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Answer **all** questions.

1 (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. *(1 mark)*

(ii) Show that $f(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where a and b are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of d. *(3 marks)*

2 A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0$$ *(3 marks)*

3 It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of R. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of θ at which this maximum value occurs. *(3 marks)*

4 On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £V, of the sculpture is modelled by the formula $V = A k^t$, where t is the time in years since 1 January 1900 and A and k are constants.

(a) Write down the value of A. *(1 mark)*

(b) Show that $k \approx 1.07664$. *(3 marks)*

(c) Use this model to:

(i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; *(2 marks)*

(ii) find the year in which the value of the sculpture will first exceed £800 000. *(3 marks)*

5 (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1-x)^2}$ up to and including the term in x^2. *(2 marks)*

(c) Given that $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$,

find the values of A, B and C. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in x^2. *(3 marks)*

Turn over for the next question

6 (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where a and b are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where a is an integer. *(5 marks)*

7 The quadrilateral $ABCD$ has vertices $A(2, 1, 3)$, $B(6, 5, 3)$, $C(6, 1, -1)$ and $D(2, -3, -1)$.

The line l_1 has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overline{A}B$. *(2 marks)*

(ii) Show that the line AB is parallel to l_1. *(1 mark)*

(iii) Verify that D lies on l_1. *(2 marks)*

(b) The line l_2 passes through $D(2, -3, -1)$ and $M(4, 1, 1)$.

(i) Find the vector equation of l_2. *(2 marks)*

(ii) Find the angle between l_2 and AC. *(3 marks)*

8 (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

to find t in terms of x, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time t minutes, the depth of fuel in the tank is x cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) The polynomial $\mathrm{p}(x)$ is defined by $\mathrm{p}(x) = 6x^3 - 19x^2 + 9x + 10$.

(i) Find $\mathrm{p}(2)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $(2x + 1)$ is a factor of $\mathrm{p}(x)$. *(3 marks)*

(iii) Write $\mathrm{p}(x)$ as the product of three linear factors. *(2 marks)*

(b) Hence simplify $\dfrac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. *(2 marks)*

2 (a) Obtain the binomial expansion of $(1 - x)^{-3}$ up to and including the term in x^2. *(2 marks)*

(b) Hence obtain the binomial expansion of $\left(1 - \dfrac{5}{2}x\right)^{-3}$ up to and including the term in x^2. *(2 marks)*

(c) Find the range of values of x for which the binomial expansion of $\left(1 - \dfrac{5}{2}x\right)^{-3}$ would be valid. *(2 marks)*

(d) Given that x is small, show that $\left(\dfrac{4}{2 - 5x}\right)^3 \approx a + bx + cx^2$, where a, b and c are integers. *(2 marks)*

3 (a) Given that $\dfrac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}$ can be written in the form $3 + \dfrac{A}{3x - 1} + \dfrac{B}{x - 1}$, where A and B are integers, find the values of A and B. *(4 marks)*

(b) Hence, or otherwise, find $\displaystyle\int \frac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}\, \mathrm{d}x$. *(4 marks)*

4 (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*

(b) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

for all values of x. *(3 marks)*

(c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0° < x < 360°$. *(4 marks)*

5 A curve is defined by the equation

$$y^2 - xy + 3x^2 - 5 = 0$$

(a) Find the y-coordinates of the two points on the curve where $x = 1$. *(3 marks)*

(b) (i) Show that $\dfrac{dy}{dx} = \dfrac{y - 6x}{2y - x}$. *(6 marks)*

(ii) Find the gradient of the curve at each of the points where $x = 1$. *(2 marks)*

(iii) Show that, at the two stationary points on the curve, $33x^2 - 5 = 0$. *(3 marks)*

6 The points A and B have coordinates $(2, 4, 1)$ and $(3, 2, -1)$ respectively. The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$, where O is the origin.

(a) Find the vectors:

(i) $\overrightarrow{OC}$; *(1 mark)*

(ii) $\overrightarrow{AB}$. *(2 marks)*

(b) (i) Show that the distance between the points A and C is 5. *(2 marks)*

(ii) Find the size of angle BAC, giving your answer to the nearest degree. *(4 marks)*

(c) The point $P(\alpha, \beta, \gamma)$ is such that BP is perpendicular to AC.

Show that $4\alpha - 3\gamma = 15$. *(3 marks)*

Turn over for the next question

7 Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6xy^2$$

given that $y = 1$ when $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$. *(6 marks)*

8 A disease is spreading through a colony of rabbits. There are 5000 rabbits in the colony. At time t hours, x is the number of rabbits infected. The rate of increase of the number of rabbits infected is proportional to the product of the number of rabbits infected and the number not yet infected.

(a) (i) Formulate a differential equation for $\frac{\mathrm{d}x}{\mathrm{d}t}$ in terms of the variables x and t and a constant of proportionality k. *(2 marks)*

(ii) Initially, 1000 rabbits are infected and the disease is spreading at a rate of 200 rabbits per hour. Find the value of the constant k.

(You are **not** required to solve your differential equation.) *(2 marks)*

(b) The solution of the differential equation in this model is

$$t = 4 \ln\left(\frac{4x}{5000 - x}\right)$$

(i) Find the time after which 2500 rabbits will be infected, giving your answer in hours to one decimal place. *(2 marks)*

(ii) Find, according to this model, the number of rabbits infected after 30 hours. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$. *(2 marks)*

(ii) Hence find $\dfrac{dy}{dx}$ in terms of t. *(2 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

2 The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where d is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$. *(2 marks)*

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$. *(2 marks)*

3 (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3 \sin x - \cos 2x = 2 \sin^2 x + 3 \sin x - 1$ for all values of x. *(2 marks)*

(ii) Solve the equation $3 \sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*

4 (a) (i) Express $\dfrac{3x-5}{x-3}$ in the form $A+\dfrac{B}{x-3}$, where A and B are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{3x-5}{x-3}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Express $\dfrac{6x-5}{4x^2-25}$ in the form $\dfrac{P}{2x+5}+\dfrac{Q}{2x-5}$, where P and Q are integers. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{6x-5}{4x^2-25}\,\mathrm{d}x$. *(3 marks)*

5 (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in x^2. *(2 marks)*

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2+\frac{1}{4}x-\frac{1}{32}x^2$ for small values of x. *(3 marks)*

(ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. *(2 marks)*

6 The points A, B and C have coordinates $(3, -2, 4)$, $(5, 4, 0)$ and $(11, 6, -4)$ respectively.

(a) (i) Find the vector $\overrightarrow{BA}$. *(2 marks)*

(ii) Show that the size of angle ABC is $\cos^{-1}\left(-\frac{5}{7}\right)$. *(5 marks)*

(b) The line l has equation $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$.

(i) Verify that C lies on l. *(2 marks)*

(ii) Show that AB is parallel to l. *(1 mark)*

(c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of D. *(3 marks)*

Turn over for the next question

7 (a) Use the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$. *(2 marks)*

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of x, $\tan 2x \neq 0$. *(4 marks)*

8 (a) (i) Solve the differential equation $\frac{dy}{dt} = y \sin t$ to obtain y in terms of t. *(4 marks)*

(ii) Given that $y = 50$ when $t = \pi$, show that $y = 50e^{-(1+\cos t)}$. *(3 marks)*

(b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation

$$\frac{dy}{dt} = y \sin t$$

(i) Given that this height is 50 cm after π seconds, find, to the nearest centimetre, the height of the water after 6 seconds. *(2 marks)*

(ii) Find $\frac{d^2y}{dt^2}$ and hence verify that the water reaches a maximum height after π seconds. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. *(2 marks)*

(b) Simplify the algebraic fraction $\dfrac{2x^2 + x - 3}{x^2 - 1}$. *(3 marks)*

2 (a) (i) Find the binomial expansion of $(1 + x)^{-1}$ up to the term in x^3. *(2 marks)*

(ii) Hence, or otherwise, obtain the binomial expansion of $\dfrac{1}{1 + 3x}$ up to the term in x^3. *(2 marks)*

(b) Express $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ in partial fractions. *(3 marks)*

(c) (i) Find the binomial expansion of $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ up to the term in x^3. *(3 marks)*

(ii) Find the range of values of x for which the binomial expansion of $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ is valid. *(2 marks)*

3 (a) Express $4\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving your value for α to the nearest 0.1°. *(3 marks)*

(b) Hence solve the equation $4\cos x + 3\sin x = 2$ in the interval $0° < x < 360°$, giving all solutions to the nearest 0.1°. *(4 marks)*

(c) Write down the minimum value of $4\cos x + 3\sin x$ and find the value of x in the interval $0° < x < 360°$ at which this minimum value occurs. *(3 marks)*

4 A biologist is researching the growth of a certain species of hamster. She proposes that the length, x cm, of a hamster t days after its birth is given by

$$x = 15 - 12\mathrm{e}^{-\frac{t}{14}}$$

(a) Use this model to find:

(i) the length of a hamster when it is born; *(1 mark)*

(ii) the length of a hamster after 14 days, giving your answer to three significant figures. *(2 marks)*

(b) (i) Show that the time for a hamster to grow to 10 cm in length is given by $t = 14\ln\left(\frac{a}{b}\right)$, where a and b are integers. *(3 marks)*

(ii) Find this time to the nearest day. *(1 mark)*

(c) (i) Show that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{14}(15 - x)$$

(3 marks)

(ii) Find the rate of growth of the hamster, in cm per day, when its length is 8 cm. *(1 mark)*

5 The point $P(1, a)$, where $a > 0$, lies on the curve $y + 4x = 5x^2y^2$.

(a) Show that $a = 1$. *(2 marks)*

(b) Find the gradient of the curve at P. *(7 marks)*

(c) Find an equation of the tangent to the curve at P. *(1 mark)*

Turn over for the next question

6 A curve is given by the parametric equations

$$x = \cos\theta \qquad y = \sin 2\theta$$

(a) (i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$. *(2 marks)*

(ii) Find the gradient of the curve at the point where $\theta = \dfrac{\pi}{6}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

where k is an integer. *(4 marks)*

7 The lines l_1 and l_2 have equations $\mathbf{r} = \begin{bmatrix} 8 \\ 6 \\ -9 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ respectively.

(a) Show that l_1 and l_2 are perpendicular. *(2 marks)*

(b) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection, P. *(5 marks)*

(c) The point $A(-4, 0, 11)$ lies on l_2. The point B on l_1 is such that $AP = BP$.

Find the length of AB. *(4 marks)*

8 (a) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1 + 2y}}{x^2}$$

given that $y = 4$ when $x = 1$. *(6 marks)*

(b) Show that the solution can be written as $y = \dfrac{1}{2}\left(15 - \dfrac{8}{x} + \dfrac{1}{x^2}\right)$. *(2 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) Given that $\dfrac{3}{9-x^2}$ can be expressed in the form $k\left(\dfrac{1}{3+x}+\dfrac{1}{3-x}\right)$, find the value of the rational number k. *(2 marks)*

(b) Show that $\displaystyle\int_1^2 \frac{3}{9-x^2}\,\mathrm{d}x = \frac{1}{2}\ln\left(\frac{a}{b}\right)$, where a and b are integers. *(3 marks)*

2 (a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 2x^3 + 3x^2 - 18x + 8$.

(i) Use the Factor Theorem to show that $(2x-1)$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(ii) Write $\mathrm{f}(x)$ in the form $(2x-1)(x^2+px+q)$, where p and q are integers. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^2+16x}{2x^3+3x^2-18x+8}$. *(2 marks)*

(b) Express the algebraic fraction $\dfrac{2x^2}{(x+5)(x-3)}$ in the form $A + \dfrac{B+Cx}{(x+5)(x-3)}$, where A, B and C are integers. *(4 marks)*

3 (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ up to and including the term in x^2. *(2 marks)*

(b) Hence obtain the binomial expansion of $\sqrt{1+\frac{3}{2}x}$ up to and including the term in x^2. *(2 marks)*

(c) Hence show that $\sqrt{\dfrac{2+3x}{8}} \approx a + bx + cx^2$ for small values of x, where a, b and c are constants to be found. *(2 marks)*

4 David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

$$P = Ak^t$$

for the selling price, £P, of this house, where t is the time in years after 1 January 1885 and A and k are constants.

(a) (i) Write down the value of A. *(1 mark)*

(ii) Show that, to six decimal places, $k = 1.079775$. *(2 marks)*

(iii) Use the model, with this value of k, to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. *(2 marks)*

(b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

$$Q = 15 \times 1.082709^t$$

for the selling price, £Q, of this house t years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price. *(4 marks)*

5 A curve is defined by the parametric equations $x = 2t + \dfrac{1}{t^2}$, $y = 2t - \dfrac{1}{t^2}$.

(a) At the point P on the curve, $t = \dfrac{1}{2}$.

(i) Find the coordinates of P. *(2 marks)*

(ii) Find an equation of the tangent to the curve at P. *(5 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where k is an integer. *(3 marks)*

Turn over for the next question

6 A curve has equation $3xy - 2y^2 = 4$.

Find the gradient of the curve at the point (2, 1). *(5 marks)*

7 (a) (i) Express $6\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value for α to the nearest 0.1°. *(2 marks)*

(ii) Hence solve the equation $6\sin 2x + 8\cos 2x = 7$, giving all solutions to the nearest 0.1° in the interval $0° < x < 360°$. *(4 marks)*

(b) (i) Prove the identity $\dfrac{\sin 2x}{1 - \cos 2x} = \dfrac{1}{\tan x}$. *(4 marks)*

(ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval $0° < x < 360°$. *(4 marks)*

8 Solve the differential equation

$$\frac{dy}{dx} = \frac{3\cos 3x}{y}$$

given that $y = 2$ when $x = \dfrac{\pi}{2}$. Give your answer in the form $y^2 = f(x)$. *(5 marks)*

9 The points A and B lie on the line l_1 and have coordinates (2, 5, 1) and (4, 1, –2) respectively.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find a vector equation of the line l_1, with parameter λ. *(1 mark)*

(b) The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(i) Show that the point $P(-2, -3, 5)$ lies on l_2. *(2 marks)*

(ii) The point Q lies on l_1 and is such that PQ is perpendicular to l_2. Find the coordinates of Q. *(6 marks)*

END OF QUESTIONS