Answer **all** questions.

1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\frac{dy}{dx} = 0$. *(5 marks)*

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2 + 1}\,dx$$

giving your answer to four significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

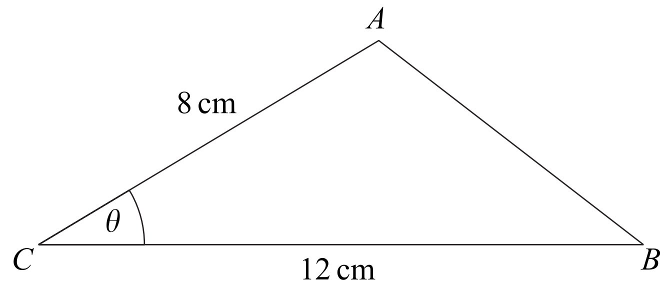
3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. *(3 marks)*

(b) An infinite geometric series has common ratio r. The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. *(3 marks)*

(ii) Given that the first term of the series is 20, find the least value of n such that the nth term of the series is less than 1. *(3 marks)*

4 The triangle ABC, shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.

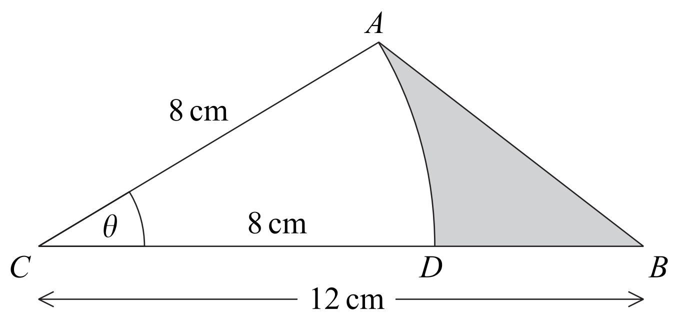


The area of triangle $ABC = 20\,\text{cm}^2$.

(a) Show that $\theta = 0.430$ correct to three significant figures. *(3 marks)*

(b) Use the cosine rule to calculate the length of AB, giving your answer to two significant figures. *(3 marks)*

(c) The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.



Calculate, to two significant figures:

(i) the length of the arc AD; *(2 marks)*

(ii) the area of the shaded region. *(3 marks)*

Turn over ▶

5 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \qquad u_2 = 150 \qquad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of q. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L. Write down an equation for L and hence find the value of L. *(3 marks)*

6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:

(i) $y = 2\sin x$; *(2 marks)*

(ii) $y = -\sin x$; *(2 marks)*

(iii) $y = \sin(x - 30°)$. *(2 marks)*

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest 0.1° in the interval $0° \leqslant \theta \leqslant 360°$. *(3 marks)*

(c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. *(4 marks)*

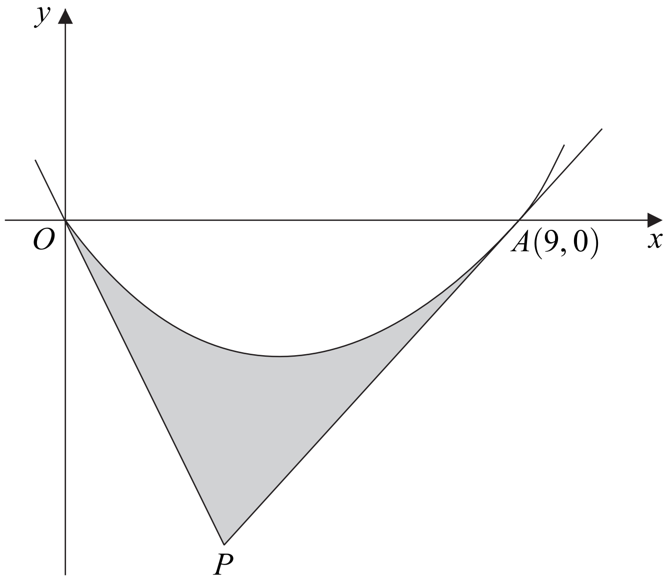
7 It is given that n satisfies the equation

$$2\log_a n - \log_a(5n - 24) = \log_a 4$$

(a) Show that $n^2 - 20n + 96 = 0$. *(3 marks)*

(b) Hence find the possible values of n. *(2 marks)*

8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\dfrac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

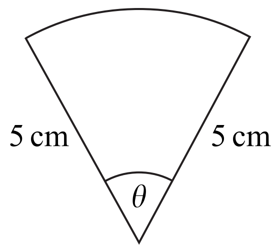
(c) Find $\displaystyle\int \left(x^{\frac{3}{2}} - 3x\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 The diagram shows a sector of a circle of radius 5 cm and angle θ radians.

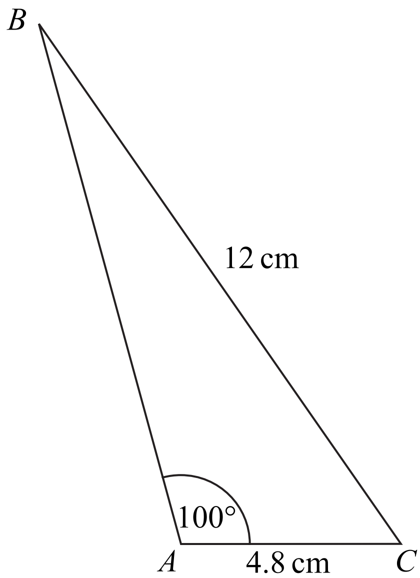


The area of the sector is $8.1\ \text{cm}^2$.

(a) Show that $\theta = 0.648$. *(2 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

2 The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

4 (a) The expression $(1 - 2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) Find the coefficient of x in the expansion of $(2 + x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$. *(3 marks)*

5 (a) Given that

$$\log_a x = 2\log_a 6 - \log_a 3$$

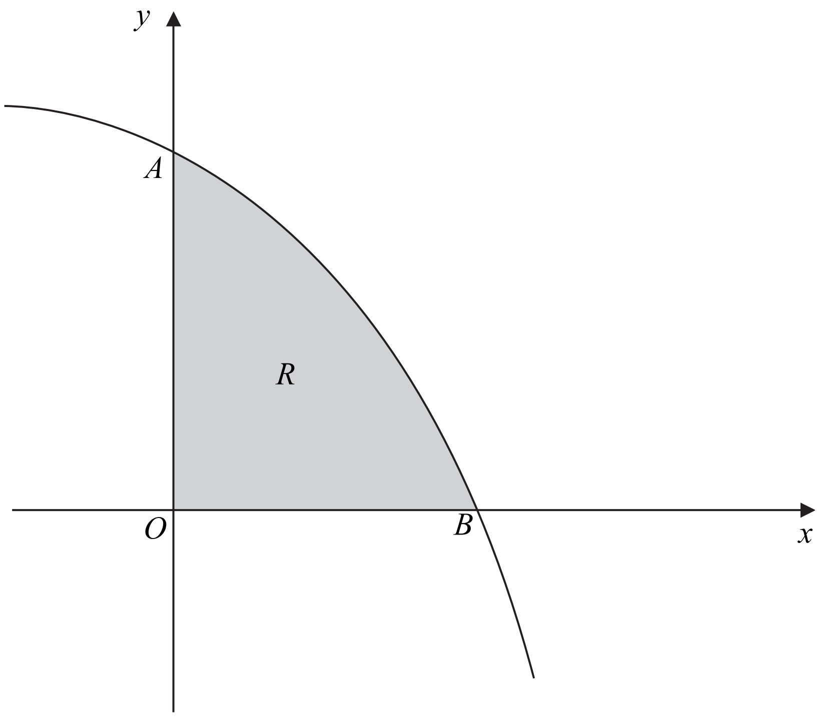
show that $x = 12$. *(3 marks)*

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express y in terms of a, giving your answer in a form not involving logarithms. *(3 marks)*

6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

(ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$. Find the value of k. *(1 mark)*

(d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. *(2 marks)*

(ii) Sketch the curve $y = -3^x$. *(2 marks)*

7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\frac{d^2y}{dx^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7)\,dx$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

8 (a) Describe the single geometrical transformation by which the curve with equation $y = \tan\frac{1}{2}x$ can be obtained from the curve $y = \tan x$. *(2 marks)*

(b) Solve the equation $\tan\frac{1}{2}x = 3$ in the interval $0 < x < 4\pi$, giving your answers in radians to three significant figures. *(4 marks)*

(c) Solve the equation

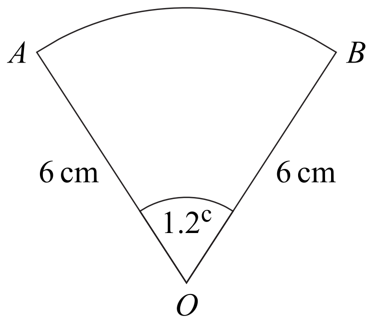
$$\cos\theta(\sin\theta - 3\cos\theta) = 0$$

in the interval $0 < \theta < 2\pi$, giving your answers in radians to three significant figures. *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) Find the perimeter of the sector OAB. *(3 marks)*

2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x}\, dx$$

giving your answer to three decimal places. *(4 marks)*

3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

(ii) $\frac{1}{64} = 8^q$;

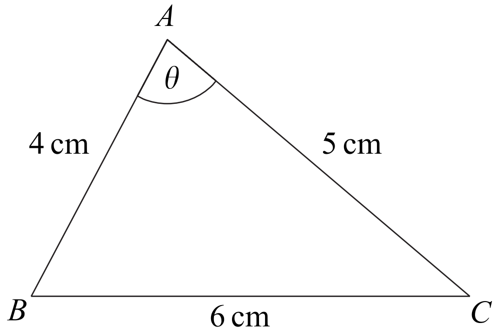
(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$

(2 marks)

4 The triangle ABC, shown in the diagram, is such that $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $AB = 4\,\text{cm}$. The angle BAC is θ.



(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(b) Hence use a trigonometrical identity to show that $\sin\theta = \frac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

5 The second term of a geometric series is 48 and the fourth term is 3.

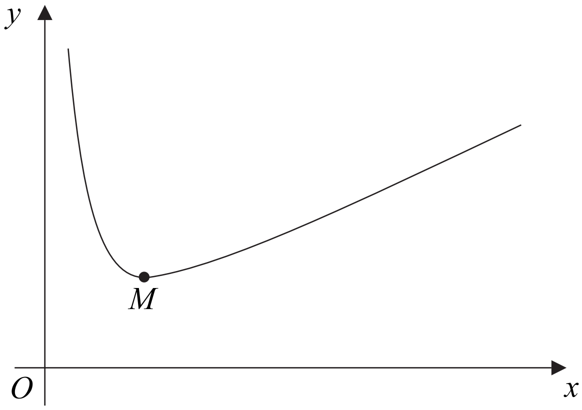
(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \dfrac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \dfrac{4}{x^2}$, find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\displaystyle\int \left(x + 1 + \frac{4}{x^2} \right) dx$. *(3 marks)*

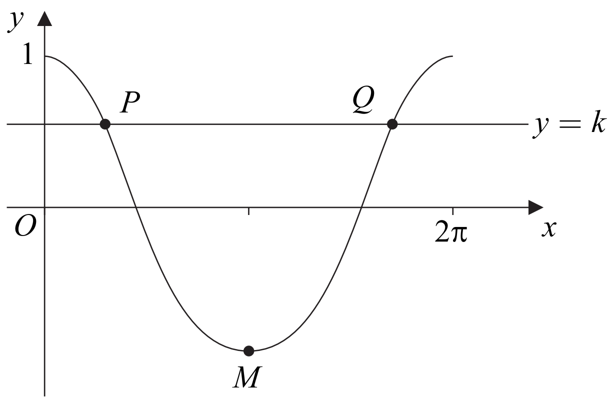
(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. *(3 marks)*

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos \dfrac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

Turn over for the next question

9 (a) Solve the equation $3\log_a x = \log_a 8$. *(2 marks)*

(b) Show that

$$3\log_a 6 - \log_a 8 = \log_a 27$$

(3 marks)

(c) (i) The point $P(3, p)$ lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that $p = \log_{10}\left(\frac{27}{8}\right)$. *(2 marks)*

(ii) The point $Q(6, q)$ also lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that the gradient of the line PQ is $\log_{10} 2$. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

(b) (i) Find $\int 3x^{\frac{1}{2}}\, dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\, dx$. *(2 marks)*

2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

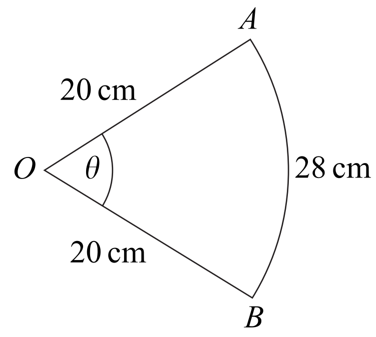
(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.

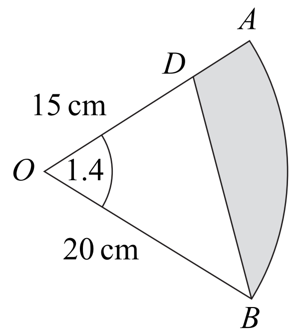


The length of the arc AB is 28 cm.

(a) Show that $\theta = 1.4$. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.



The length of OD is 15 cm.

(i) Find the area of the shaded region, giving your answer to three significant figures. *(3 marks)*

(ii) Use the cosine rule to calculate the length of BD, giving your answer to three significant figures. *(3 marks)*

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

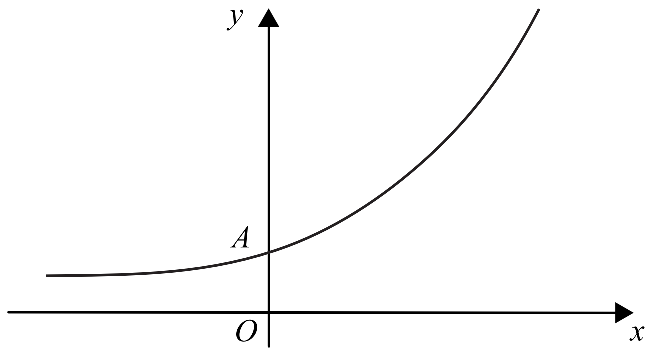
(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

(c) Find $\frac{dy}{dx}$. *(3 marks)*

(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,dx$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

Turn over for the next question

7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

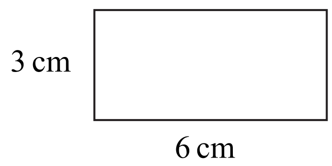
(i) express x in terms of a; *(1 mark)*

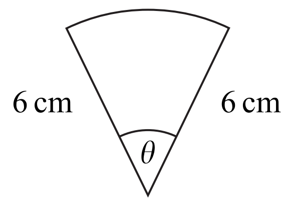
(ii) express xy in terms of a. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.





The area of the rectangle is twice the area of the sector.

(a) Show that $\theta = 0.5$. *(3 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

2 The arithmetic series

$$51 + 58 + 65 + 72 + \ldots + 1444$$

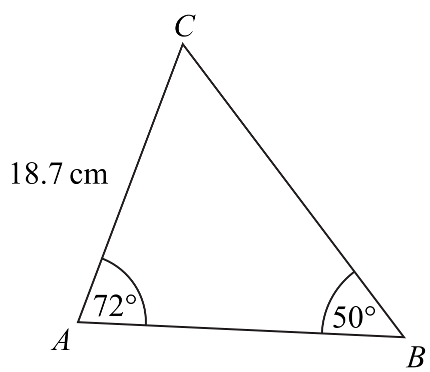
has 200 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 101st term of the series. *(2 marks)*

(c) Find the sum of **the last** 100 terms of the series. *(2 marks)*

3 The diagram shows a triangle *ABC*. The length of *AC* is 18.7 cm, and the sizes of angles *BAC* and *ABC* are 72° and 50° respectively.



(a) Show that the length of $BC = 23.2$ cm, correct to the nearest 0.1 cm. *(3 marks)*

(b) Calculate the area of triangle *ABC*, giving your answer to the nearest cm^2. *(3 marks)*

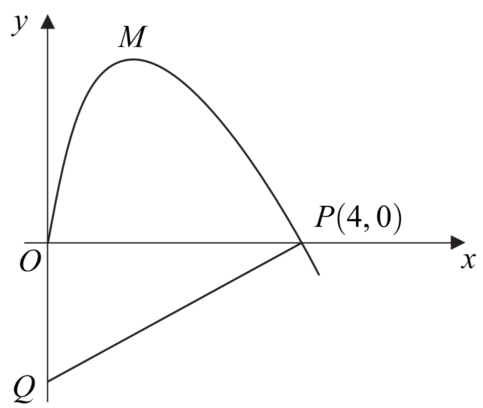
4 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{x^2+3}\, dx$$

giving your answer to three decimal places. *(4 marks)*

5 A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.

The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Show that the gradient of the curve at $P(4, 0)$ is -2. *(2 marks)*

(iii) Find an equation of the normal to the curve at $P(4, 0)$. *(3 marks)*

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. *(3 marks)*

(v) The curve has a maximum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$. *(3 marks)*

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO. *(3 marks)*

6 (a) Using the binomial expansion, or otherwise:

(i) express $(1+x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1+x)^4$ in ascending powers of x. *(2 marks)*

(b) Hence, or otherwise:

(i) express $(1+4x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1+3x)^4$ in ascending powers of x. *(2 marks)*

(c) Show that the expansion of

$$(1+3x)^4 - (1+4x)^3$$

can be written in the form

$$px^2 + qx^3 + rx^4$$

where p, q and r are integers. *(2 marks)*

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x. *(1 mark)*

(b) Given that

$$\log_a y = 2\log_a 3 + \log_a 4 + 1$$

express y in terms of a, giving your answer in a form **not** involving logarithms. *(3 marks)*

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y-axis. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; *(2 marks)*

(ii) onto the graph of $y = 3^{x+1}$. *(2 marks)*

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y - 1)(Y - 2) = 0$$ *(2 marks)*

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. *(4 marks)*

9 (a) Given that

$$\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$$

show that

$$\cos \theta = -\frac{1}{2}$$ *(4 marks)*

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3 \cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

END OF QUESTIONS