Answer **all** questions.

1 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x+4y=7$.

(a) (i) Show that $k=-2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

3 (a) (i) Express x^2-4x+9 in the form $(x-p)^2+q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y=x^2-4x+9$. *(2 marks)*

(b) The line L has equation $y+2x=12$ and the curve C has equation $y=x^2-4x+9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2-2x-3=0$$

(1 mark)

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

4 The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO. *(2 marks)*

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

6 The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Hence express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

7 The volume, V m^3, of water in a tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2\,, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{dV}{dt}$; *(3 marks)*

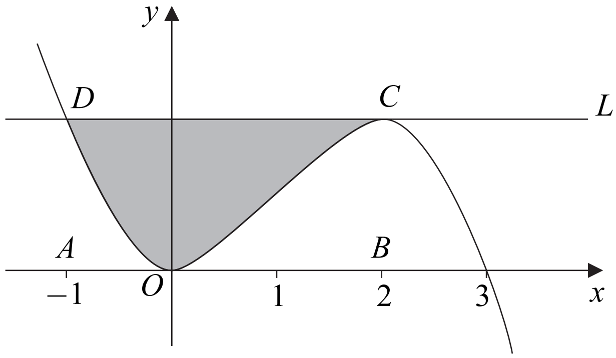
(ii) $\dfrac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in m^3 s^{-1}, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{dy}{dx}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

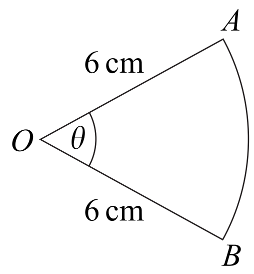
(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

END OF QUESTIONS

Answer **all** questions in the spaces provided.

1 The diagram shows a sector OAB of a circle with centre O and radius 6 cm.



The angle between the radii OA and OB is θ radians.

The area of the sector OAB is $21.6\,\text{cm}^2$.

(a) Find the value of θ. *(2 marks)*

(b) Find the length of the arc AB. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Turn over ▶

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{2^x}{x+1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Turn over ▶

3 (a) Write $\sqrt[4]{x^3}$ in the form x^k. *(1 mark)*

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$. *(2 marks)*

QUESTION PART REFERENCE

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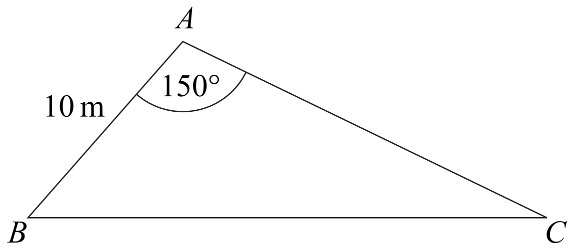
Turn over ▶

4 The triangle ABC, shown in the diagram, is such that AB is 10 metres and angle BAC is 150°.



The area of triangle ABC is $40\,\text{m}^2$.

(a) Show that the length of AC is 16 metres. *(2 marks)*

(b) Calculate the length of BC, giving your answer, in metres, to two decimal places. *(3 marks)*

(c) Calculate the smallest angle of triangle ABC, giving your answer to the nearest 0.1°. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

5 (a) (i) Describe the geometrical transformation that maps the graph of $y = \left(1 + \frac{x}{3}\right)^6$ onto the graph of $y = (1 + 2x)^6$. *(2 marks)*

(ii) The curve $y = \left(1 + \frac{x}{3}\right)^6$ is translated by the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give the curve $y = \mathrm{g}(x)$. Find an expression for $\mathrm{g}(x)$, simplifying your answer. *(2 marks)*

(b) The first four terms in the binomial expansion of $\left(1 + \frac{x}{3}\right)^6$ are $1 + ax + bx^2 + cx^3$.

Find the values of the constants a, b and c, giving your answers in their simplest form. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Turn over ▶

6 An arithmetic series has first term a and common difference d.

The sum of the first 25 terms of the series is 3500.

(a) Show that $a + 12d = 140$. *(3 marks)*

(b) The fifth term of this series is 100.

Find the value of d and the value of a. *(4 marks)*

(c) The nth term of this series is u_n. Given that

$$33 \quad \sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n) = 67 \sum_{n=1}^{k} u_n$$

find the value of $\sum_{n=1}^{k} u_n$. *(3 marks)*

QUESTION PART REFERENCE

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7 (a) Sketch the graph of $y = \frac{1}{2^x}$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $\frac{1}{2^x} = \frac{5}{4}$, giving your answer to three significant figures. *(3 marks)*

(c) Given that

$$\log_a(b^2) + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$$

express y in terms of a and b.

Give your answer in a form not involving logarithms. *(5 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

8 (a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$6\sin^2 x = 4 + \cos x$$

can be written as

$$6\cos^2 x + \cos x - 2 = 0$$

(2 marks)

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

QUESTION PART REFERENCE

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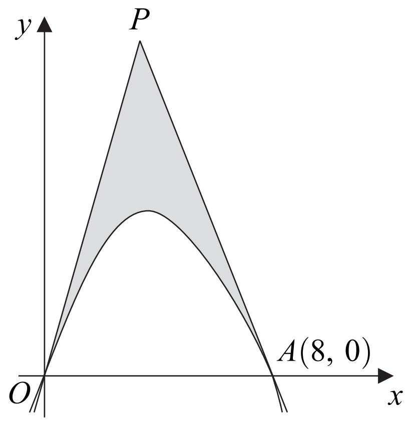
9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*

QUESTION PART REFERENCE	

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QUESTION PART REFERENCE

END OF QUESTIONS

1 **(i)** Evaluate 9^0. **[1]**

(ii) Express $9^{-\frac{1}{2}}$ as a fraction. **[2]**

2 **(i)** Sketch the curve $y = -\frac{1}{x^2}$. **[2]**

(ii) Sketch the curve $y = 3 - \frac{1}{x^2}$. **[2]**

(iii) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor 2. State the equation of the transformed curve. **[1]**

3 **(i)** Express $\frac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers. **[3]**

(ii) Express $\sqrt{18} - \sqrt{2}$ in simplified surd form. **[2]**

4 **(i)** Expand $(x-2)^2(x+1)$, simplifying your answer. **[3]**

(ii) Sketch the curve $y = (x-2)^2(x+1)$, indicating the coordinates of all intercepts with the axes. **[3]**

5 Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. **[5]**

6 Find the gradient of the curve $y = 2x + \frac{6}{\sqrt{x}}$ at the point where $x = 4$. **[5]**

7 Solve the simultaneous equations

$$x + 2y - 6 = 0, \qquad 2x^2 + y^2 = 57.$$ **[6]**

8 **(i)** Express $2x^2 + 5x$ in the form $2(x+p)^2 + q$. **[3]**

(ii) State the coordinates of the minimum point of the curve $y = 2x^2 + 5x$. **[2]**

(iii) State the equation of the normal to the curve at its minimum point. **[1]**

(iv) Solve the inequality $2x^2 + 5x > 0$. **[4]**

9 **(i)** The line joining the points A (4, 5) and B (p, q) has mid-point M (−1, 3). Find p and q. **[3]**

AB is the diameter of a circle.

(ii) Find the radius of the circle. **[2]**

(iii) Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

(iv) Find an equation of the tangent to the circle at the point (4, 5). **[5]**

10 **(i)** Find the coordinates of the stationary points of the curve $y = 2x^3 + 5x^2 - 4x$. **[6]**

(ii) State the set of values for x for which $2x^3 + 5x^2 - 4x$ is a decreasing function. **[2]**

(iii) Show that the equation of the tangent to the curve at the point where $x = \frac{1}{2}$ is $10x - 4y - 7 = 0$. **[4]**

(iv) Hence, with the aid of a sketch, show that the equation $2x^3 + 5x^2 - 4x = \frac{5}{2}x - \frac{7}{4}$ has two distinct real roots. **[2]**

1 (i)	
1 (ii)	
2 (i)	
2 (ii)	

2 (iii)	
3 (i)	
3 (ii)	

Turn over

4 (i)	
4 (ii)	

5	
6	

Turn over

7	

8 (i)	
8 (ii)	
8 (iii)	
8 (iv)	

Turn over

9 (i)	
9 (ii)	
9 (iii)	

9 (iv)	

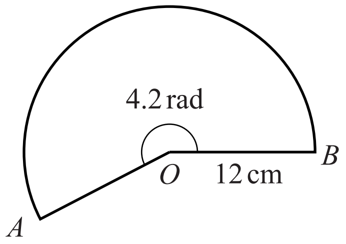
Turn over

10 (i)	
10 (ii)	

10 (iii)	

10 (iv)

1

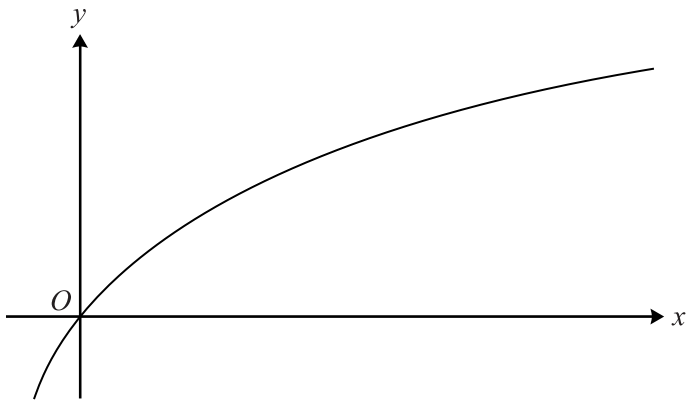


The diagram shows a sector AOB of a circle with centre O and radius 12 cm. The reflex angle AOB is 4.2 radians.

(i) Find the perimeter of the sector. **[3]**

(ii) Find the area of the sector. **[2]**

2



The diagram shows the curve $y = \log_{10}(2x + 1)$.

(i) Use the trapezium rule with 4 strips each of width 1.5 to find an approximation to the area of the region bounded by the curve, the x-axis and the lines $x = 4$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

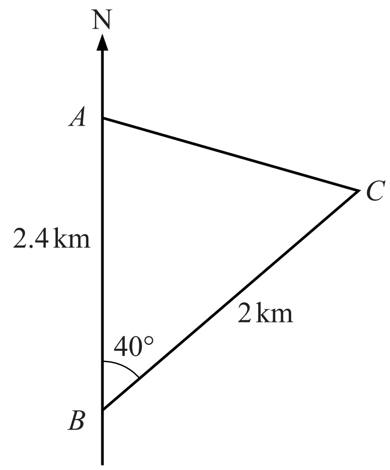
(ii) Explain why this approximation is an under-estimate. **[1]**

3 One of the terms in the binomial expansion of $(4 + ax)^6$ is $160x^3$.

(i) Find the value of a. **[4]**

(ii) Using this value of a, find the first two terms in the expansion of $(4 + ax)^6$ in ascending powers of x. **[2]**

4



The diagram shows two points A and B on a straight coastline, with A being 2.4 km due north of B. A stationary ship is at point C, on a bearing of 040° and at a distance of 2 km from B.

(i) Find the distance AC, giving your answer correct to 3 significant figures. **[2]**

(ii) Find the bearing of C from A. **[3]**

(iii) Find the shortest distance from the ship to the coastline. **[2]**

5 The cubic polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 17x + 6$.

(i) Find the remainder when $f(x)$ is divided by $(x - 3)$. **[2]**

(ii) Given that $f(2) = 0$, express $f(x)$ as the product of a linear factor and a quadratic factor. **[4]**

(iii) Determine the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

6 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 85 - 5n$ for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3. **[2]**

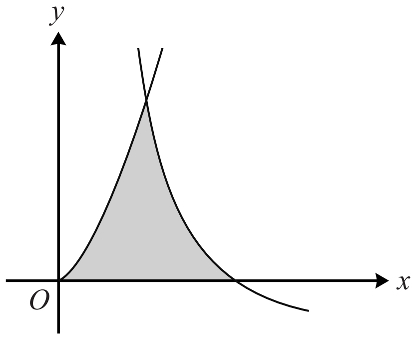
(ii) Find $\sum_{n=1}^{20} u_n$. **[3]**

(iii) Given that u_1, u_5 and u_p are, respectively, the first, second and third terms of a geometric progression, find the value of p. **[4]**

(iv) Find the sum to infinity of the geometric progression in part **(iii)**. **[2]**

7 **(a)** Find $\int (x^2 + 4)(x - 6)\,dx$. **[3]**

(b)



The diagram shows the curve $y = 6x^{\frac{3}{2}}$ and part of the curve $y = \frac{8}{x^2} - 2$, which intersect at the point (1, 6). Use integration to find the area of the shaded region enclosed by the two curves and the x-axis. **[8]**

8 **(a)** Use logarithms to solve the equation $7^{w-3} - 4 = 180$, giving your answer correct to 3 significant figures. **[4]**

(b) Solve the simultaneous equations

$$\log_{10}x + \log_{10}y = \log_{10}3, \qquad \log_{10}(3x + y) = 1.$$ **[6]**

9 **(i)** Sketch the graph of $y = \tan(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$ on the axes provided.

On the same axes, sketch the graph of $y = 3\cos(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$, indicating the point of intersection with the y-axis. **[3]**

(ii) Show that the equation $\tan(\frac{1}{2}x) = 3\cos(\frac{1}{2}x)$ can be expressed in the form

$$3\sin^2(\tfrac{1}{2}x) + \sin(\tfrac{1}{2}x) - 3 = 0.$$

Hence solve the equation $\tan(\frac{1}{2}x) = 3\cos(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$. **[6]**

Answer **all** questions.

1 The point A has coordinates $(1, 7)$ and the point B has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x + 2y = 17$. *(2 marks)*

(b) The line AB intersects the line with equation $x - 4y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

(c) Find an equation of the line through A which is perpendicular to AB. *(3 marks)*

2 (a) Express $x^2 + 8x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

3 A curve has equation $y = 7 - 2x^5$.

(a) Find $\frac{dy}{dx}$. *(2 marks)*

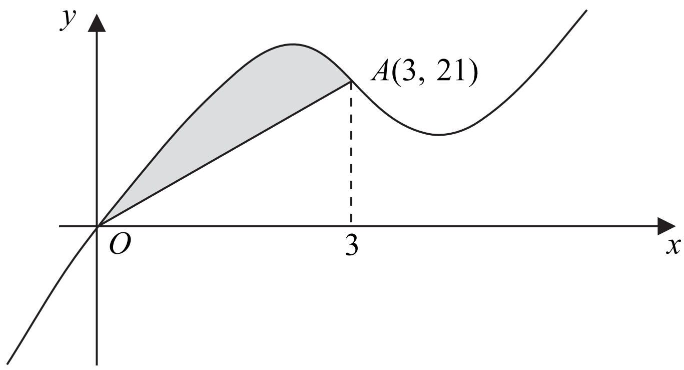
(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

4 (a) Express $\left(4\sqrt{5} - 1\right)\left(\sqrt{5} + 3\right)$ in the form $p + q\sqrt{5}$, where p and q are integers. *(3 marks)*

(b) Show that $\frac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*

5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\, dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\, dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when $p(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $p(x)$ in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

Turn over for the next question

7 A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

END OF QUESTIONS