Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when $y = \tan 3x$. *(2 marks)*

(b) Given that $y = \frac{3x+1}{2x+1}$, show that $\frac{dy}{dx} = \frac{1}{(2x+1)^2}$. *(3 marks)*

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\, dx$$

giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\int \frac{2x^3+1}{x^4+2x}\, dx$. *(2 marks)*

(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1}\, dx = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$ *(3 marks)*

(ii) Hence show that $\int_0^4 x\sqrt{2x+1}\, dx = 19.9$ correct to three significant figures. *(4 marks)*

4 It is given that $2\operatorname{cosec}^2 x = 5 - 5\cot x$.

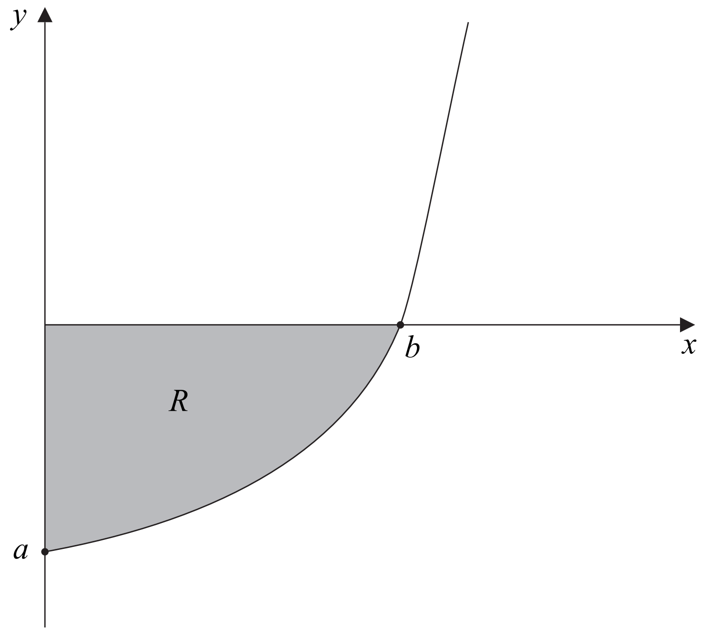
(a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

(d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$. *(2 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

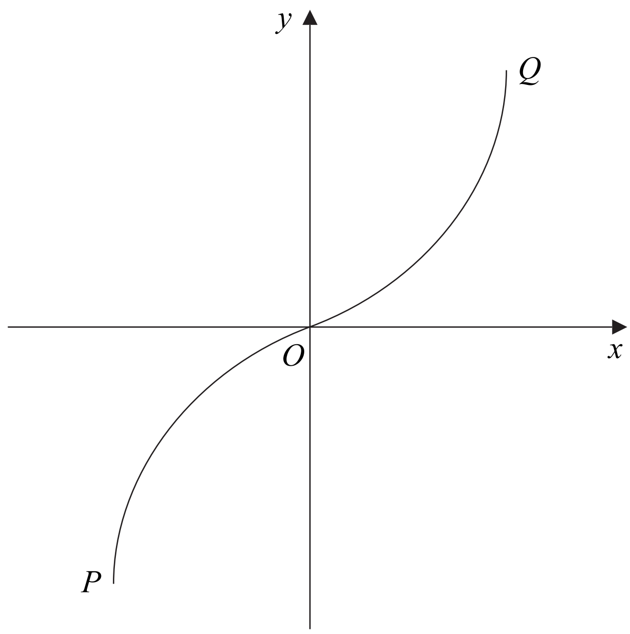
(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \dfrac{3 - x^3}{4}$. *(1 mark)*

(c) (i) Use the iteration $x_{n+1} = \dfrac{3 - x_n{}^3}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \dfrac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*

7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q, the end-points of the graph.

(2 marks)

(b) Sketch the graph of $y = -\sin^{-1}(x-1)$. *(3 marks)*

8 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 4$. *(4 marks)*

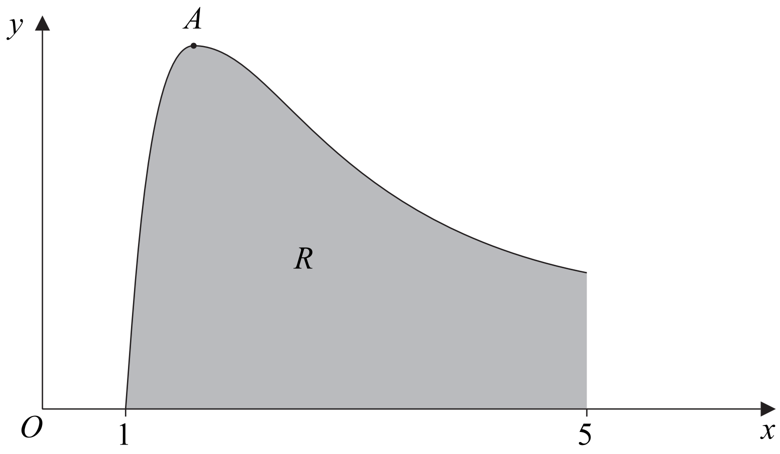
(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

9 (a) Given that $y = x^{-2} \ln x$, show that $\frac{dy}{dx} = \frac{1 - 2\ln x}{x^3}$. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



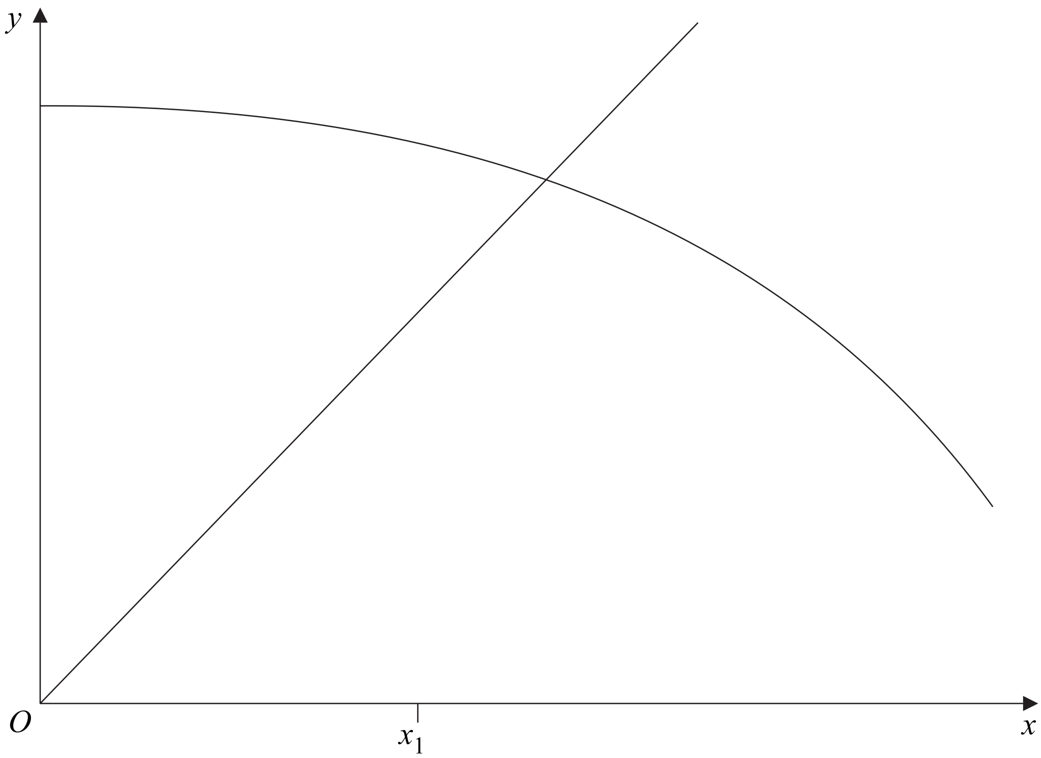
(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

END OF QUESTIONS

Figure 1 (for Question 6)



Answer **all** questions.

1 The curve $y = x^3 - x - 7$ intersects the x-axis at the point where $x = \alpha$.

(a) Show that α lies between 2.0 and 2.1. *(2 marks)*

(b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x+7}$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \sqrt[3]{x_n + 7}$ with $x_1 = 2$ to find the values of x_2, x_3 and x_4, giving your answers to three significant figures. *(3 marks)*

2 (a) Find $\dfrac{dy}{dx}$ when $y = (3x - 1)^{10}$. *(2 marks)*

(b) Use the substitution $u = 2x + 1$ to find $\int x(2x+1)^8 \, dx$, giving your answer in terms of x . *(4 marks)*

3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation $\tan^2 x = 3\sec x + 9$ can be written as

$$\sec^2 x - 3\sec x - 10 = 0$$ *(2 marks)*

(c) Solve the equation $\tan^2 x = 3\sec x + 9$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

4 (a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; *(1 mark)*

(ii) $y = |2x - 4|$. *(2 marks)*

(b) (i) Solve the equation $|x| = |2x - 4|$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. *(2 marks)*

5 (a) A curve has equation $y = e^{2x} - 10e^{x} + 12x$.

(i) Find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) The points P and Q are the stationary points of the curve.

(i) Show that the x-coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^{x} + 6 = 0$$ *(1 mark)*

(ii) By using the substitution $z = e^{x}$, or otherwise, show that the x-coordinates of P and Q are $\ln 2$ and $\ln 3$. *(3 marks)*

(iii) Find the y-coordinates of P and Q, giving each of your answers in the form $m + 12\ln n$, where m and n are integers. *(3 marks)*

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point. *(3 marks)*

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\displaystyle\int_1^5 \ln x \, dx$, giving your answer to three significant figures. *(3 marks)*

(b) (i) Given that $y = x\ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Hence, or otherwise, find $\displaystyle\int \ln x \, dx$. *(2 marks)*

(iii) Find the exact value of $\displaystyle\int_1^5 \ln x \, dx$. *(2 marks)*

7 (a) Given that $z = \dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{dz}{dx} = \sec^2 x$. *(3 marks)*

(b) Sketch the curve with equation $y = \sec x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. *(2 marks)*

(c) The region R is bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(3 marks)*

Turn over ▶

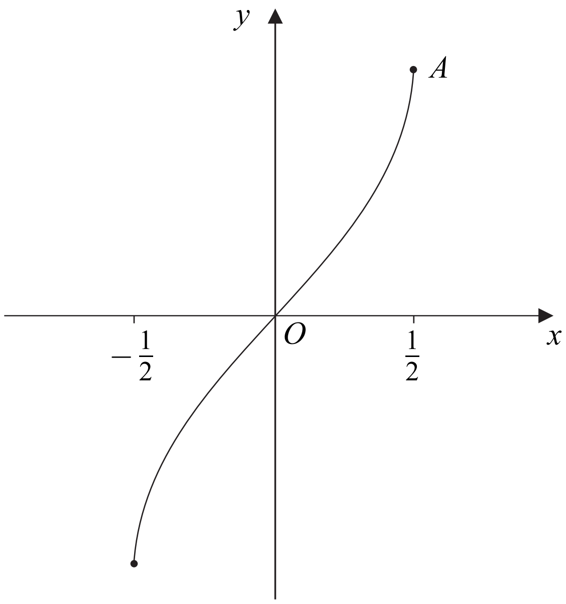
8 A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x.

(a) Find the range of f. *(2 marks)*

(b) Show that $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x+1}{2}\right)$. *(3 marks)*

(c) Find the gradient of the curve $y = f^{-1}(x)$ when $x = 0$. *(4 marks)*

9 The diagram shows the curve with equation $y = \sin^{-1} 2x$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.



(a) Find the y-coordinate of the point A, where $x = \frac{1}{2}$. *(1 mark)*

(b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2}\sin y$. *(1 mark)*

(ii) Given that $x = \frac{1}{2}\sin y$, find $\frac{dx}{dy}$ in terms of y. *(1 mark)*

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$ *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 Use the mid-ordinate rule with four strips of equal width to find an estimate for $\int_1^5 \frac{1}{1+\ln x}\,dx$, giving your answer to three significant figures. *(4 marks)*

2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. *(4 marks)*

3 The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \quad \text{for real values of } x, \ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

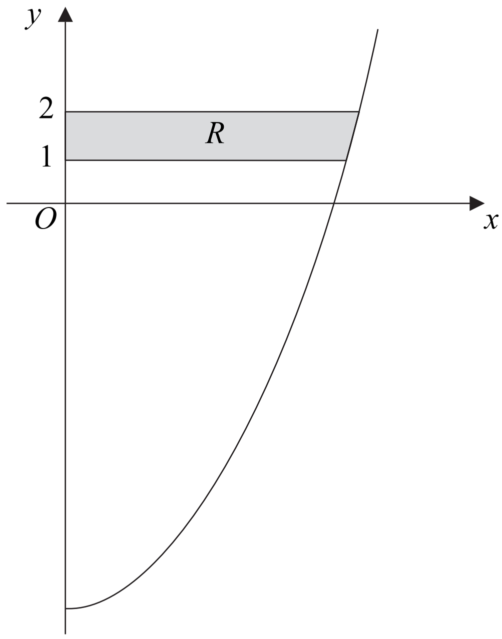
(i) Find $h(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2 + 5}\, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the *y*-axis**. *(4 marks)*

5 (a) (i) Show that the equation

$$2\cot^2 x + 5\,\text{cosec}\, x = 10$$

can be written in the form $2\,\text{cosec}^2 x + 5\,\text{cosec}\, x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

$$2\cot^2(\theta - 0.1) + 5\,\text{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. *(3 marks)*

6 (a) Find $\dfrac{dy}{dx}$ when:

(i) $y = (4x^2 + 3x + 2)^{10}$; *(2 marks)*

(ii) $y = x^2 \tan x$. *(2 marks)*

(b) (i) Find $\dfrac{dx}{dy}$ when $x = 2y^3 + \ln y$. *(1 mark)*

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. *(3 marks)*

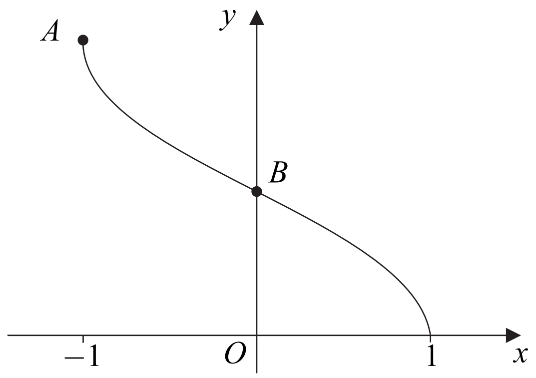
7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

8 The diagram shows the curve $y = \cos^{-1} x$ for $-1 \leqslant x \leqslant 1$.

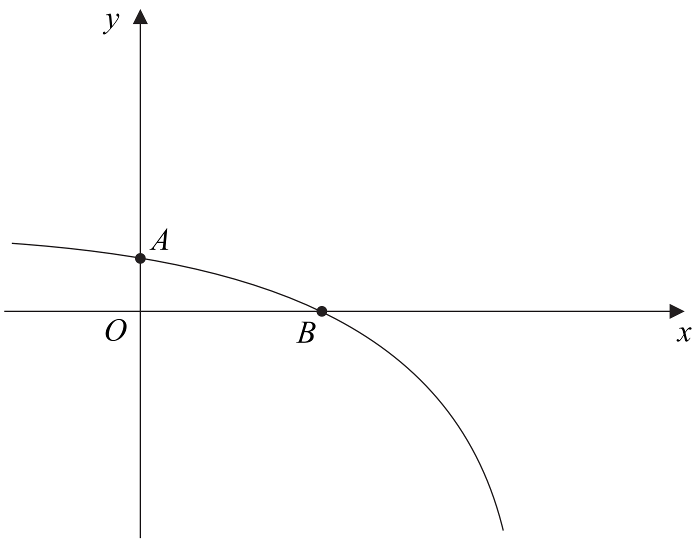


(a) Write down the exact coordinates of the points A and B. *(2 marks)*

(b) The equation $\cos^{-1} x = 3x + 1$ has only one root. Given that the root of this equation is α, show that $0.1 \leqslant \alpha \leqslant 0.2$. *(2 marks)*

(c) Use the iteration $x_{n+1} = \frac{1}{3}(\cos^{-1} x_n - 1)$ with $x_1 = 0.1$ to find the values of x_2, x_3 and x_4, giving your answers to three decimal places. *(3 marks)*

9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.



(a) (i) Find $\int (4 - e^{2x})\,dx$. *(2 marks)*

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x})\,dx = 4\ln 2 - \frac{3}{2}$. *(2 marks)*

(b) (i) Write down the y-coordinate of A. *(1 mark)*

(ii) Show that $x = \ln 2$ at B. *(2 marks)*

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B. *(4 marks)*

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y-axis. *(3 marks)*

END OF QUESTIONS

Answer **all** questions.

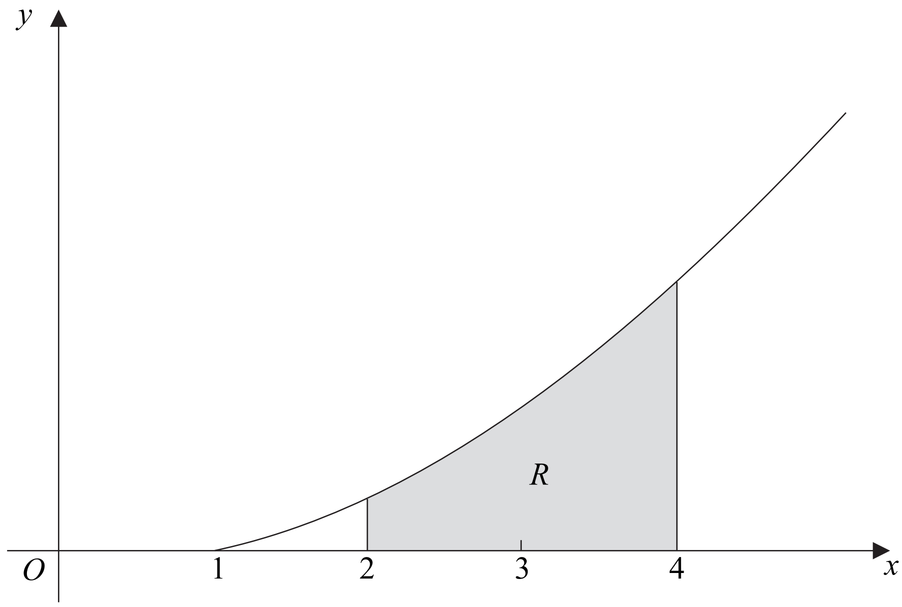
1 (a) Differentiate $\ln x$ with respect to x. *(1 mark)*

(b) Given that $y = (x + 1)\ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Find an equation of the normal to the curve $y = (x + 1)\ln x$ at the point where $x = 1$. *(4 marks)*

2 (a) Differentiate $(x - 1)^4$ with respect to x. *(1 mark)*

(b) The diagram shows the curve with equation $y = 2\sqrt{(x-1)^3}$ for $x \geqslant 1$.



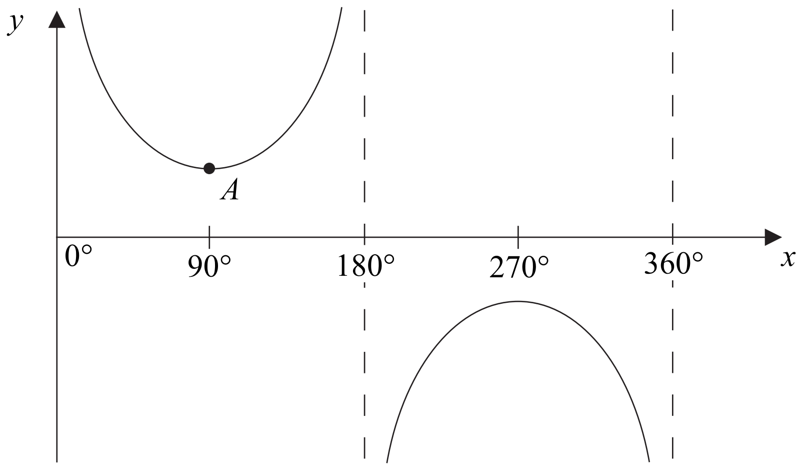
The shaded region R is bounded by the curve $y = 2\sqrt{(x-1)^3}$, the lines $x = 2$ and $x = 4$, and the x-axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

(c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sqrt{x^3}$ onto the graph of $y = 2\sqrt{(x-1)^3}$. *(4 marks)*

3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

(b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0° < x < 360°$.



(i) The point A on the curve is where $x = 90°$. State the y-coordinate of A.
(1 mark)

(ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0° < x < 360°$. *(2 marks)*

(c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

Turn over for the next question

4 [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^2 3^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(b) The curve $y = 3^x$ intersects the line $y = x + 3$ at the point where $x = \alpha$.

(i) Show that α lies between 0.5 and 1.5. *(2 marks)*

(ii) Show that the equation $3^x = x + 3$ can be rearranged into the form

$$x = \frac{\ln(x+3)}{\ln 3}$$ *(2 marks)*

(iii) Use the iteration $x_{n+1} = \frac{\ln(x_n + 3)}{\ln 3}$ with $x_1 = 0.5$ to find x_3 to two significant figures. *(2 marks)*

(iv) The sketch on **Figure 1** shows part of the graphs of $y = \frac{\ln(x+3)}{\ln 3}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x-2} \quad \text{for } x \geqslant 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \; x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 1$. *(3 marks)*

(c) The inverse of f is f^{-1}. Find $f^{-1}(x)$. *(3 marks)*

6 (a) Use integration by parts to find $\int x\mathrm{e}^{5x}\,\mathrm{d}x$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x = \int \frac{2}{1+u}\,\mathrm{d}u$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x$. *(3 marks)*

7 (a) A curve has equation $y = (x^2 - 3)\mathrm{e}^x$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(2 marks)*

(b) (i) Find the x-coordinate of each of the stationary points of the curve. *(4 marks)*

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. *(2 marks)*

8 (a) Write down $\int \sec^2 x\,\mathrm{d}x$. *(1 mark)*

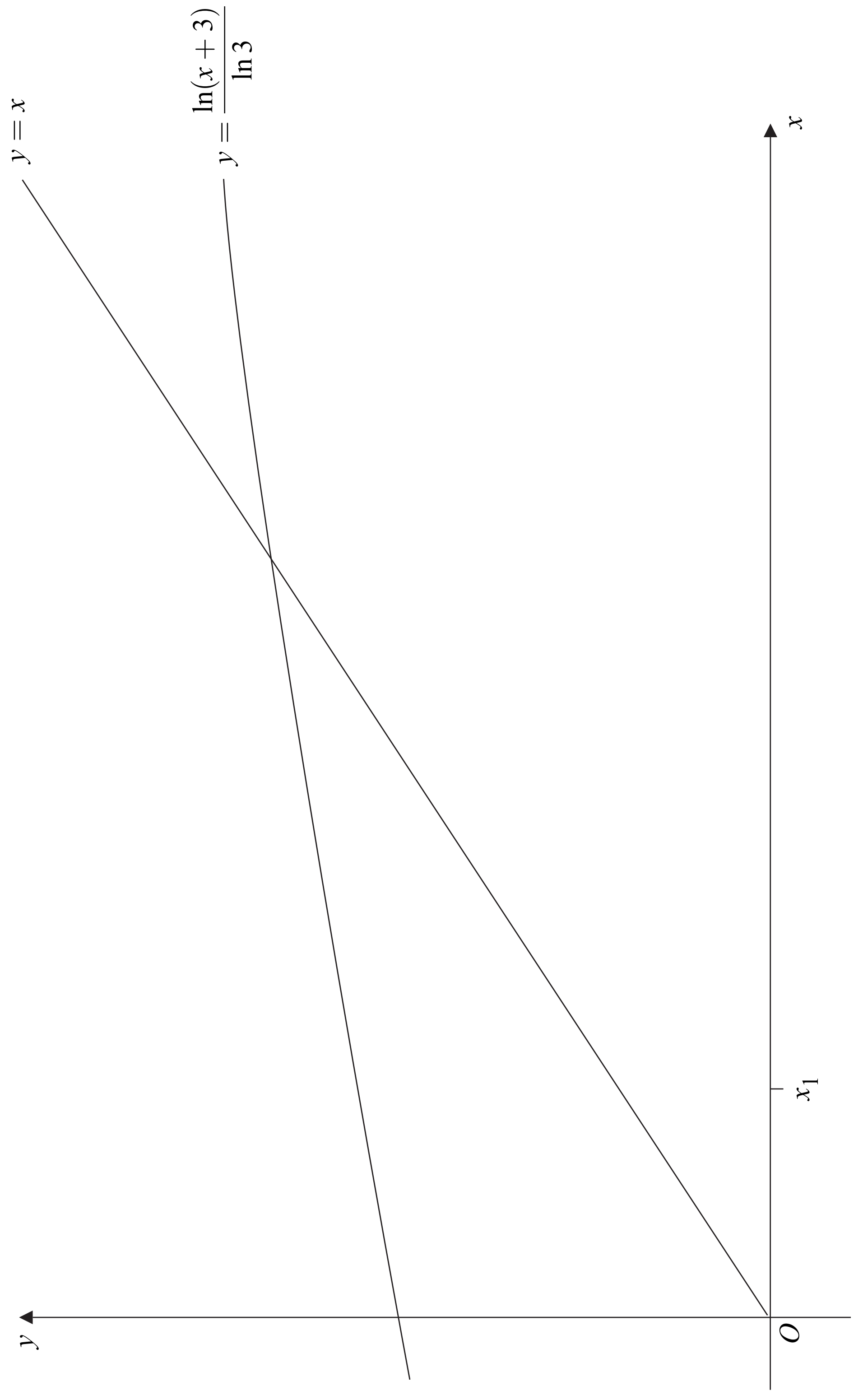
(b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\mathrm{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \mathrm{cosec}^2 x$. *(3 marks)*

(d) Hence find $\int_{0.5}^{1} (\tan x + \cot x)^2\,\mathrm{d}x$, giving your answer to two significant figures. *(4 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 4)



Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when:

(i) $y = (2x^2 - 5x + 1)^{20}$; *(2 marks)*

(ii) $y = x \cos x$. *(2 marks)*

(b) Given that

$$y = \frac{x^3}{x-2}$$

show that

$$\frac{dy}{dx} = \frac{kx^2(x-3)}{(x-2)^2}$$

where k is a positive integer. *(3 marks)*

2 (a) Solve the equation $\cot x = 2$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(2 marks)*

(b) Show that the equation $\operatorname{cosec}^2 x = \frac{3\cot x + 4}{2}$ can be written as

$$2\cot^2 x - 3\cot x - 2 = 0$$ *(2 marks)*

(c) Solve the equation $\operatorname{cosec}^2 x = \frac{3\cot x + 4}{2}$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

3 The equation

$$x + (1 + 3x)^{\frac{1}{4}} = 0$$

has a single root, α.

(a) Show that α lies between -0.33 and -0.32. *(2 marks)*

(b) Show that the equation $x + (1 + 3x)^{\frac{1}{4}} = 0$ can be rearranged into the form

$$x = \tfrac{1}{3}(x^4 - 1)$$ *(2 marks)*

(c) Use the iteration $x_{n+1} = \dfrac{(x_n{}^4 - 1)}{3}$ with $x_1 = -0.3$ to find x_4, giving your answer to three significant figures. *(3 marks)*

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^3, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x-3}, \qquad \text{for real values of } x,\ x \neq 3$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 64$. *(3 marks)*

(c) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

5 (a) (i) Given that $y = 2x^2 - 8x + 3$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(1 mark)*

(ii) Hence, or otherwise, find

$$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\,\mathrm{d}x$$

giving your answer in the form $k \ln 3$, where k is a rational number. *(4 marks)*

(b) Use the substitution $u = 3x - 1$ to find $\int x\sqrt{3x-1}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

Turn over for the next question

6 (a) Sketch the curve with equation $y = \operatorname{cosec} x$ for $0 < x < \pi$. *(2 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.1}^{0.5} \operatorname{cosec} x \, dx$, giving your answer to three significant figures. *(4 marks)*

7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 - 5$. *(4 marks)*

(b) Sketch the graph of $y = |4x^2 - 5|$, indicating the coordinates of the point where the curve crosses the y-axis. *(3 marks)*

(c) (i) Solve the equation $|4x^2 - 5| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4x^2 - 5| \geqslant 4$. *(2 marks)*

8 (a) Given that $e^{-2x} = 3$, find the exact value of x. *(2 marks)*

(b) Use integration by parts to find $\int x e^{-2x} \, dx$. *(4 marks)*

(c) A curve has equation $y = e^{-2x} + 6x$.

(i) Find the exact values of the coordinates of the stationary point of the curve. *(4 marks)*

(ii) Determine the nature of the stationary point. *(2 marks)*

(iii) The region R is bounded by the curve $y = e^{-2x} + 6x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(5 marks)*

END OF QUESTIONS