n.
(3)

Question 11 continued

Turn over

Question 11 continued

Question 11 continued

(Total 7 marks)

Q11

TOTAL FOR PAPER: 75 MARKS

END

mock papers 3

1. Find $\int (2 + 5x^2)\,dx$.

(3)

Q1

(Total 3 marks)

2. Factorise completely

$$x^3 - 9x.$$

(3)

(Total 3 marks)

Q2

3.

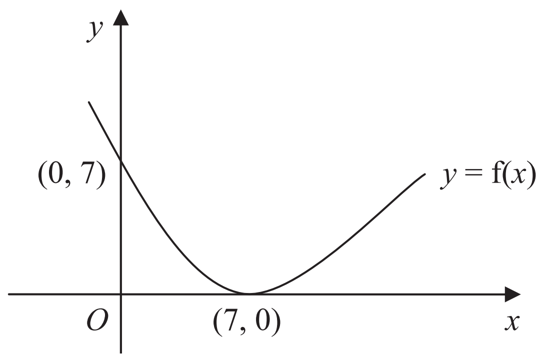


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the point (0, 7) and has a minimum point at (7, 0).

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 3$,

(3)

(b) $y = \mathrm{f}(2x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

Question 3 continued

Q3

(Total 5 marks)

Turn over

4. $f(x) = 3x + x^3, \qquad x > 0.$

(a) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(b) find the value of x.

(3)

Question 4 continued

(Total 5 marks)

Q4

Turn over

5. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geqslant 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a.

(3)

Question 5 continued

(Total 6 marks)

Q5

Turn over

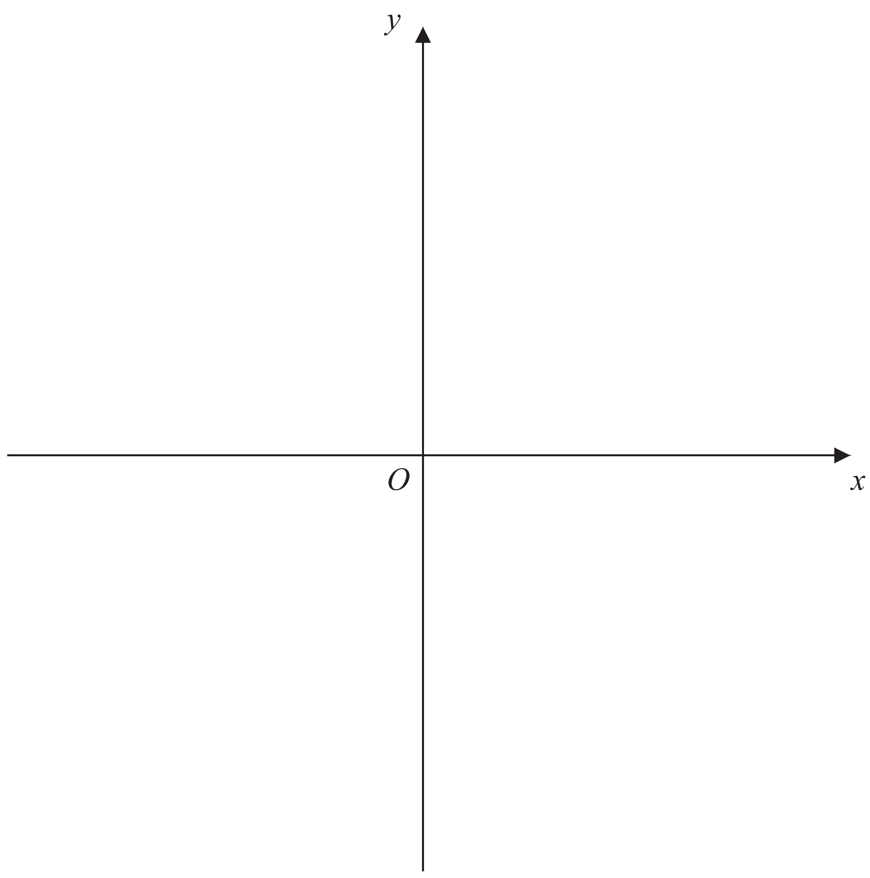
6. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(a) On the axes below, sketch the graphs of C and l, indicating clearly the coordinates of any intersections with the axes.

(3)

(b) Find the coordinates of the points of intersection of C and l.

(6)



Question 6 continued

Q6

(Total 9 marks)

7. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(a) Show that on the 4th Saturday of training she runs 11 km.
(1)

(b) Find an expression, in terms of n, for the length of her training run on the nth Saturday.
(2)

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.
(3)

On the nth Saturday Sue runs 43 km.

(d) Find the value of n.
(2)

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.
(2)

Question 7 continued

Turn over

Question 7 continued

Question 7 continued

(Total 10 marks)

Q7

8. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

Question 8 continued

Q8

(Total 5 marks)

Turn over

9. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y-coordinate of A.

(2)

Question 9 continued

Turn over

Question 9 continued

Question 9 continued

(Total 8 marks)

Q9

Turn over

10.

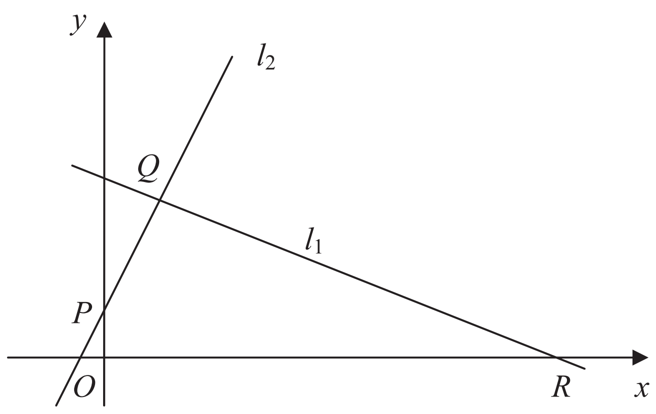


Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(a) Find the value of a.

(3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2.

Find

(b) an equation for l_2,

(5)

(c) the coordinates of P,

(1)

(d) the area of ΔPQR.

(4)

Question 10 continued

Turn over

Question 10 continued

Question 10 continued

(Total 13 marks)

Q10

Turn over

11. The gradient of a curve C is given by $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

Question 11 continued

Turn over

Question 11 continued

(Total 8 marks)

Q11

TOTAL FOR PAPER: 75 MARKS

END

mock papers 4

1. (a) Write down the value of $125^{\frac{1}{3}}$.

(1)

(b) Find the value of $125^{-\frac{2}{3}}$.

(2)

Q1

(Total 3 marks)

Turn over

2. Find $\int (12x^5 - 8x^3 + 3)\,dx$, giving each term in its simplest form.

(4)

(Total 4 marks)

Q2

3. Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(2)

Q3

(Total 2 marks)

4. A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find f(x), giving each term in its simplest form.

(5)

Question 4 continued

Q4

(Total 5 marks)

Turn over

5.

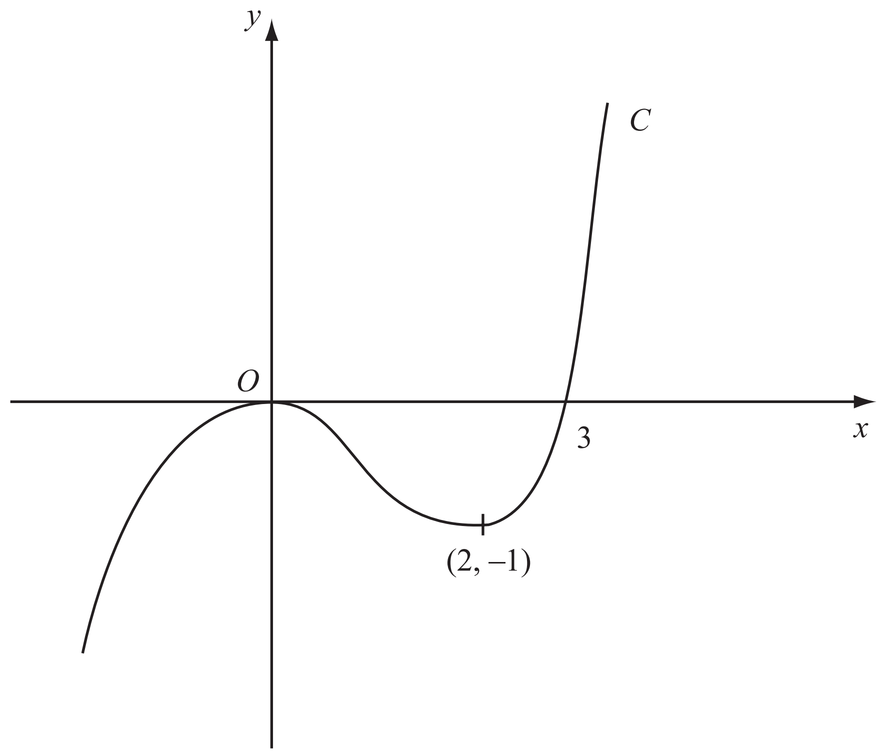


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$. There is a maximum at (0, 0), a minimum at (2, –1) and C passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x + 3)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

Question 5 continued

Q5

(Total 6 marks)

6. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

Question 6 continued

(Total 6 marks)

Q6

Turn over

7. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

Question 7 continued

Q7

(Total 7 marks)

Turn over

8. The point P (1, a) lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \frac{2}{x}$.

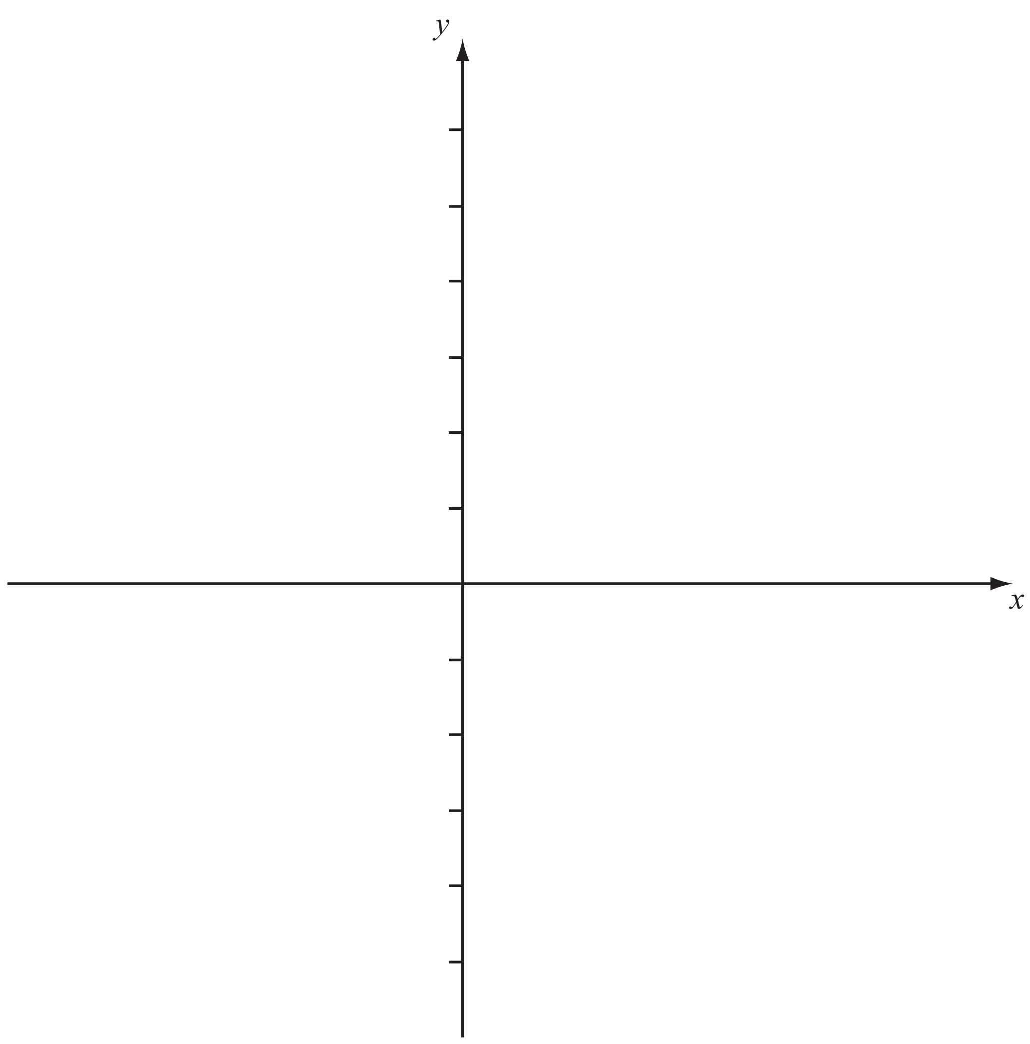
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)



Question 8 continued

Q8

(Total 7 marks)

9. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.

(2)

(b) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.

(3)

Question 9 continued

Turn over

Question 9 continued

Question 9 continued

(Total 11 marks)

Q9

10. The line l_1 passes through the point A (2, 5) and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$.
(3)

The point B has coordinates (–2, 7).

(b) Show that B lies on l_1.
(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.
(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

Question 10 continued

Turn over

Question 10 continued

Question 10 continued

Q10

(Total 11 marks)

Turn over

11. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0 .$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.

(4)

Question 11 continued

Turn over

Question 11 continued

Question 11 continued

(Total 13 marks)

Q11

TOTAL FOR PAPER: 75 MARKS

END

mock papers 5

1. Simplify

(a) $\left(3\sqrt{7}\right)^2$

(1)

(b) $\left(8+\sqrt{5}\right)\left(2-\sqrt{5}\right)$

(3)

(Total 4 marks)

Q1

2. Given that $32\sqrt{2} = 2^a$, find the value of a.

(3)

(Total 3 marks)

Q2

3. Given that $y = 2x^3 + \dfrac{3}{x^2}$, $x \neq 0$, find

(a) $\dfrac{dy}{dx}$ **(3)**

(b) $\int y\,dx$, simplifying each term. **(3)**

Question 3 continued

(Total 6 marks)

Q3

Turn over

4. Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

Question 4 continued

(Total 7 marks)

Q4

Turn over

5. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d,

(3)

(b) the value of a,

(2)

(c) the total number of houses built in Oldtown over the 40-year period.

(3)

Question 5 continued

(Total 8 marks)

Q5

Turn over

6. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

Question 6 continued

(Total 4 marks)

Q6

Turn over

7. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k.

(4)

Question 7 continued

(Total 7 marks)

Q7

8.

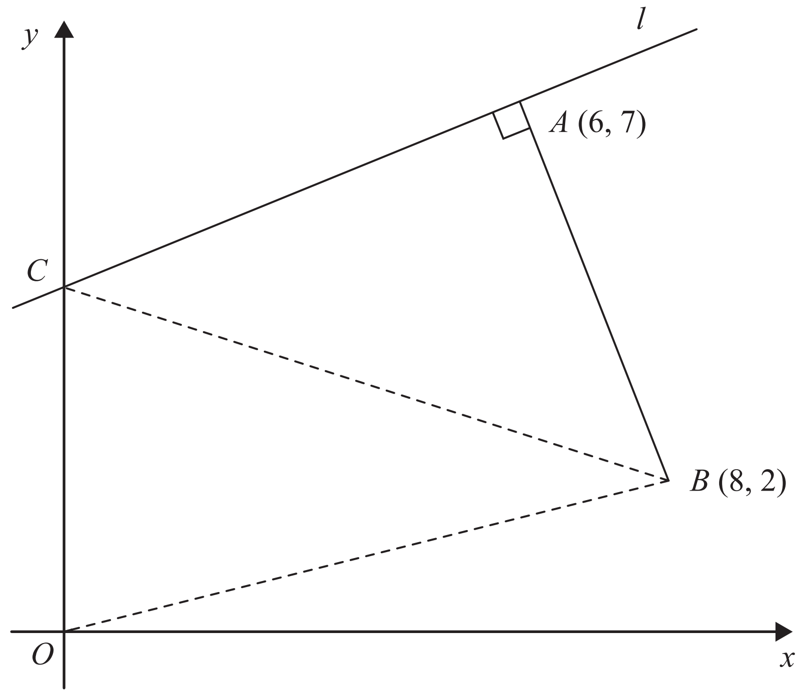


Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,
(2)

(c) the area of ΔOCB, where O is the origin.
(2)

Question 8 continued

Turn over

Question 8 continued

Question 8 continued

(Total 8 marks)

Q8

Turn over

9.

$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(b) Find $f'(x)$.

(3)

(c) Evaluate $f'(9)$.

(2)

Question 9 continued

Turn over

Question 9 continued

Question 9 continued

(Total 8 marks)

Q9

10. (a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

Question 10 continued

Turn over

Question 10 continued

Question 10 continued

Q10

(Total 9 marks)

11. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point P has coordinates (2, 7).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}(2 + \sqrt{6})$.

(5)

Question 11 continued

Turn over

Question 11 continued

(Total 11 marks)

Q11

TOTAL FOR PAPER: 75 MARKS

END