1. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

(a) the 20th term of the series,

(2)

(b) the sum of the first 20 terms of the series,

(2)

(c) the sum to infinity of the series.

(2)

Question 1 continued

Q1

(Total 6 marks)

2. A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C.

(4)

Question 2 continued

Q2

(Total 4 marks)

3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

Question 3 continued

(Total 7 marks)

Q3

4. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3(28x - 9)$$

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total 6 marks)

Q4

5. $f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants.

Given that when $f(x)$ is divided by $(x+2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when $f(x)$ is divided by $(x-1)$ the remainder is 4,

(b) find the value of a and the value of b.

(4)

Question 5 continued

(Total 6 marks)

Q5

6.

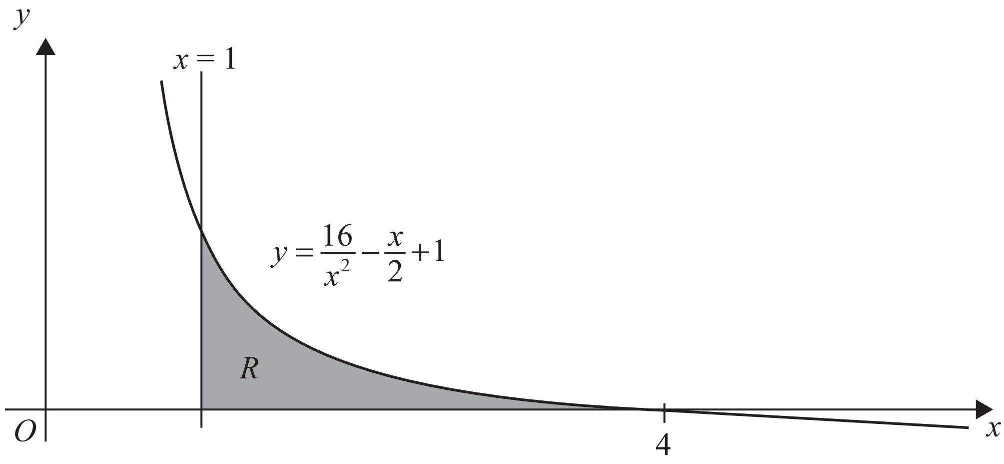


Figure 1

Figure 1 shows the graph of the curve with equation

$$y=\frac{16}{x^2}-\frac{x}{2}+1, \qquad x>0$$

The finite region R, bounded by the lines $x=1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x=2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

Q6

(Total 11 marks)

7.

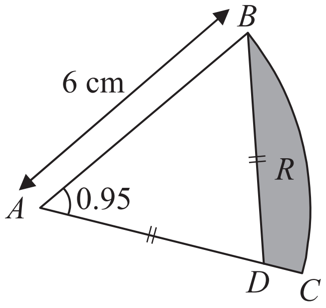


Figure 2

Figure 2 shows *ABC*, a sector of a circle of radius 6 cm with centre *A*. Given that the size of angle *BAC* is 0.95 radians, find

(a) the length of the arc *BC*,

(2)

(b) the area of the sector *ABC*.

(2)

The point *D* lies on the line *AC* and is such that $AD = BD$. The region *R*, shown shaded in Figure 2, is bounded by the lines *CD*, *DB* and the arc *BC*.

(c) Show that the length of *AD* is 5.16 cm to 3 significant figures.

(2)

Find

(d) the perimeter of *R*,

(2)

(e) the area of *R*, giving your answer to 2 significant figures.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

Q7

(Total 12 marks)

8.

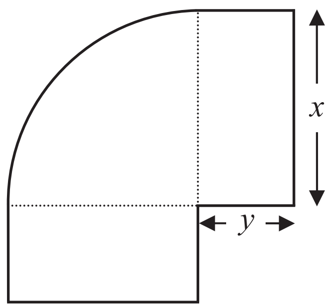


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total 13 marks)

Q8

9. (i) Find the solutions of the equation $\sin(3x - 15^\circ) = \frac{1}{2}$, for which $0 \leqslant x \leqslant 180^\circ$

(ii)

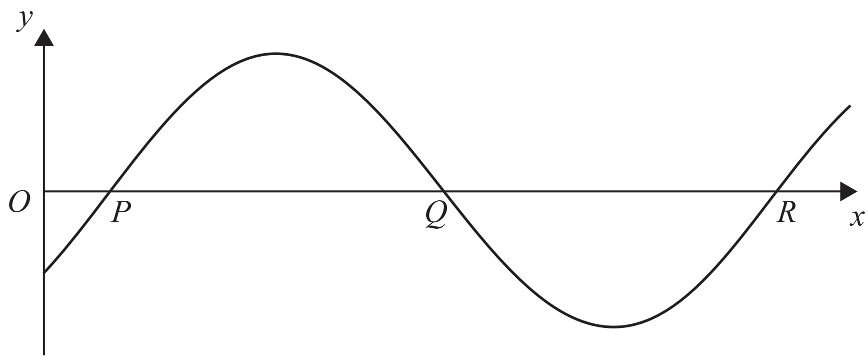


Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, \; 0 < b < \pi$$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b.

(4)

Question 9 continued

Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5$$

giving each term in its simplest form.

(4)

Q1

(Total 4 marks)

2. Find the values of x such that

$$2\log_3 x - \log_3(x-2) = 2$$

(5)

Question 2 continued

Q2

(Total 5 marks)

3.

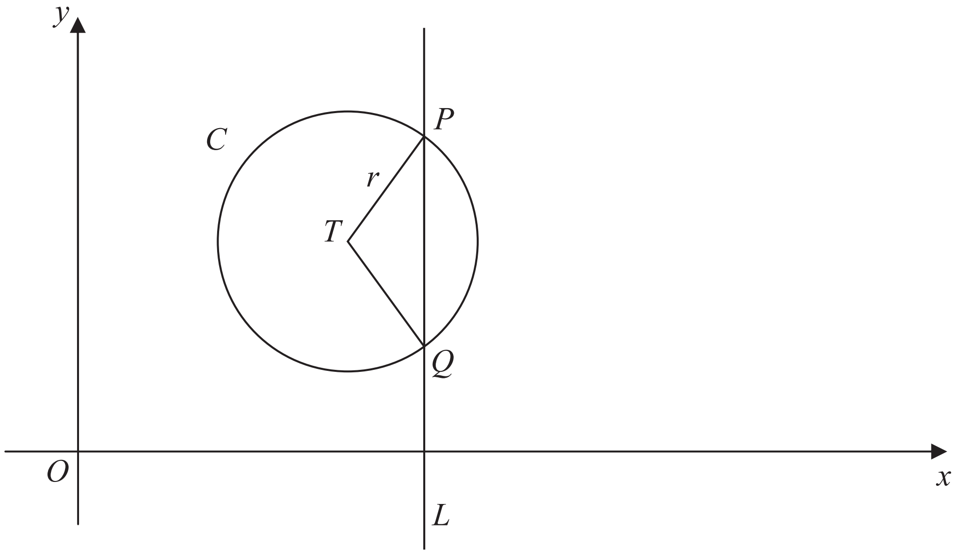


Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q.

(3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

(d) find the perimeter of the sector PTQ.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total 11 marks)

Q3

4. $$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

Question 4 continued

Q4

(Total 6 marks)

5.

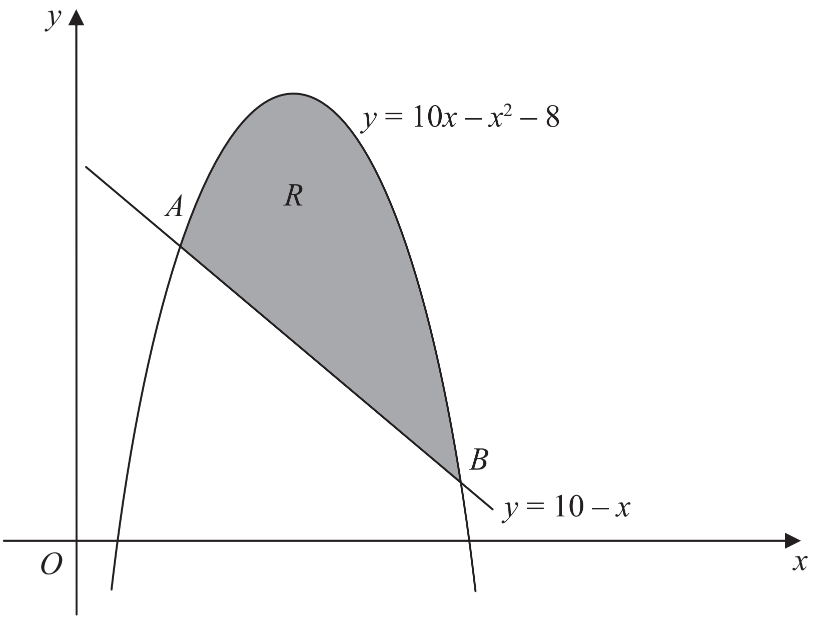


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

Question 5 continued

Question 5 continued

Question 5 continued

(Total 12 marks)

Q5

6. (a) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leqslant x \leqslant 180°$,

$$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

Q6

(Total 7 marks)

7.

$$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of $\int_0^1 \sqrt{(3^x + x)}\,dx$

You must show clearly how you obtained your answer.

(4)

Question 7 continued

Q7

(Total 6 marks)

8.

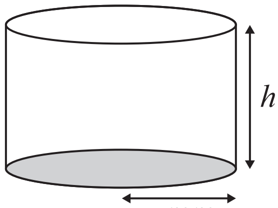


Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total 13 marks)

Q8

9. A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term,

(2)

(d) the sum to infinity.

(3)

Question 9 continued

Question 9 continued

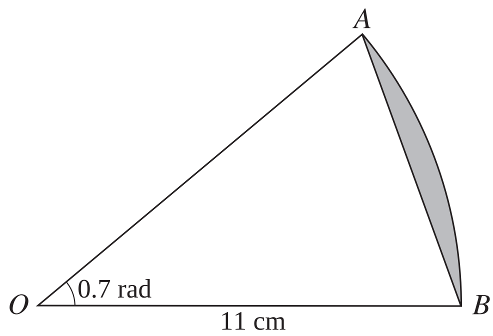
Q9

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

1



The diagram shows a sector AOB of a circle with centre O and radius 11 cm. The angle AOB is 0.7 radians. Find the area of the segment shaded in the diagram. [4]

2 Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

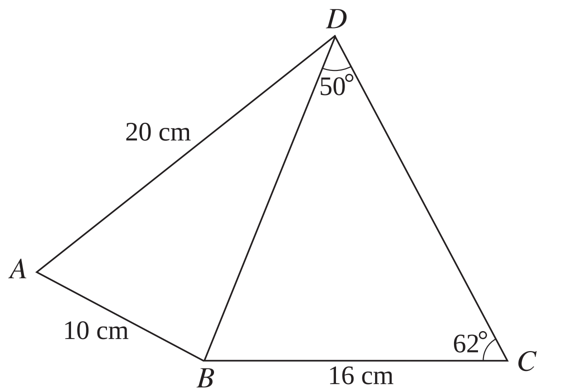
$$\int_1^7 \sqrt{x^2+3}\,dx. \qquad [4]$$

3 Express each of the following as a single logarithm:

(i) $\log_a 2 + \log_a 3$, [1]

(ii) $2\log_{10} x - 3\log_{10} y$. [3]

4



In the diagram, angle $BDC = 50°$ and angle $BCD = 62°$. It is given that $AB = 10$ cm, $AD = 20$ cm and $BC = 16$ cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

5 The gradient of a curve is given by $\frac{dy}{dx} = 12\sqrt{x}$. The curve passes through the point (4, 50). Find the equation of the curve. [6]

6 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

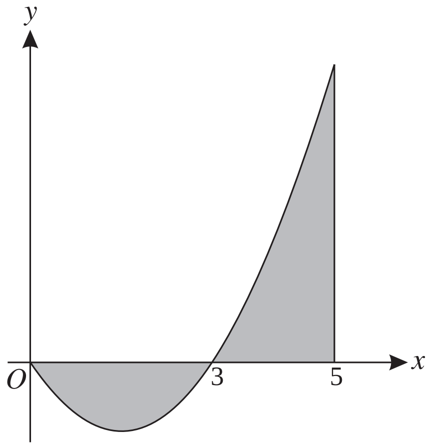
$$u_n = 2n + 5, \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_1, u_2 and u_3. [2]

(ii) State what type of sequence it is. [1]

(iii) Given that $\sum_{n=1}^{N} u_n = 2200$, find the value of N. [5]

7



The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 (x^2 - 3x)\,dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

8 The first term of a geometric progression is 10 and the common ratio is 0.8.

(i) Find the fourth term. [2]

(ii) Find the sum of the first 20 terms, giving your answer correct to 3 significant figures. [2]

(iii) The sum of the first N terms is denoted by S_N, and the sum to infinity is denoted by S_∞.

Show that the inequality $S_\infty - S_N < 0.01$ can be written as

$$0.8^N < 0.0002,$$

and use logarithms to find the smallest possible value of N. [7]

9 **(i)**

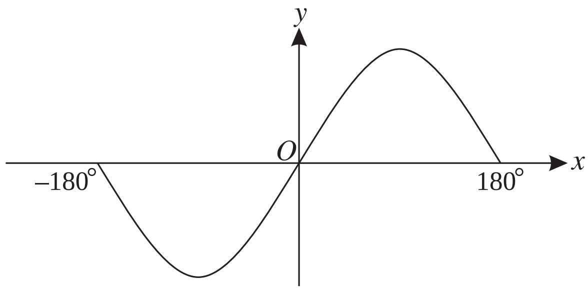


Fig. 1

Fig. 1 shows the curve $y = 2\sin x$ for values of x such that $-180° \leqslant x \leqslant 180°$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

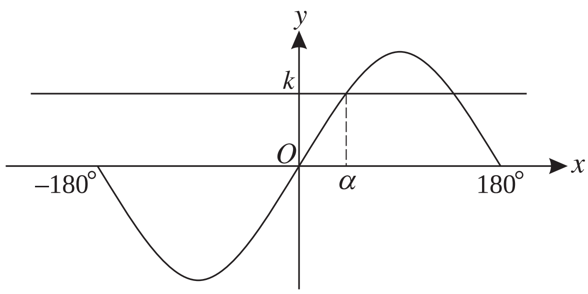


Fig. 2

Fig. 2 shows the curve $y = 2\sin x$ and the line $y = k$. The smallest positive solution of the equation $2\sin x = k$ is denoted by α. State, in terms of α, and in the range $-180° \leqslant x \leqslant 180°$,

(a) another solution of the equation $2\sin x = k$, [1]

(b) one solution of the equation $2\sin x = -k$. [1]

(iii) Find the x-coordinates of the points where the curve $y = 2\sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of x such that $-180° \leqslant x \leqslant 180°$. [6]

10 **(i)** Find the binomial expansion of $(2x + 5)^4$, simplifying the terms. [4]

(ii) Hence show that $(2x + 5)^4 - (2x - 5)^4$ can be written as

$$320x^3 + kx,$$

where the value of the constant k is to be stated. [2]

(iii) Verify that $x = 2$ is a root of the equation

$$(2x + 5)^4 - (2x - 5)^4 = 3680x - 800,$$

and find the other possible values of x. [6]

1 **(i)** Find and simplify the first three terms, in ascending powers of x, in the binomial expansion of $(1+2x)^7$. **[3]**

(ii) Hence find the coefficient of x^2 in the expansion of $(2-5x)(1+2x)^7$. **[3]**

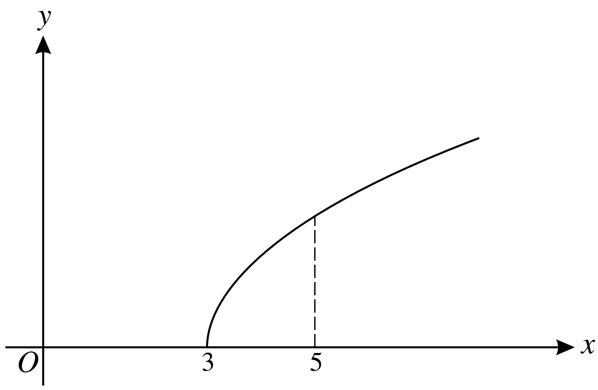
2 A sequence S has terms $u_1, u_2, u_3, \ldots$ defined by $u_n = 3n+2$ for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3. **[2]**

(ii) State what type of sequence S is. **[1]**

(iii) Find $\sum_{n=101}^{200} u_n$. **[3]**

3



The diagram shows the curve $y = \sqrt{x-3}$.

(i) Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for the area of the region bounded by the curve, the x-axis and the line $x = 5$. Give your answer correct to 3 significant figures. **[4]**

(ii) State, with a reason, whether this approximation is an underestimate or an overestimate. **[2]**

4 **(a)** Use logarithms to solve the equation $5^{x-1} = 120$, giving your answer correct to 3 significant figures. **[4]**

(b) Solve the equation $\log_2 x + 2\log_2 3 = \log_2(x+5)$. **[4]**

5 In a geometric progression, the sum to infinity is four times the first term.

(i) Show that the common ratio is $\frac{3}{4}$. **[3]**

(ii) Given that the third term is 9, find the first term. **[3]**

(iii) Find the sum of the first twenty terms. **[2]**

6 **(a)** Find $\int \frac{x^3 + 3x^{\frac{1}{2}}}{x}\,dx$. **[4]**

(b) **(i)** Find, in terms of a, the value of $\int_2^a 6x^{-4}\,dx$, where a is a constant greater than 2. **[3]**

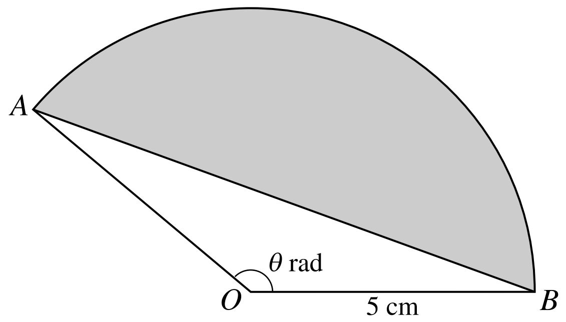
(ii) Deduce the value of $\int_2^{\infty} 6x^{-4}\,dx$. **[1]**

7 Solve each of the following equations for $0^\circ \leqslant x \leqslant 180^\circ$.

(i) $3\tan 2x = 1$ **[3]**

(ii) $3\cos^2 x + 2\sin x - 3 = 0$ **[5]**

8



The diagram shows a sector AOB of a circle with centre O and radius 5 cm. Angle AOB is θ radians. The area of triangle AOB is 8 cm^2.

(i) Given that the angle θ is obtuse, find θ. **[3]**

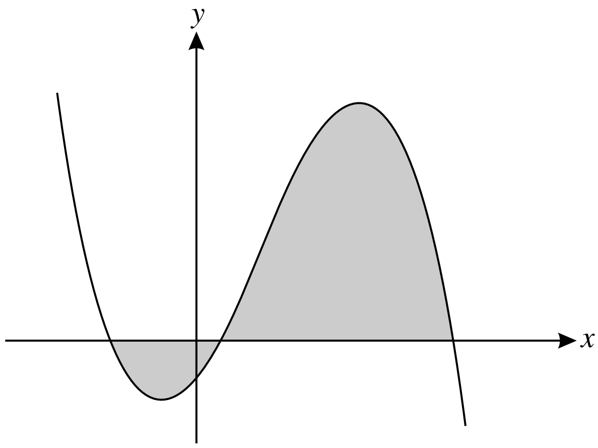
The shaded segment in the diagram is bounded by the chord AB and the arc AB.

(ii) Find the area of the segment, giving your answer correct to 3 significant figures. **[3]**

(iii) Find the perimeter of the segment, giving your answer correct to 3 significant figures. **[4]**

[Question 9 is printed overleaf.]

9



The diagram shows the curve $y = f(x)$, where $f(x) = -4x^3 + 9x^2 + 10x - 3$.

(i) Verify that the curve crosses the x-axis at $(3, 0)$ and hence state a factor of $f(x)$. **[2]**

(ii) Express $f(x)$ as the product of a linear factor and a quadratic factor. **[3]**

(iii) Hence find the other two points of intersection of the curve with the x-axis. **[2]**

(iv) The region enclosed by the curve and the x-axis is shaded in the diagram. Use integration to find the total area of this region. **[5]**

1 (i)	
1 (ii)	

2 (i)	
2 (ii)	
2 (iii)	

Turn over

3 (i)	
3 (ii)	

4 (a)	
4 (b)	

Turn over

5 (i)	
5 (ii)	
5 (iii)	

6 (a)	
6(b)(i)	
6(b)(ii)	

Turn over

7 (i)	
7 (ii)	

8 (i)	
8 (ii)	

Turn over

8 (iii)	

9 (i)	
9 (ii)	
9 (iii)	

Turn over

9 (iv)	

1 A geometric progression $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 15 \quad \text{and} \quad u_{n+1} = 0.8u_n \ \text{ for } n \geqslant 1.$$

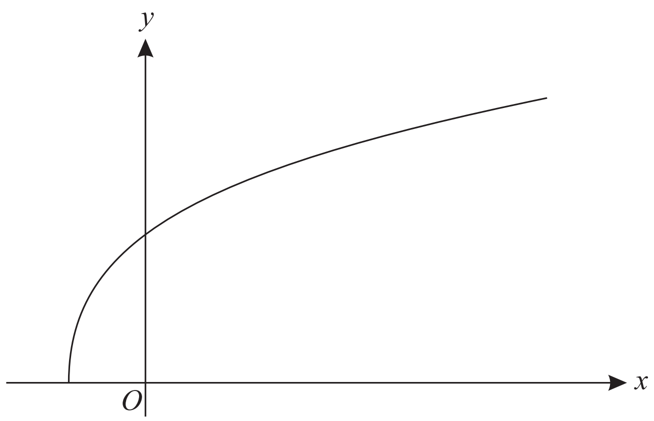
(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{20} u_n$. [3]

2 Expand $\left(x + \frac{2}{x}\right)^4$ completely, simplifying the terms. [5]

3 Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures. [5]

4



The diagram shows the curve $y = \sqrt{4x+1}$.

(i) Use the trapezium rule, with strips of width 0.5, to find an approximate value for the area of the region bounded by the curve $y = \sqrt{4x+1}$, the x-axis, and the lines $x = 1$ and $x = 3$. Give your answer correct to 3 significant figures. [4]

(ii) State with a reason whether this approximation is an under-estimate or an over-estimate. [2]

5 **(i)** Show that the equation

$$3\cos^2\theta = \sin\theta + 1$$

can be expressed in the form

$$3\sin^2\theta + \sin\theta - 2 = 0.$$ [2]

(ii) Hence solve the equation

$$3\cos^2\theta = \sin\theta + 1,$$

giving all values of θ between 0° and 360°. [5]

6 **(a)** **(i)** Find $\int x(x^2 - 4)\,dx$. [3]

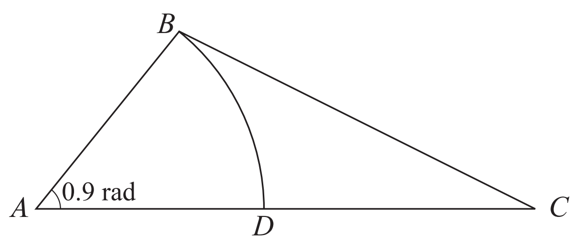
(ii) Hence evaluate $\int_1^6 x(x^2 - 4)\,dx$. [2]

(b) Find $\int \frac{6}{x^3}\,dx$. [3]

7 **(a)** In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

(b) In a geometric progression, the second term is −4 and the sum to infinity is 9. Find the common ratio. [7]

8



The diagram shows a triangle ABC, where angle BAC is 0.9 radians. BAD is a sector of the circle with centre A and radius AB.

(i) The area of the sector BAD is $16.2\,\text{cm}^2$. Show that the length of AB is 6 cm. [2]

(ii) The area of triangle ABC is twice the area of sector BAD. Find the length of AC. [3]

(iii) Find the perimeter of the region BCD. [6]

9 The polynomial $f(x)$ is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) **(a)** Show that $(x + 1)$ is a factor of $f(x)$. [1]

(b) Hence find the exact roots of the equation $f(x) = 0$. [6]

(ii) **(a)** Show that the equation

$$2\log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

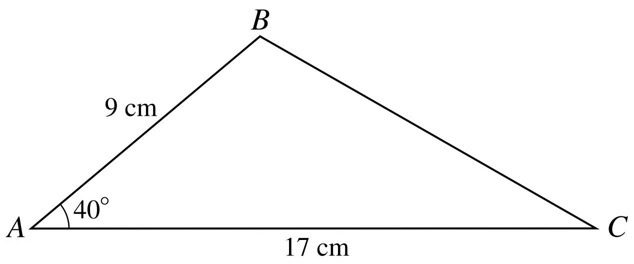
can be written in the form $f(x) = 0$. [5]

(b) Explain why the equation

$$2\log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

has only one real root and state the exact value of this root. [2]

1



The diagram shows triangle ABC, with $AB = 9$ cm, $AC = 17$ cm and angle $BAC = 40°$.

(i) Find the length of BC. **[2]**

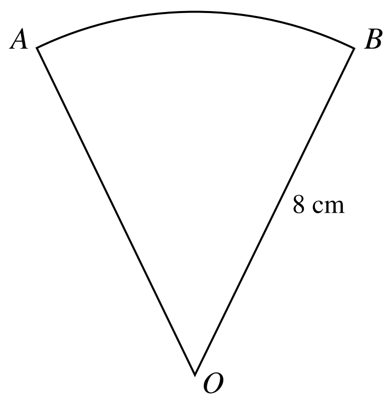
(ii) Find the area of triangle ABC. **[2]**

(iii) D is the point on AC such that angle $BDA = 63°$. Find the length of BD. **[3]**

2 **(i)** Find $\int \left(6x^{\frac{1}{2}} - 1\right) \mathrm{d}x$. **[3]**

(ii) Hence find the equation of the curve for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{\frac{1}{2}} - 1$ and which passes through the point (4, 17). **[3]**

3

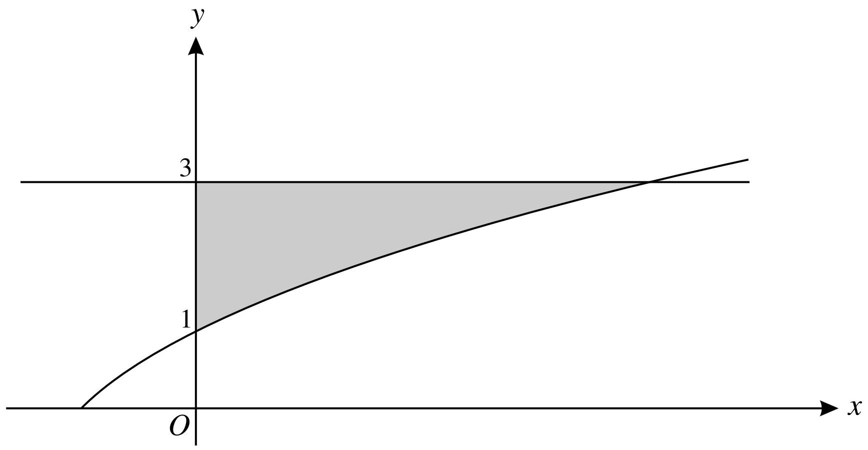


The diagram shows a sector AOB of a circle, centre O and radius 8 cm. The perimeter of the sector is 23.2 cm.

(i) Find angle AOB in radians. **[3]**

(ii) Find the area of the sector. **[2]**

4



The diagram shows the curve $y = -1 + \sqrt{x+4}$ and the line $y = 3$.

(i) Show that $y = -1 + \sqrt{x+4}$ can be rearranged as $x = y^2 + 2y - 3$. **[2]**

(ii) Hence find by integration the exact area of the shaded region enclosed between the curve, the y-axis and the line $y = 3$. **[5]**

5 The first four terms in the binomial expansion of $(3 + kx)^5$, in ascending powers of x, can be written as $a + bx + cx^2 + dx^3$.

(i) State the value of a. **[1]**

(ii) Given that $b = c$, find the value of k. **[5]**

(iii) Hence find the value of d. **[2]**

6 The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 + x^2 - 11x + 10$.

(i) Use the factor theorem to find a factor of $\mathrm{f}(x)$. **[2]**

(ii) Hence solve the equation $\mathrm{f}(x) = 0$, giving each root in an exact form. **[6]**

7 **(a)** The first term of a geometric progression is 7 and the common ratio is −2.

(i) Find the ninth term. **[2]**

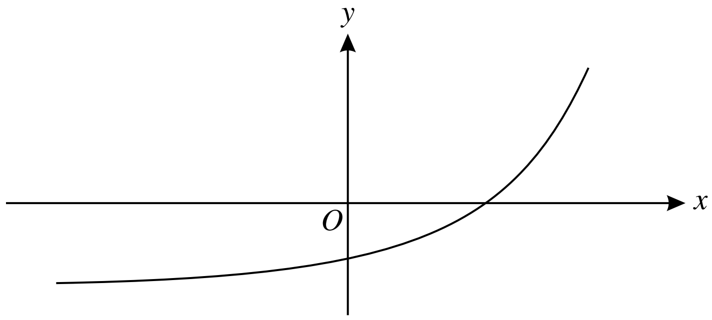
(ii) Find the sum of the first 15 terms. **[2]**

(b) The first term of an arithmetic progression is 7 and the common difference is −2. The sum of the first N terms is −2900. Find the value of N. **[5]**

[Questions 8 and 9 are printed overleaf.]

Turn over

8



The diagram shows the curve $y = 2^x - 3$.

(i) Describe the geometrical transformation that transforms the curve $y = 2^x$ to the curve $y = 2^x - 3$. **[2]**

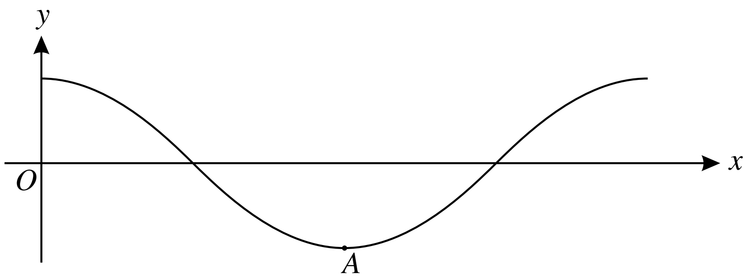
(ii) State the y-coordinate of the point where the curve $y = 2^x - 3$ crosses the y-axis. **[1]**

(iii) Find the x-coordinate of the point where the curve $y = 2^x - 3$ crosses the x-axis, giving your answer in the form $\log_a b$. **[2]**

(iv) The curve $y = 2^x - 3$ passes through the point $(p, 62)$. Use logarithms to find the value of p, correct to 3 significant figures. **[3]**

(v) Use the trapezium rule, with 2 strips each of width 0.5, to find an estimate for $\int_3^4 (2^x - 3)\,dx$. Give your answer correct to 3 significant figures. **[3]**

9 **(a)**



The diagram shows part of the curve $y = \cos 2x$, where x is in radians. The point A is the minimum point of this part of the curve.

(i) State the period of $y = \cos 2x$. **[1]**

(ii) State the coordinates of A. **[2]**

(iii) Solve the inequality $\cos 2x \leqslant 0.5$ for $0 \leqslant x \leqslant \pi$, giving your answers exactly. **[4]**

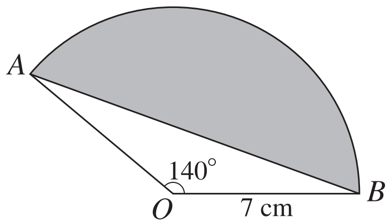
(b) Solve the equation $\cos 2x = \sqrt{3} \sin 2x$ for $0 \leqslant x \leqslant \pi$, giving your answers exactly. **[4]**

1 Find

(i) $\int (x^3 + 8x - 5)\,dx$, **[3]**

(ii) $\int 12\sqrt{x}\,dx$. **[3]**

2



The diagram shows a sector OAB of a circle, centre O and radius 7 cm. The angle AOB is 140°.

(i) Express 140° in radians, giving your answer in an exact form as simply as possible. **[2]**

(ii) Find the perimeter of the segment shaded in the diagram, giving your answer correct to 3 significant figures. **[4]**

3 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

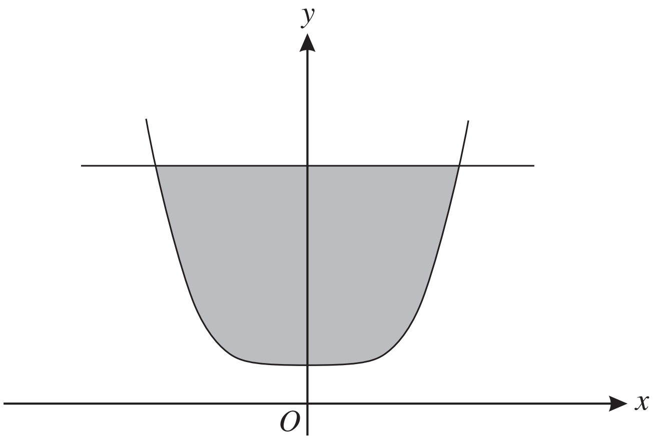
$$u_n = 24 - \tfrac{2}{3}n.$$

(i) Write down the exact values of u_1, u_2 and u_3. **[2]**

(ii) Find the value of k such that $u_k = 0$. **[2]**

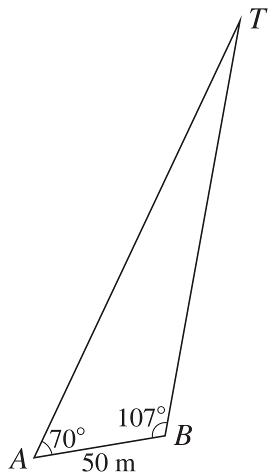
(iii) Find $\sum_{n=1}^{20} u_n$. **[3]**

4



The diagram shows the curve $y = x^4 + 3$ and the line $y = 19$ which intersect at $(-2, 19)$ and $(2, 19)$. Use integration to find the exact area of the shaded region enclosed by the curve and the line. **[7]**

5



Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from a point A and then walk for 50 m in a straight line before taking another bearing from a point B. They find that angle TAB is 70° and angle TBA is 107° (see diagram).

(i) Find the distance of the tower from A. **[2]**

(ii) They continue walking in the same direction for another 100 m to a point C, so that AC is 150 m. What is the distance of the tower from C? **[3]**

(iii) Find the shortest distance of the walkers from the tower as they walk from A to C. **[2]**

6 A geometric progression has first term 20 and common ratio 0.9.

(i) Find the sum to infinity. **[2]**

(ii) Find the sum of the first 30 terms. **[2]**

(iii) Use logarithms to find the smallest value of p such that the pth term is less than 0.4. **[4]**

Turn over

7 In the binomial expansion of $(k + ax)^4$ the coefficient of x^2 is 24.

(i) Given that a and k are both positive, show that $ak = 2$. **[3]**

(ii) Given also that the coefficient of x in the expansion is 128, find the values of a and k. **[4]**

(iii) Hence find the coefficient of x^3 in the expansion. **[2]**

8 **(a)** Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of p and q.

(i) $\log_a(xy)$ **[1]**

(ii) $\log_a\left(\frac{a^2x^3}{y}\right)$ **[3]**

(b) **(i)** Express $\log_{10}(x^2 - 10) - \log_{10} x$ as a single logarithm. **[1]**

(ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10} x = 2\log_{10} 3$. **[5]**

9 **(i)** The polynomial $f(x)$ is defined by

$$f(x) = x^3 - x^2 - 3x + 3.$$

Show that $x = 1$ is a root of the equation $f(x) = 0$, and hence find the other two roots. **[6]**

(ii) Hence solve the equation

$$\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$$

for $0 \leqslant x \leqslant 2\pi$. Give each solution for x in an exact form. **[6]**

1 The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + ax^2 - ax - 14$, where a is a constant.

(i) Given that $(x - 2)$ is a factor of $f(x)$, find the value of a. **[3]**

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

2 **(i)** Use the trapezium rule, with 3 strips each of width 3, to estimate the area of the region bounded by the curve $y = \sqrt[3]{7 + x}$, the x-axis, and the lines $x = 1$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate of the area. **[1]**

3 **(i)** Find and simplify the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{10}$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of x^3 in the expansion of $(3 + 4x + 2x^2)(1 + \frac{1}{2}x)^{10}$. **[3]**

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5n + 1$.

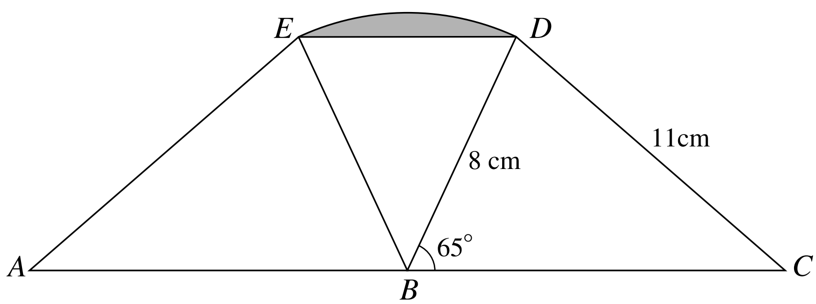
(i) State the values of u_1, u_2 and u_3. **[1]**

(ii) Evaluate $\sum_{n=1}^{40} u_n$. **[3]**

Another sequence $w_1, w_2, w_3, \ldots$ is defined by $w_1 = 2$ and $w_{n+1} = 5w_n + 1$.

(iii) Find the value of p such that $u_p = w_3$. **[3]**

5



The diagram shows two congruent triangles, BCD and BAE, where ABC is a straight line. In triangle BCD, $BD = 8$ cm, $CD = 11$ cm and angle $CBD = 65°$. The points E and D are joined by an arc of a circle with centre B and radius 8 cm.

(i) Find angle BCD. **[2]**

(ii) **(a)** Show that angle EBD is 0.873 radians, correct to 3 significant figures. **[2]**

(b) Hence find the area of the shaded segment bounded by the chord ED and the arc ED, giving your answer correct to 3 significant figures. **[4]**

6 **(a)** Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. **[4]**

(b) Find $\int \left(2 - 6\sqrt{y}\right) \mathrm{d}y$. **[3]**

(c) Evaluate $\int_1^{\infty} \frac{8}{x^3} \mathrm{d}x$. **[4]**

7 **(i)** Show that $\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. **[2]**

(ii) Hence solve the equation

$$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x,$$

for $0° \leqslant x \leqslant 360°$. **[6]**

8 **(a)** Use logarithms to solve the equation $5^{3w-1} = 4^{250}$, giving the value of w correct to 3 significant figures. **[5]**

(b) Given that $\log_x(5y + 1) - \log_x 3 = 4$, express y in terms of x. **[4]**

9 A geometric progression has first term a and common ratio r, and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

(i) Show that $r^3 - 2r + 1 = 0$. **[3]**

(ii) Given that the geometric progression converges, find the exact value of r. **[5]**

(iii) Given also that the sum to infinity of this geometric progression is $3 + \sqrt{5}$, find the value of the integer a. **[4]**

1 Find and simplify the first three terms in the expansion of $(2 - 3x)^6$ in ascending powers of x. [4]

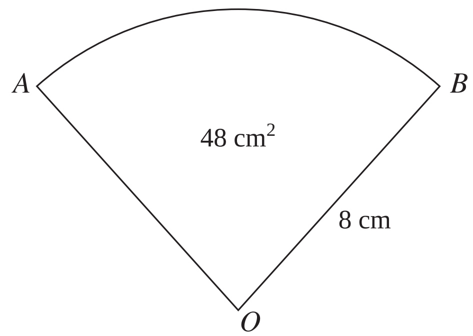
2 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = 1 - \frac{1}{u_n} \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [3]

(ii) Describe the behaviour of the sequence. [1]

3



The diagram shows a sector AOB of a circle with centre O and radius 8 cm. The area of the sector is 48 cm^2.

(i) Find angle AOB, giving your answer in radians. [2]

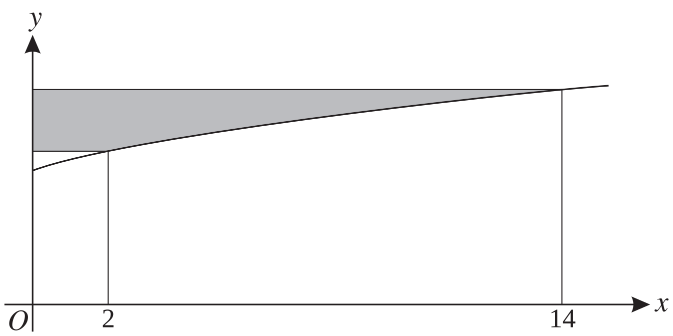
(ii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

4 The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $\mathrm{f}(x)$.

(i) Given that $(x - 3)$ is a factor of $\mathrm{f}(x)$, find the value of the constant a. [3]

(ii) Using this value of a, find the remainder when $\mathrm{f}(x)$ is divided by $(x + 2)$. [2]

5



The diagram shows the curve $y = 3 + \sqrt{x + 2}$.

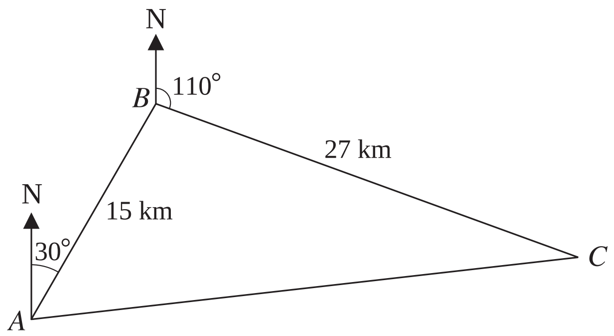
The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

(i) Show that the area of the shaded region is given by

$$\int_5^7 (y^2 - 6y + 7)\,dy.$$ [3]

(ii) Hence find the exact area of the shaded region. [4]

6



In the diagram, a lifeboat station is at point A. A distress call is received and the lifeboat travels 15 km on a bearing of 030° to point B. A second call is received and the lifeboat then travels 27 km on a bearing of 110° to arrive at point C. The lifeboat then travels back to the station at A.

(i) Show that angle ABC is 100°. [1]

(ii) Find the distance that the lifeboat has to travel to get from C back to A. [2]

(iii) Find the bearing on which the lifeboat has to travel to get from C to A. [4]

7 **(a)** Find $\int x^3(x^2 - x + 5)\,dx$. [4]

(b) **(i)** Find $\int 18x^{-4}\,dx$. [2]

(ii) Hence evaluate $\int_2^\infty 18x^{-4}\,dx$. [2]

[Turn over

8 **(i)** Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]

(ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point P. Show that the x-coordinate of P may be written as

$$\frac{1}{3 - \log_2 3}.$$ [5]

9 **(a)** **(i)** Show that the equation

$$2\sin x \tan x - 5 = \cos x$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ [3]

(ii) Hence solve the equation

$$2\sin x \tan x - 5 = \cos x,$$

giving all values of x, in radians, for $0 \leqslant x \leqslant 2\pi$. [4]

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \cos x \, dx,$$

where x is in radians. Give your answer correct to 3 significant figures. [4]

10 Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

(i) Find how far Jamie runs on Day 15. [2]

(ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

(iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

(iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]

1 **(i)** Show that the equation

$$2\sin^2 x = 5\cos x - 1$$

can be expressed in the form

$$2\cos^2 x + 5\cos x - 3 = 0. \quad \textbf{[2]}$$

(ii) Hence solve the equation

$$2\sin^2 x = 5\cos x - 1,$$

giving all values of x between 0° and 360°. **[4]**

2 The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points (2, 5) and (p, 5).

(i) Find the equation of the curve. **[4]**

(ii) Find the value of p. **[3]**

3 **(i)** Find and simplify the first four terms in the expansion of $(2 - x)^7$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of w^6 in the expansion of $\left(2 - \frac{1}{4}w^2\right)^7$. **[2]**

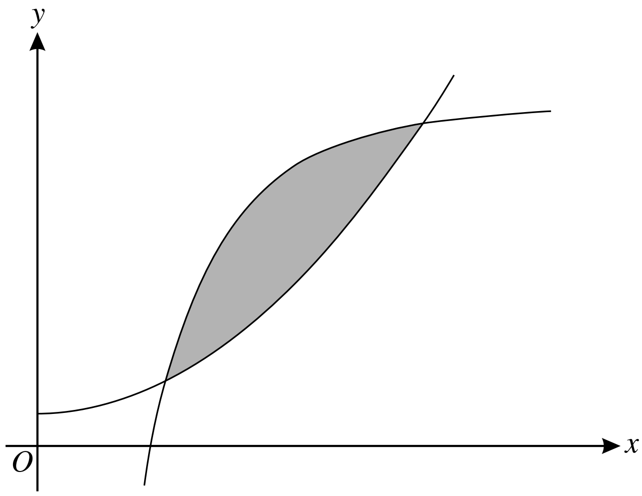
4 **(i)** Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for

$$\int_3^5 \log_{10}(2 + x)\,dx,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Use your answer to part **(i)** to deduce an approximate value for $\int_3^5 \log_{10}\sqrt{2 + x}\,dx$, showing your method clearly. **[2]**

5



The diagram shows parts of the curves $y = x^2 + 1$ and $y = 11 - \frac{9}{x^2}$, which intersect at (1, 2) and (3, 10). Use integration to find the exact area of the shaded region enclosed between the two curves. **[7]**

6 The cubic polynomial f(x) is given by

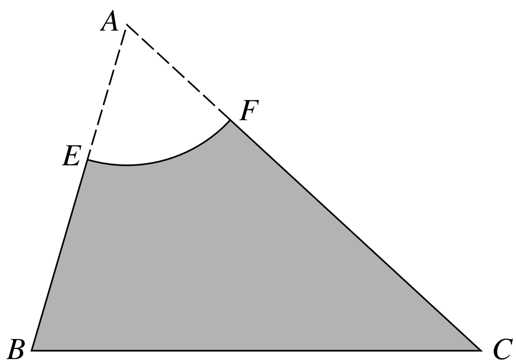
$$f(x) = 2x^3 + ax^2 + bx + 15,$$

where a and b are constants. It is given that $(x + 3)$ is a factor of f(x) and that, when f(x) is divided by $(x - 2)$, the remainder is 35.

(i) Find the values of a and b. **[6]**

(ii) Using these values of a and b, divide f(x) by $(x + 3)$. **[3]**

7



The diagram shows triangle ABC, with $AB = 10$ cm, $BC = 13$ cm and $CA = 14$ cm. E and F are points on AB and AC respectively such that $AE = AF = 4$ cm. The sector AEF of a circle with centre A is removed to leave the shaded region $EBCF$.

(i) Show that angle CAB is 1.10 radians, correct to 3 significant figures. **[2]**

(ii) Find the perimeter of the shaded region $EBCF$. **[3]**

(iii) Find the area of the shaded region $EBCF$. **[5]**

Turn over

8 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 8 \qquad \text{and} \qquad u_{n+1} = u_n + 3.$$

(i) Show that $u_5 = 20$. **[2]**

(ii) The nth term of the sequence can be written in the form $u_n = pn + q$. State the values of p and q. **[2]**

(iii) State what type of sequence it is. **[1]**

(iv) Find the value of N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$. **[5]**

9 **(i)** Sketch the curve $y = 6 \times 5^x$, stating the coordinates of any points of intersection with the axes. **[3]**

(ii) The point P on the curve $y = 9^x$ has y-coordinate equal to 150. Use logarithms to find the x-coordinate of P, correct to 3 significant figures. **[3]**

(iii) The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q. Show that the x-coordinate of Q can be written as $x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$. **[5]**

1 The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. **[3]**

(ii) Find the area of the triangle. **[2]**

2 The tenth term of an arithmetic progression is equal to twice the fourth term. The twentieth term of the progression is 44.

(i) Find the first term and the common difference. **[4]**

(ii) Find the sum of the first 50 terms. **[2]**

3 Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of x correct to 3 significant figures. **[5]**

4 **(i)** Find the binomial expansion of $(x^2 - 5)^3$, simplifying the terms. **[4]**

(ii) Hence find $\int (x^2 - 5)^3 \, dx$. **[4]**

5 Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

6 The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve. **[8]**

7 The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. **[2]**

(ii) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. **[2]**

(iii) Express $f(x)$ as a product of a linear factor and a quadratic factor. **[3]**

(iv) State the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

8

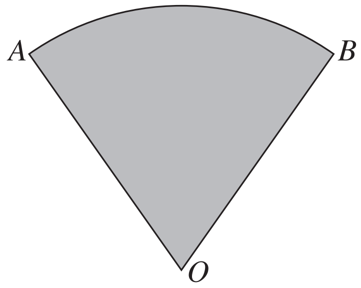


Fig. 1

Fig. 1 shows a sector AOB of a circle, centre O and radius OA. The angle AOB is 1.2 radians and the area of the sector is $60\,\text{cm}^2$.

(i) Find the perimeter of the sector. **[4]**

A pattern on a T-shirt, the start of which is shown in Fig. 2, consists of a sequence of similar sectors. The first sector in the pattern is sector AOB from Fig. 1, and the area of each successive sector is $\frac{3}{5}$ of the area of the previous one.

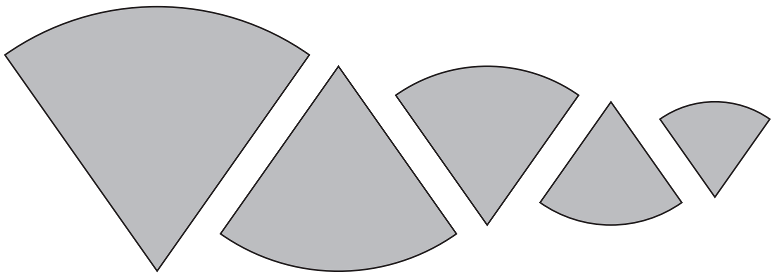


Fig. 2

(ii) **(a)** Find the area of the fifth sector in the pattern. **[2]**

(b) Find the total area of the first ten sectors in the pattern. **[2]**

(c) Explain why the total area will never exceed a certain limit, no matter how many sectors are used, and state the value of this limit. **[3]**

9 **(i)** Sketch the graph of $y = 4k^x$, where k is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. **[2]**

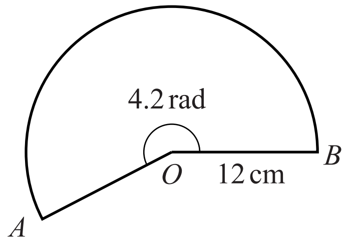
(ii) The point P on the curve $y = 4k^x$ has its y-coordinate equal to $20k^2$. Show that the x-coordinate of P may be written as $2 + \log_k 5$. **[4]**

(iii) **(a)** Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of

$$\int_0^1 4k^x \, \mathrm{d}x.$$ **[3]**

(b) Given that this approximate value is equal to 16, find the value of k. **[3]**

1

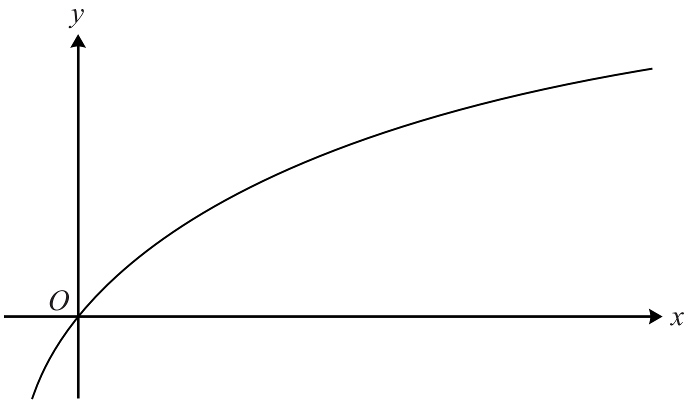


The diagram shows a sector AOB of a circle with centre O and radius 12 cm. The reflex angle AOB is 4.2 radians.

(i) Find the perimeter of the sector. **[3]**

(ii) Find the area of the sector. **[2]**

2



The diagram shows the curve $y = \log_{10}(2x + 1)$.

(i) Use the trapezium rule with 4 strips each of width 1.5 to find an approximation to the area of the region bounded by the curve, the x-axis and the lines $x = 4$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

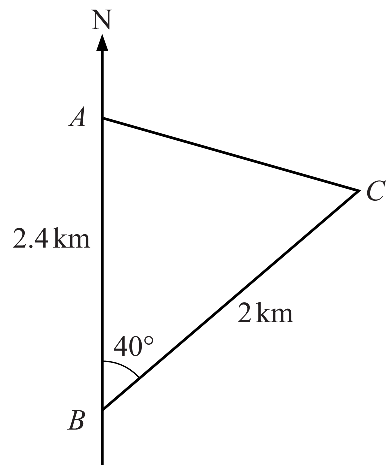
(ii) Explain why this approximation is an under-estimate. **[1]**

3 One of the terms in the binomial expansion of $(4 + ax)^6$ is $160x^3$.

(i) Find the value of a. **[4]**

(ii) Using this value of a, find the first two terms in the expansion of $(4 + ax)^6$ in ascending powers of x. **[2]**

4



The diagram shows two points A and B on a straight coastline, with A being 2.4 km due north of B. A stationary ship is at point C, on a bearing of 040° and at a distance of 2 km from B.

(i) Find the distance AC, giving your answer correct to 3 significant figures. **[2]**

(ii) Find the bearing of C from A. **[3]**

(iii) Find the shortest distance from the ship to the coastline. **[2]**

5 The cubic polynomial f(x) is defined by $f(x) = 2x^3 + 3x^2 - 17x + 6$.

(i) Find the remainder when f(x) is divided by $(x - 3)$. **[2]**

(ii) Given that $f(2) = 0$, express f(x) as the product of a linear factor and a quadratic factor. **[4]**

(iii) Determine the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

6 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 85 - 5n$ for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3. **[2]**

(ii) Find $\sum_{n=1}^{20} u_n$. **[3]**

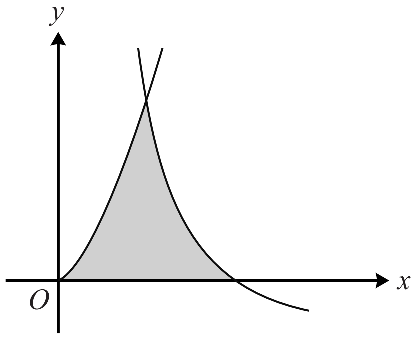
(iii) Given that u_1, u_5 and u_p are, respectively, the first, second and third terms of a geometric progression, find the value of p. **[4]**

(iv) Find the sum to infinity of the geometric progression in part **(iii)**. **[2]**

Turn over

7 **(a)** Find $\int (x^2 + 4)(x - 6)\,dx$. **[3]**

(b)



The diagram shows the curve $y = 6x^{\frac{3}{2}}$ and part of the curve $y = \frac{8}{x^2} - 2$, which intersect at the point (1, 6). Use integration to find the area of the shaded region enclosed by the two curves and the x-axis. **[8]**

8 **(a)** Use logarithms to solve the equation $7^{w-3} - 4 = 180$, giving your answer correct to 3 significant figures. **[4]**

(b) Solve the simultaneous equations

$$\log_{10}x + \log_{10}y = \log_{10}3, \qquad \log_{10}(3x + y) = 1.$$ **[6]**

9 **(i)** Sketch the graph of $y = \tan(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$ on the axes provided.

On the same axes, sketch the graph of $y = 3\cos(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$, indicating the point of intersection with the y-axis. **[3]**

(ii) Show that the equation $\tan(\frac{1}{2}x) = 3\cos(\frac{1}{2}x)$ can be expressed in the form

$$3\sin^2(\tfrac{1}{2}x) + \sin(\tfrac{1}{2}x) - 3 = 0.$$

Hence solve the equation $\tan(\frac{1}{2}x) = 3\cos(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$. **[6]**