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Answer **all** questions.

1 (a) Draw a bipartite graph representing the following adjacency matrix. *(2 marks)*

	U	***V***	***W***	***X***	***Y***	***Z***
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(b) Given that initially A is matched to W, B is matched to X, C is matched to V, and E is matched to Y, use the alternating path algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

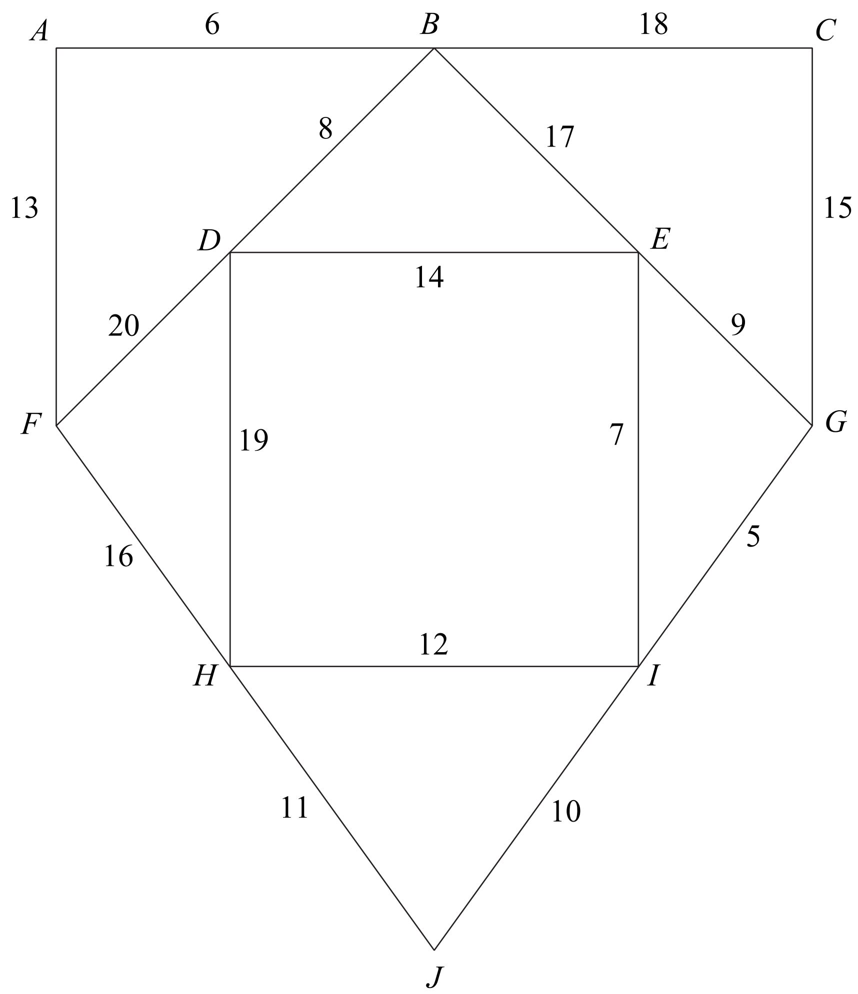
2 Use the quicksort algorithm to rearrange the following numbers into ascending order. Indicate clearly the pivots that you use.

18 23 12 7 26 19 16 24 *(5 marks)*

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, …, J. The numbers on each edge represent the distances, in miles, between pairs of vertices.

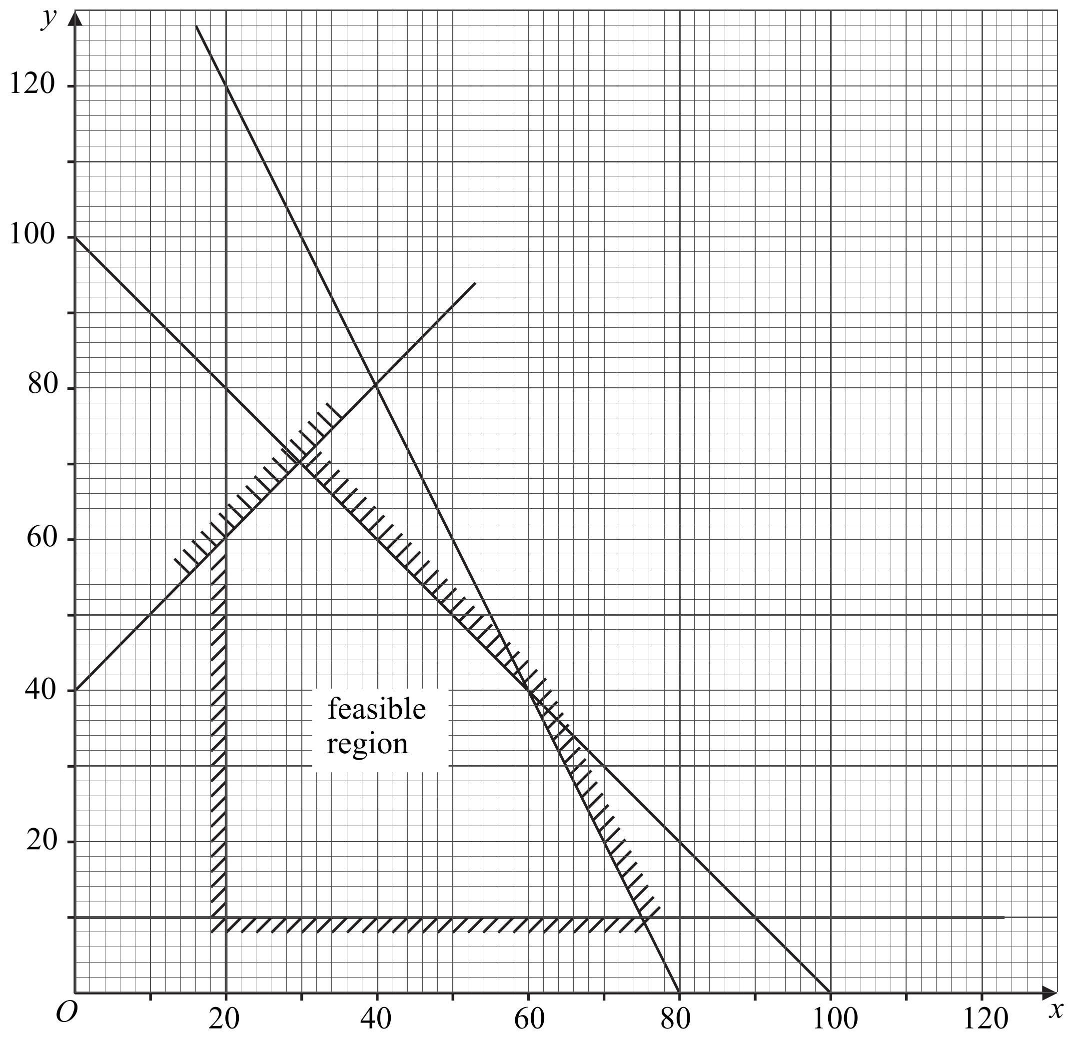


(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your spanning tree. *(1 mark)*

(iii) Draw your spanning tree. *(2 marks)*

4 The diagram shows the feasible region of a linear programming problem.



(a) On the feasible region, find:

(i) the maximum value of $2x + 3y$; *(2 marks)*

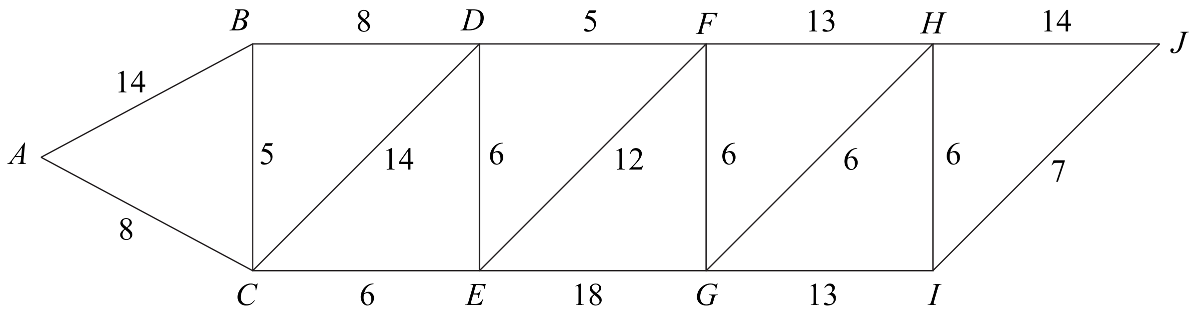
(ii) the maximum value of $3x + 2y$; *(2 marks)*

(iii) the minimum value of $-2x + y$. *(2 marks)*

(b) Find the 5 inequalities that define the feasible region. *(6 marks)*

5 [**Figure 1**, printed on the insert, is provided for use in this question.]

The network shows the times, in minutes, to travel between 10 towns.



(a) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to travel from A to J. *(6 marks)*

(b) State the corresponding route. *(1 mark)*

6 Two algorithms are shown.

Algorithm 1

Line 10	Input P
Line 20	Input R
Line 30	Input T
Line 40	Let $I = (P * R * T)/100$
Line 50	Let $A = P + I$
Line 60	Let $M = A/(12 * T)$
Line 70	Print M
Line 80	Stop

Algorithm 2

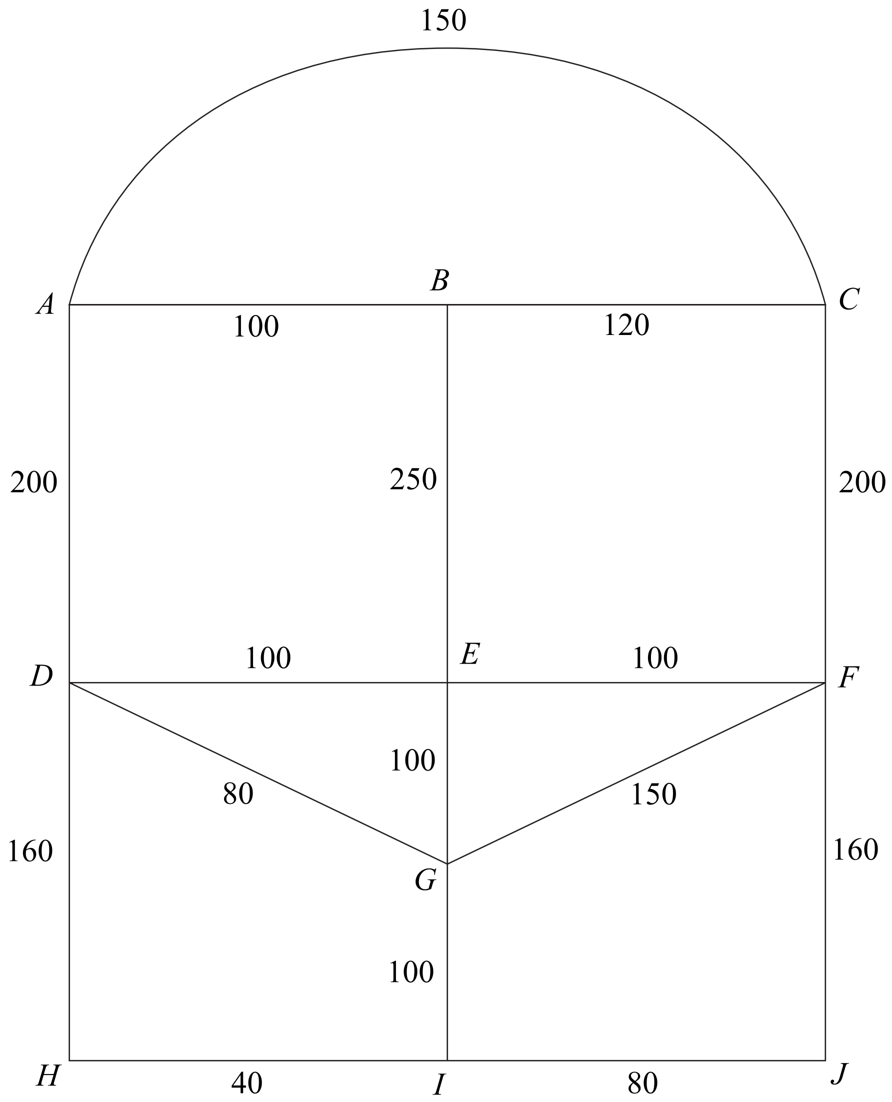
Line 10	Input P
Line 20	Input R
Line 30	Input T
Line 40	Let $A = P$
Line 50	$K = 0$
Line 60	Let $K = K + 1$
Line 70	Let $I = (A * R)/100$
Line 80	Let $A = A + I$
Line 90	If $K < T$ then goto Line 60
Line 100	Let $M = A/(12 * T)$
Line 110	Print M
Line 120	Stop

In the case where the input values are $P = 400$, $R = 5$ and $T = 3$:

(a) trace **Algorithm 1**; *(3 marks)*

(b) trace **Algorithm 2**. *(4 marks)*

7 Stella is visiting Tijuana on a day trip. The diagram shows the lengths, in metres, of the roads near the bus station.



Total = 2090

Stella leaves the bus station at A. She decides to walk along all of the roads at least once before returning to A.

(a) Explain why it is not possible to start from A, travel along each road only once and return to A. *(1 mark)*

(b) Find the length of an optimal 'Chinese postman' route around the network, starting and finishing at A. *(5 marks)*

(c) At each of the 9 places B, C, . . ., J, there is a statue. Find the number of times that Stella will pass a statue if she follows her optimal route. *(2 marks)*

8 Salvadore is visiting six famous places in Barcelona: La Pedrera (L), Nou Camp (N), Olympic Village (O), Park Guell (P), Ramblas (R) and Sagrada Familia (S). Owing to the traffic system the time taken to travel between two places may vary according to the direction of travel.

The table shows the times, in minutes, that it will take to travel between the six places.

From \ To	**La Pedrera** (L)	**Nou Camp** (N)	**Olympic Village** (O)	**Park Guell** (P)	**Ramblas** (R)	**Sagrada Familia** (S)
La Pedrera (L)	—	35	30	30	37	35
Nou Camp (N)	25	—	20	21	25	40
Olympic Village (O)	15	40	—	25	30	29
Park Guell (P)	30	35	25	—	35	20
Ramblas (R)	20	30	17	25	—	25
Sagrada Familia (S)	25	35	29	20	30	—

(a) Find the total travelling time for:

(i) the route $LNOL$; *(1 mark)*

(ii) the route $LONL$. *(1 mark)*

(b) Give an example of a Hamiltonian cycle in the context of the above situation. *(1 mark)*

(c) Salvadore intends to travel from one place to another until he has visited all of the places before returning to his starting place.

(i) Show that, using the nearest neighbour algorithm starting from Sagrada Familia (S), the total travelling time for Salvadore is 145 minutes. *(3 marks)*

(ii) Explain why your answer to part (c)(i) is an upper bound for the minimum travelling time for Salvadore. *(2 marks)*

(iii) Salvadore starts from Sagrada Familia (S) and then visits Ramblas (R). Given that he visits Nou Camp (N) before Park Guell (P), find an improved upper bound for the total travelling time for Salvadore. *(3 marks)*

Turn over for the next question

9 A factory makes three different types of widget: plain, bland and ordinary. Each widget is made using three different machines: A, B and C.

Each plain widget needs 5 minutes on machine A, 12 minutes on machine B and 24 minutes on machine C.

Each bland widget needs 4 minutes on machine A, 8 minutes on machine B and 12 minutes on machine C.

Each ordinary widget needs 3 minutes on machine A, 10 minutes on machine B and 18 minutes on machine C.

Machine A is available for 3 hours a day, machine B for 4 hours a day and machine C for 9 hours a day.

The factory must make:

more plain widgets than bland widgets;

more bland widgets than ordinary widgets.

At least 40% of the total production must be plain widgets.

Each day, the factory makes x plain, y bland and z ordinary widgets.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, writing your answers with simplified integer coefficients. *(8 marks)*

END OF QUESTIONS

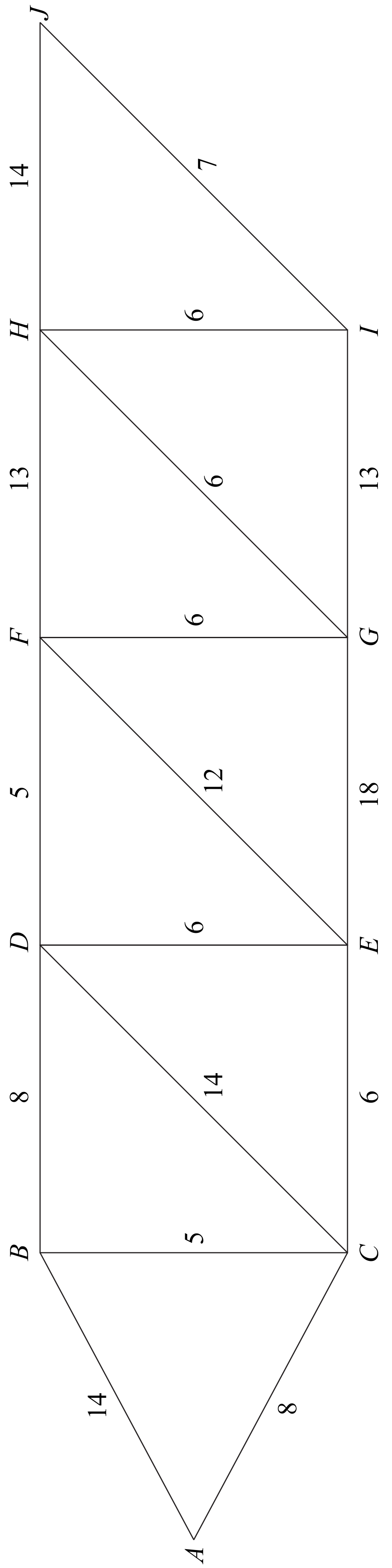


Figure 1 (for Question 5)

Answer **all** questions.

1 Five people, A, B, C, D and E, are to be matched to five tasks, 1, 2, 3, 4 and 5. The table shows which tasks each person can do.

Person	**Tasks**
A	1, 3, 5
B	2, 4
C	2
D	4, 5
E	3, 5

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially A is matched to task 3, B to task 4, C to task 2 and E to task 5.

Use an alternating path from this initial matching to find a complete matching. *(4 marks)*

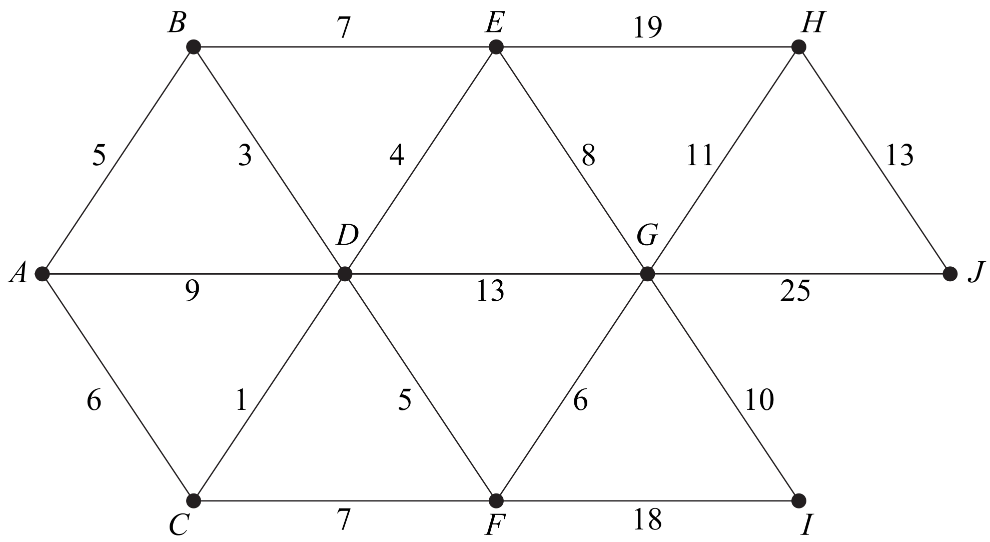
2 (a) Use a shuttle sort to rearrange the following numbers into ascending order.

18 2 12 7 26 19 16 24 *(5 marks)*

(b) State the number of comparisons and swaps (exchanges) for each of the first three passes. *(3 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in **part (b)** of this question.]

The diagram shows a network of roads. The number on each edge is the length, in kilometres, of the road.



(a) (i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

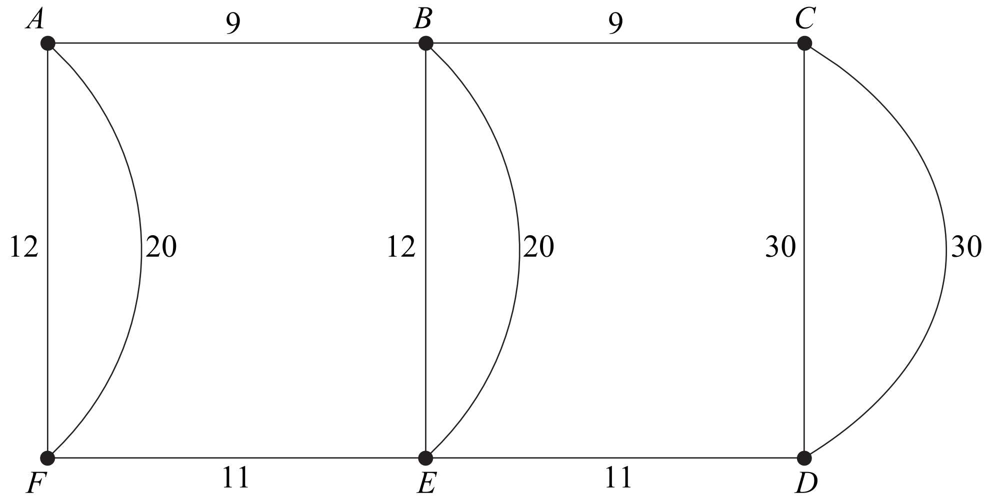
(ii) State the length of your minimum spanning tree. *(1 mark)*

(b) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest distance from A to J. *(6 marks)*

(ii) A new road, of length x km, is built connecting I to J. The minimum distance from A to J is reduced by using this new road. Find, and solve, an inequality for x. *(2 marks)*

Turn over for the next question

4 The diagram shows a network of roads connecting 6 villages. The number on each edge is the length, in miles, of the road.



Total length of the roads = 164 miles

(a) A police patrol car based at village A has to travel along each road at least once before returning to A. Find the length of an optimal 'Chinese postman' route for the police patrol car. *(6 marks)*

(b) A council worker starts from A and travels along each road at least once before finishing at C. Find the length of an optimal route for the council worker. *(2 marks)*

(c) A politician is to travel along all the roads at least once. He can start his journey at any village and can finish his journey at any village.

(i) Find the length of an optimal route for the politician. *(2 marks)*

(ii) State the vertices from which the politician could start in order to achieve this optimal route. *(1 mark)*

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

(a) Gill is solving a travelling salesperson problem.

(i) She finds the following upper bounds: 7.5, 8, 7, 7.5, 8.5 .

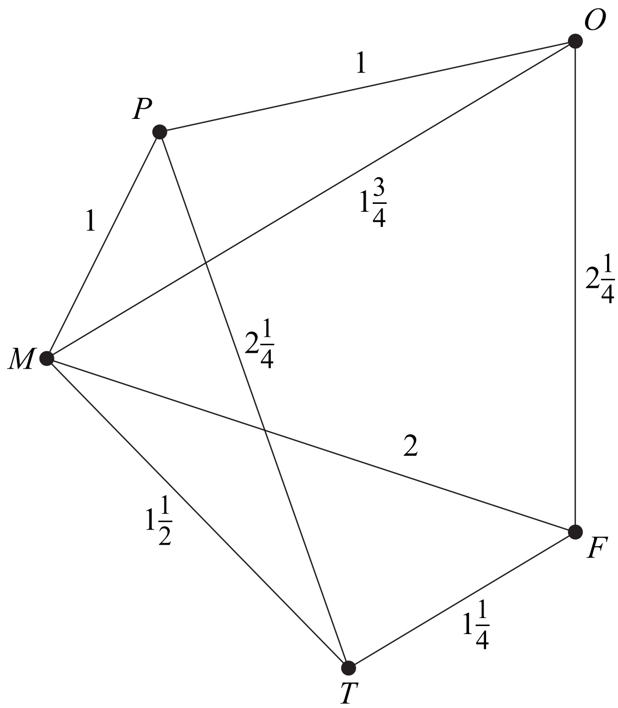
Write down the best upper bound. *(1 mark)*

(ii) She finds the following lower bounds: 6.5, 7, 6.5, 5, 7.

Write down the best lower bound. *(1 mark)*

(b) George is travelling by plane to a number of cities. He is to start at F and visit each of the other cities at least once before returning to F.

The diagram shows the times of flights, in hours, between cities. Where no time is shown, there is no direct flight available.



(i) Complete **Figure 2** to show the minimum times to travel between all pairs of cities. *(2 marks)*

(ii) Find an upper bound for the minimum total flying time by using the route *FTPOMF*. *(1 mark)*

(iii) Using the nearest neighbour algorithm starting from F, find an upper bound for the minimum total flying time. *(4 marks)*

(iv) By deleting F, find a lower bound for the minimum total flying time. *(5 marks)*

6 [**Figure 3**, printed on the insert, is provided for use in this question.]

Ernesto is to plant a garden with two types of tree: palms and conifers.

He is to plant at least 10, but not more than 80 palms.

He is to plant at least 5, but not more than 40 conifers.

He cannot plant more than 100 trees in total.

Each palm needs 20 litres of water each day and each conifer needs 60 litres of water each day. There are 3000 litres of water available each day.

Ernesto makes a profit of £2 on each palm and £1 on each conifer that he plants and he wishes to maximise his profit.

Ernesto plants x palms and y conifers.

(a) Formulate Ernesto's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 3**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(7 marks)*

(c) Find the maximum profit for Ernesto. *(2 marks)*

(d) Ernesto introduces a new pricing structure in which he makes a profit of £1 on each palm and £4 on each conifer.

Find Ernesto's new maximum profit and the number of each type of tree that he should plant to obtain this maximum profit. *(2 marks)*

7 A connected graph **G** has m vertices and n edges.

(a) (i) Write down the number of edges in a minimum spanning tree of **G**. *(1 mark)*

(ii) Hence write down an inequality relating m and n. *(2 marks)*

(b) The graph **G** contains a Hamiltonian cycle. Write down the number of edges in this cycle. *(1 mark)*

(c) In the case where **G** is Eulerian, draw a graph of **G** for which $m = 6$ and $n = 12$. *(2 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 3(b))

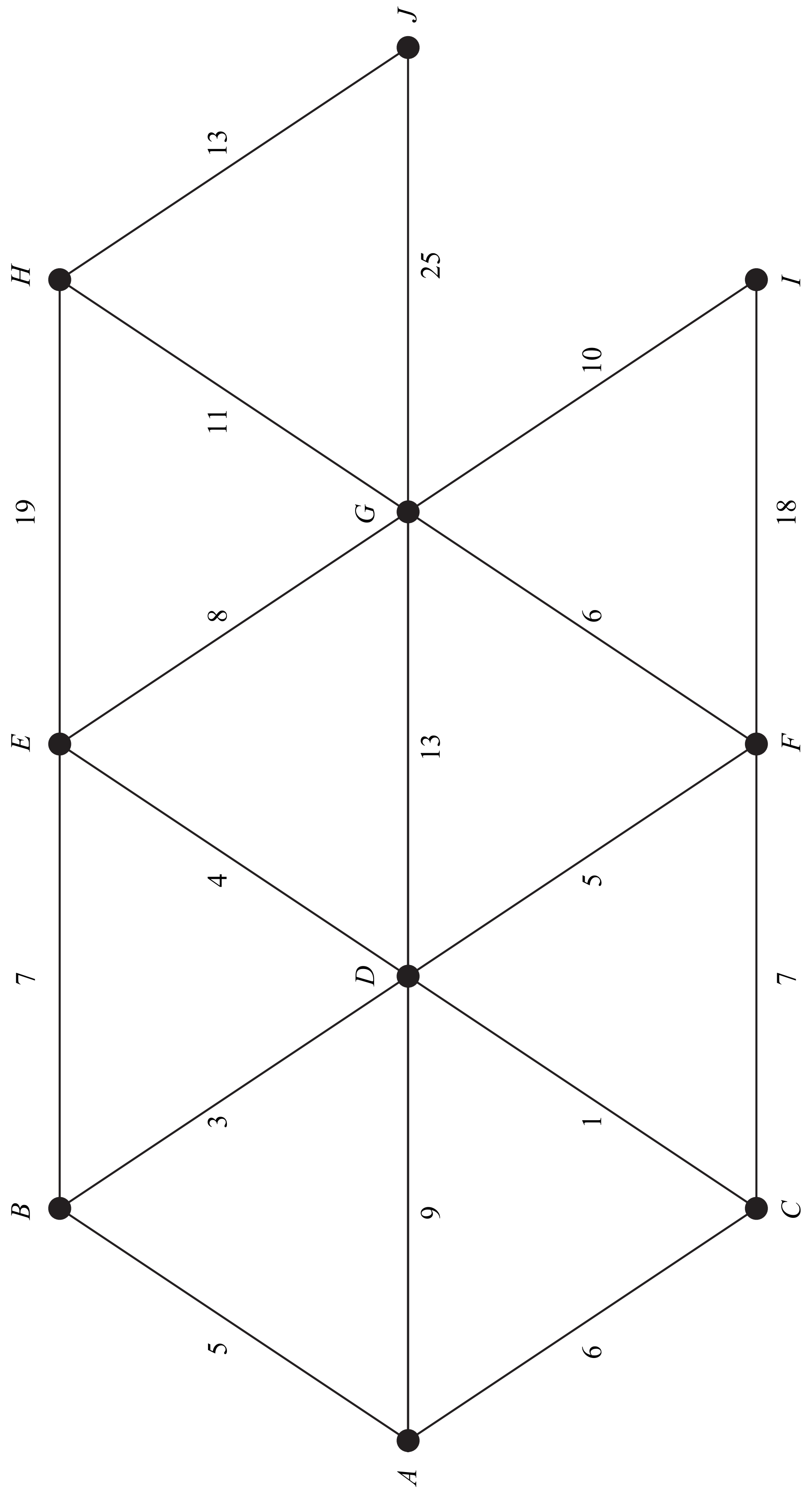
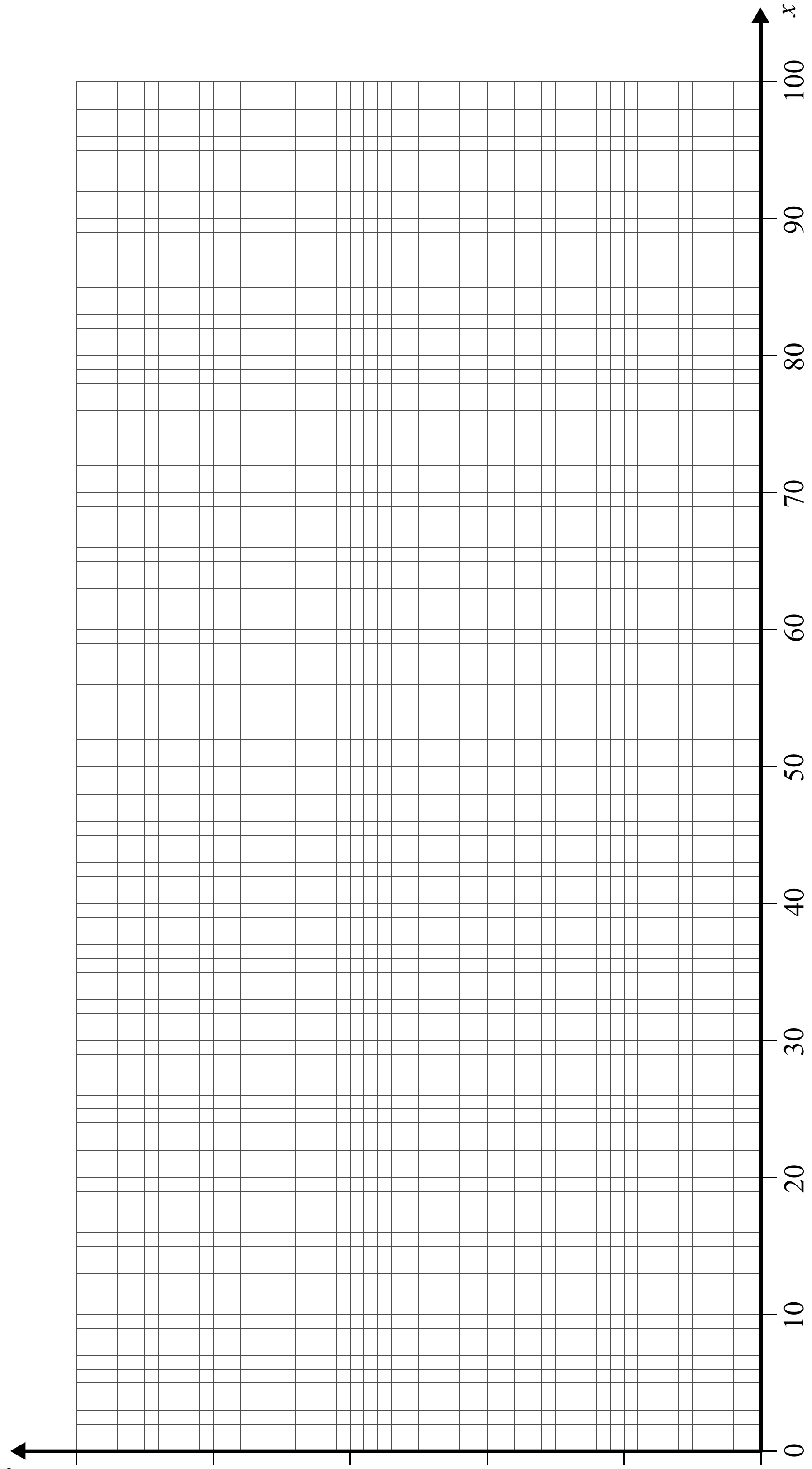


Figure 2 (for use in Question 5)

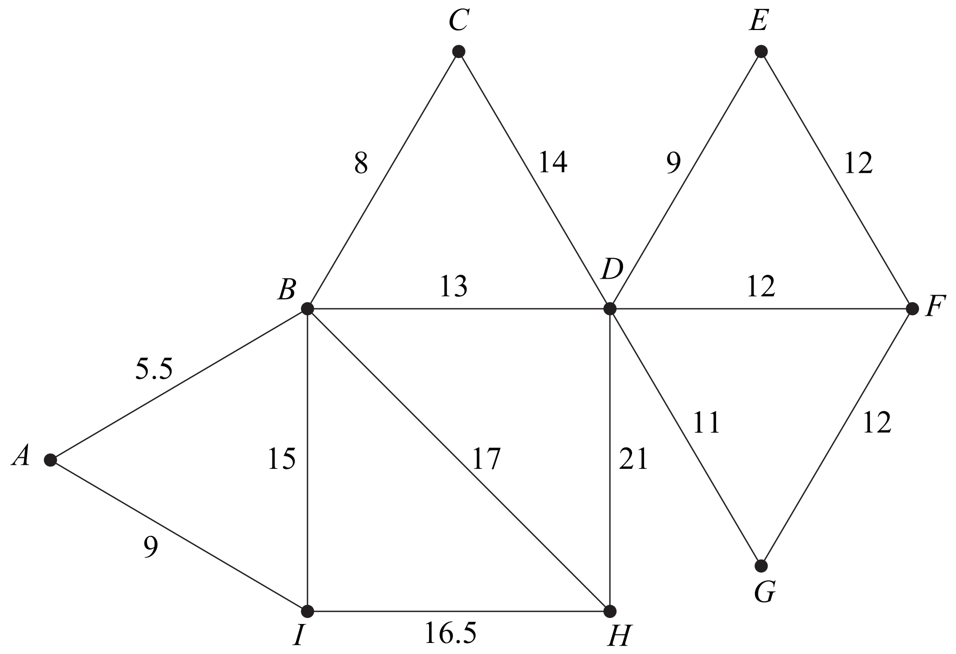
	M	P	O	T	F
M	–	1	$1\frac{3}{4}$	$1\frac{1}{2}$	2
P	1	–	1	$2\frac{1}{4}$	
O	$1\frac{3}{4}$	1	–		$2\frac{1}{4}$
T	$1\frac{1}{2}$	$2\frac{1}{4}$		–	$1\frac{1}{4}$
F	2		$2\frac{1}{4}$	$1\frac{1}{4}$	–

Figure 3 (for use in Question 6)



Answer **all** questions.

1 The following network shows the lengths, in miles, of roads connecting nine villages.



(a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(b) Find the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(d) State the number of other spanning trees that are of the same length as your answer in part (a). *(1 mark)*

2 Five people A, B, C, D and E are to be matched to five tasks R, S, T, U and V.

The table shows the tasks that each person is able to undertake.

Person	**Tasks**
A	R, V
B	R, T
C	T, V
D	U, V
E	S, U

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task V, B to task R, C to task T, and E to task U.

Demonstrate, by using an alternating path from this initial matching, how each person can be matched to a task. *(4 marks)*

3 Mark is driving around the one-way system in Leicester. The following table shows the times, in minutes, for Mark to drive between four places: A, B, C and D. Mark decides to start from A, drive to the other three places and then return to A.

Mark wants to keep his driving time to a minimum.

From \ To	***A***	***B***	***C***	***D***
A	–	8	6	11
B	14	–	13	25
C	14	9	–	17
D	26	10	18	–

(a) Find the length of the tour $ABCDA$. *(2 marks)*

(b) Find the length of the tour $ADCBA$. *(1 mark)*

(c) Find the length of the tour using the nearest neighbour algorithm starting from A. *(4 marks)*

(d) Write down which of your answers to parts (a), (b) and (c) gives the best upper bound for Mark's driving time. *(1 mark)*

4 (a) A student is using a bubble sort to rearrange seven numbers into ascending order.

Her correct solution is as follows:

Initial list	18	17	13	26	10	14	24
After 1st pass	17	13	18	10	14	24	26
After 2nd pass	13	17	10	14	18	24	26
After 3rd pass	13	10	14	17	18	24	26
After 4th pass	10	13	14	17	18	24	26
After 5th pass	10	13	14	17	18	24	26

Write down the number of comparisons and swaps on each of the five passes.

(6 marks)

(b) Find the maximum number of comparisons and the maximum number of swaps that might be needed in a bubble sort to rearrange seven numbers into ascending order.

(2 marks)

5 A student is using the following algorithm with different values of A and B.

Line 10	Input A, B
Line 20	Let $C = 0$ and let $D = 0$
Line 30	Let $C = C + A$
Line 40	Let $D = D + B$
Line 50	If $C = D$ then go to Line 110
Line 60	If $C > D$ then go to Line 90
Line 70	Let $C = C + A$
Line 80	Go to Line 50
Line 90	Let $D = D + B$
Line 100	Go to Line 50
Line 110	Print C
Line 120	End

(a) (i) Trace the algorithm in the case where $A = 2$ and $B = 3$. *(3 marks)*

(ii) Trace the algorithm in the case where $A = 6$ and $B = 8$. *(3 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

(c) Write down the final value of C in the case where $A = 200$ and $B = 300$. *(1 mark)*

Turn over for the next question

Turn over ▶

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

Dino is to have a rectangular swimming pool at his villa.

He wants its width to be at least 2 metres and its length to be at least 5 metres.

He wants its length to be at least twice its width.

He wants its length to be no more than three times its width.

Each metre of the width of the pool costs £1000 and each metre of the length of the pool costs £500.

He has £9000 available.

Let the width of the pool be x metres and the length of the pool be y metres.

(a) Show that one of the constraints leads to the inequality

$$2x + y \leqslant 18$$ *(1 mark)*

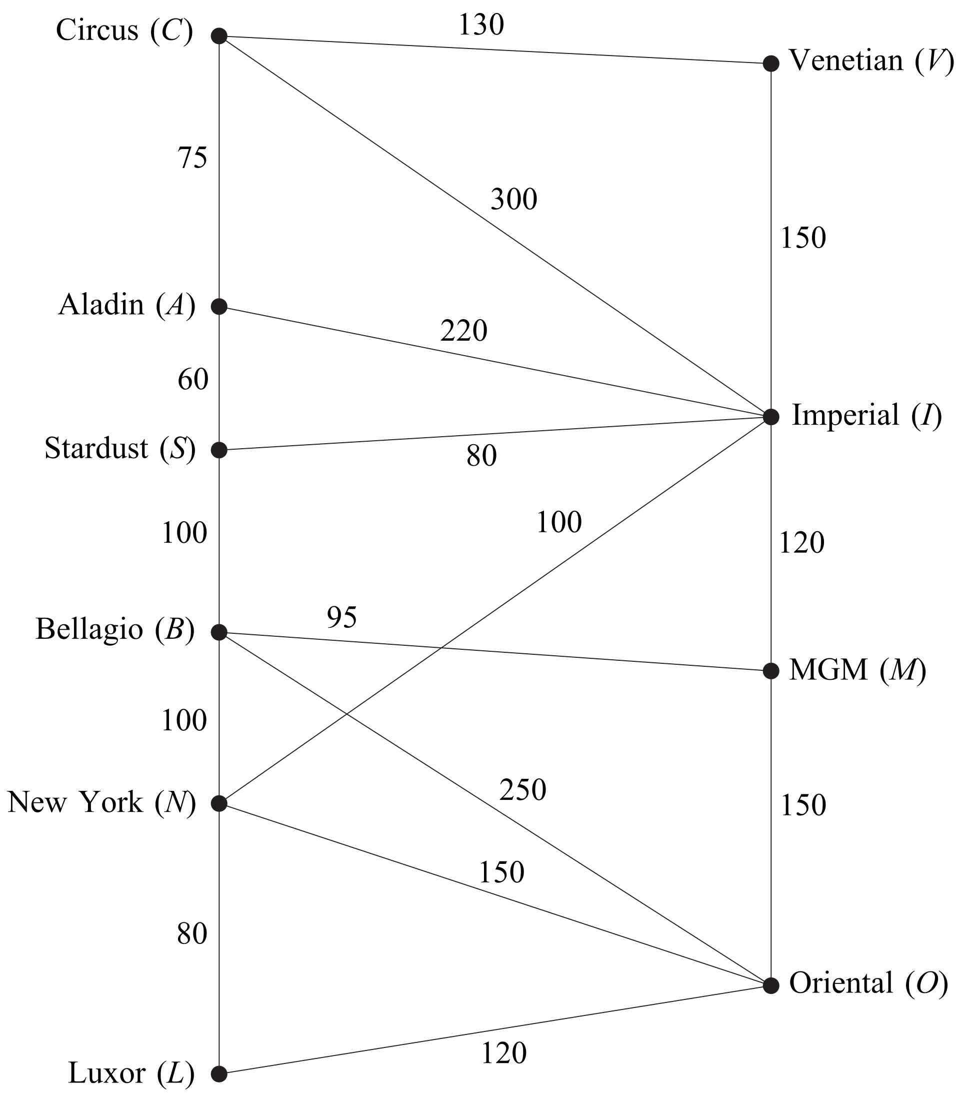
(b) Find four further inequalities. *(3 marks)*

(c) On **Figure 1**, draw a suitable diagram to show the feasible region. *(6 marks)*

(d) Use your diagram to find the maximum width of the pool. State the corresponding length of the pool. *(3 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows the times, in seconds, taken by Craig to walk along walkways connecting ten hotels in Las Vegas.



The total of all the times in the diagram is 2280 seconds.

(a) (i) Craig is staying at the Circus (C) and has to visit the Oriental (O).

Use Dijkstra's algorithm on **Figure 2** to find the minimum time to walk from C to O. *(6 marks)*

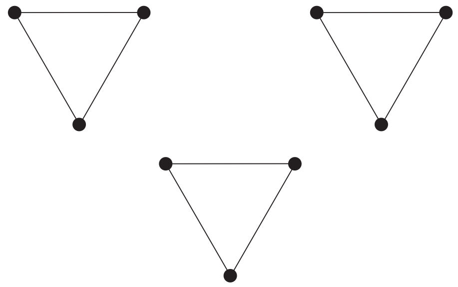
(ii) Write down the corresponding route. *(1 mark)*

(b) (i) Find, by inspection, the shortest time to walk from A to M. *(1 mark)*

(ii) Craig intends to walk along all the walkways. Find the minimum time for Craig to walk along every walkway and return to his starting point. *(6 marks)*

Turn over for the next question

8 (a) The diagram shows a graph **G** with 9 vertices and 9 edges.



(i) State the minimum number of edges that need to be added to **G** to make a connected graph. Draw an example of such a graph. *(2 marks)*

(ii) State the minimum number of edges that need to be added to **G** to make the graph Hamiltonian. Draw an example of such a graph. *(2 marks)*

(iii) State the minimum number of edges that need to be added to **G** to make the graph Eulerian. Draw an example of such a graph. *(2 marks)*

(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

(ii) In addition, the number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(1 mark)*

END OF QUESTIONS

Figure 1 (for use in Question 6)

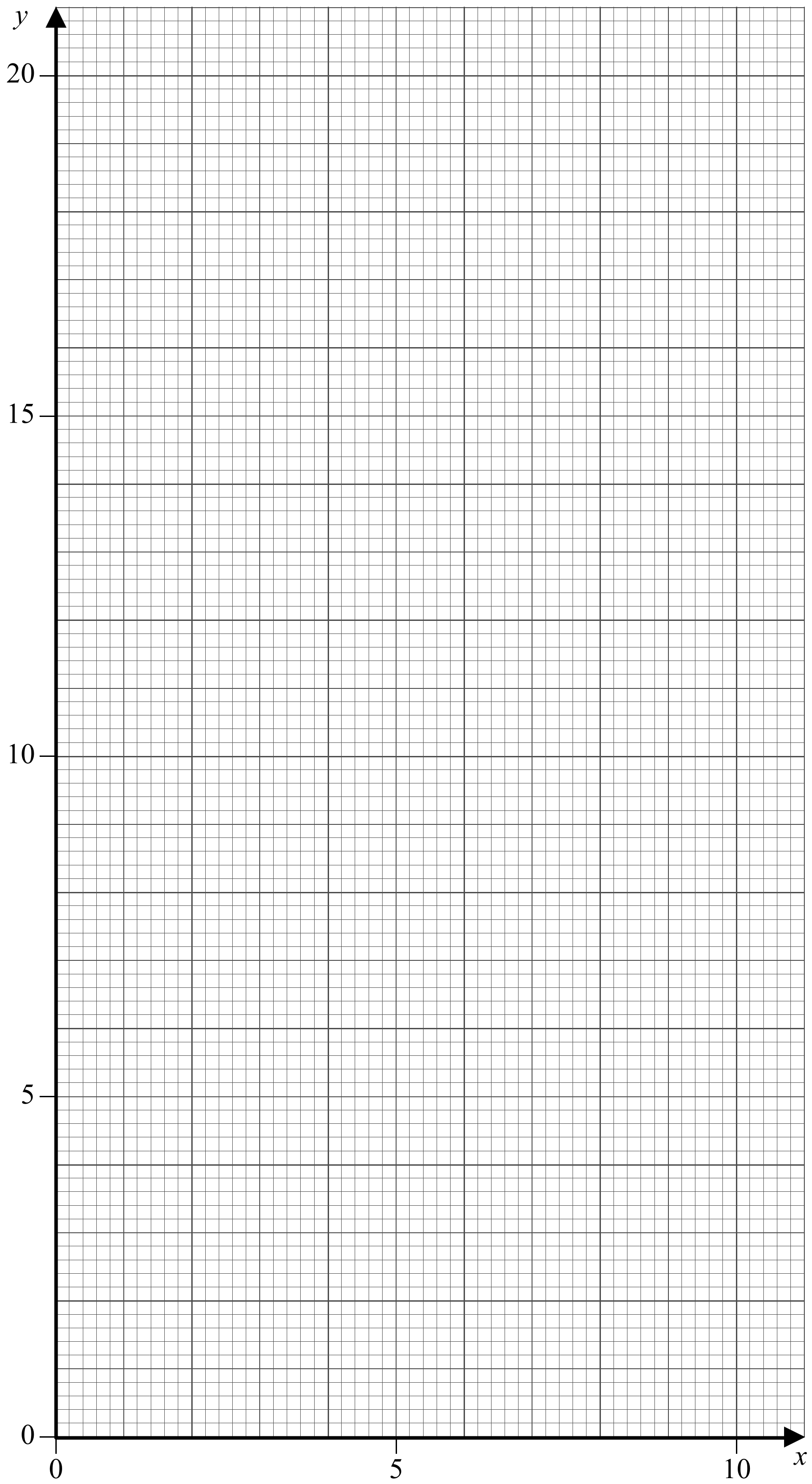
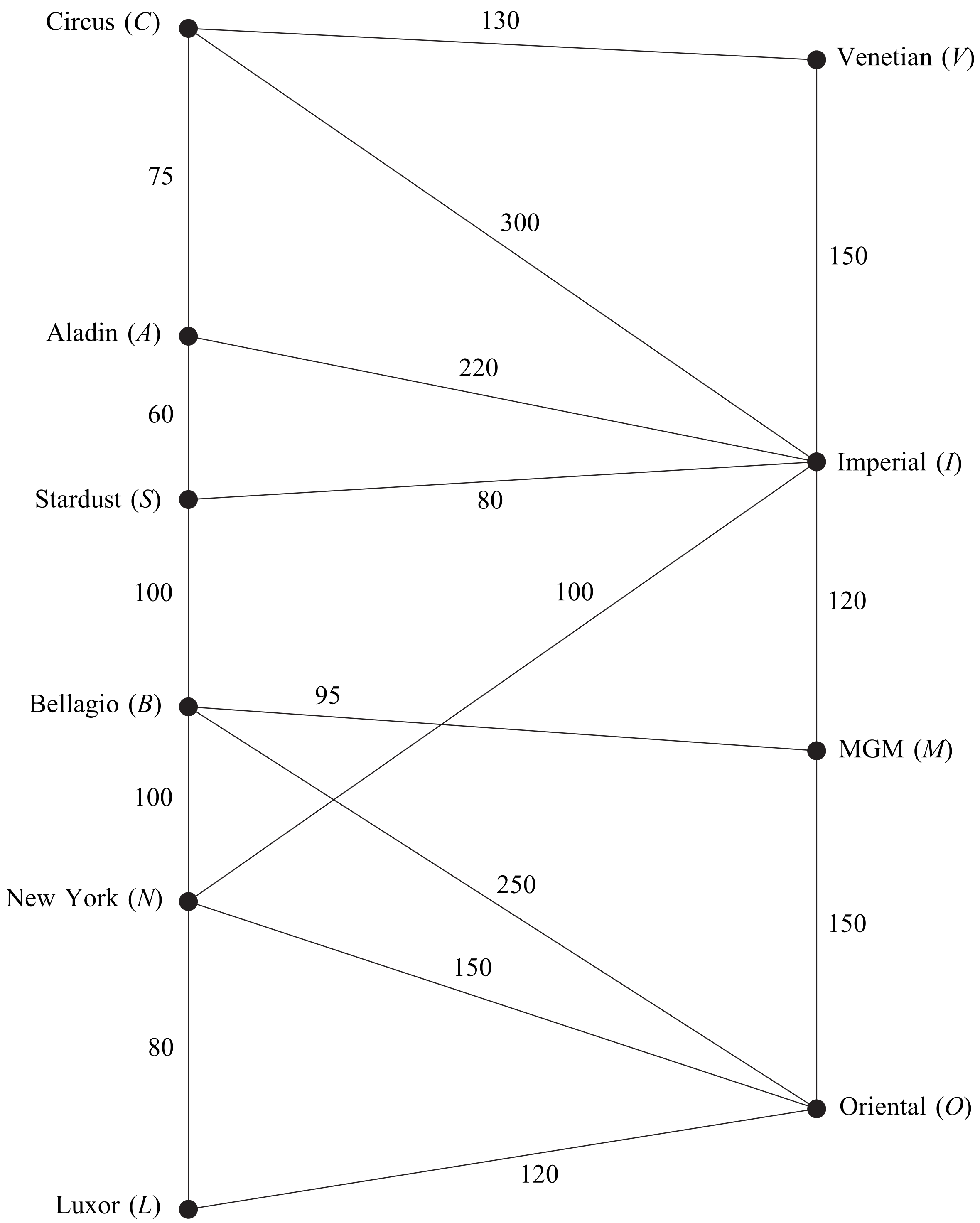


Figure 2 (for use in Question 7)



Answer **all** questions.

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**	**Task 6**
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	0	1	0	1	1
D	0	0	0	1	0	0
E	0	1	0	0	0	1
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) At first F insists on being matched to task 4. Explain why, in this case, a complete matching is impossible. *(1 mark)*

(c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*

2 (a) Use a Shell sort to rearrange the following numbers into ascending order, showing the new arrangement after each pass.

28 22 20 17 14 11 6 5 *(5 marks)*

(b) (i) Write down the number of comparisons on the first pass. *(1 mark)*

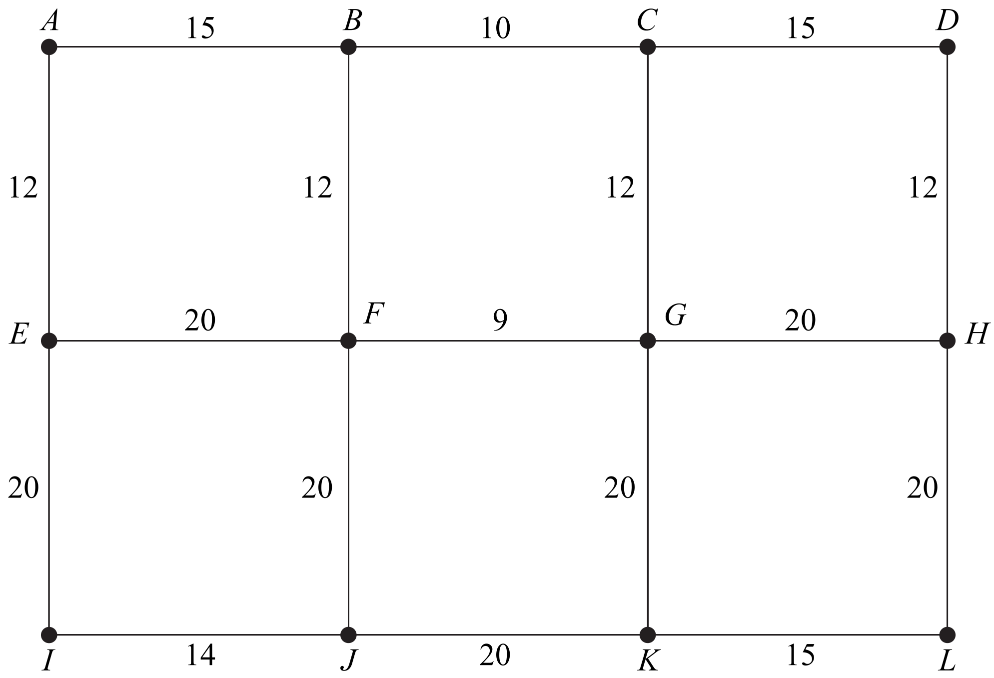
(ii) Write down the number of swaps on the first pass. *(1 mark)*

(c) Find the total number of comparisons needed to rearrange the original list of 8 numbers into ascending order using a shuttle sort.

(You do not need to perform a shuttle sort.) *(1 mark)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The following network represents the footpaths connecting 12 buildings on a university campus. The number on each edge represents the time taken, in minutes, to walk along a footpath.



(a) (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from A to L. *(7 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) A new footpath is to be constructed. There are two possibilities:

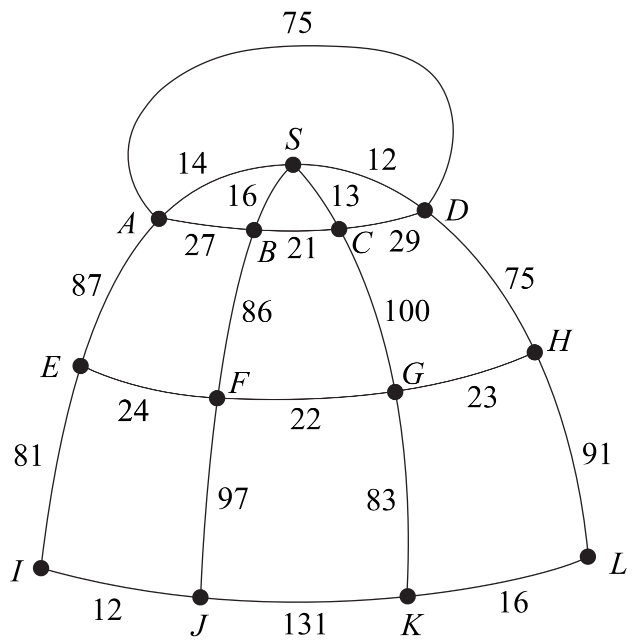
from A to D, with a walking time of 30 minutes; or

from A to I, with a walking time of 20 minutes.

Determine which of the two alternative new footpaths would reduce the walking time from A to L by the greater amount. *(3 marks)*

4 The diagram shows the various ski-runs at a ski resort. There is a shop at S. The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points A, B,...,L and at the shop at S.

The number on each edge represents the distance, in metres, between two points.



Total of all edges $= 1135$

(a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points A, B,...,L and the shop at S.

(i) Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(3 marks)*

(iv) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree. *(2 marks)*

(b) At the end of each day a snow plough has to drive at least once along each edge shown in the diagram in preparation for the following day's skiing. The snow plough must start and finish at the point L.

Use the Chinese Postman algorithm to find the minimum distance that the snow plough must travel. *(6 marks)*

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

The Jolly Company sells two types of party pack: excellent and luxury.

Each excellent pack has five balloons and each luxury pack has ten balloons.

Each excellent pack has 32 sweets and each luxury pack has 8 sweets.

The company has 1500 balloons and 4000 sweets available.

The company sells at least 50 of each type of pack and at least 140 packs in total.

The company sells x excellent packs and y luxury packs.

(a) Show that the above information can be modelled by the following inequalities.

$x + 2y \leqslant 300$, $\quad 4x + y \leqslant 500$, $\quad x \geqslant 50$, $\quad y \geqslant 50$, $\quad x + y \geqslant 140$ *(4 marks)*

(b) The company sells each excellent pack for 80p and each luxury pack for £1.20. The company needs to find its minimum and maximum total income.

(i) On **Figure 2**, draw a suitable diagram to enable this linear programming problem to be solved graphically, indicating the feasible region and an objective line. *(8 marks)*

(ii) Find the company's maximum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

(iii) Find the company's minimum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

Turn over for the next question

6 (a) Mark is staying at the Grand Hotel (G) in Oslo. He is going to visit four famous places in Oslo: Aker Brygge (A), the National Theatre (N), Parliament House (P) and the Royal Palace (R).

The figures in the table represent the walking times, in seconds, between the places.

	Grand Hotel (G)	**Aker Brygge (A)**	**National Theatre (N)**	**Parliament House (P)**	**Royal Palace (R)**
Grand Hotel (G)	–	165	185	65	160
Aker Brygge (A)	165	–	155	115	275
National Theatre (N)	185	155	–	205	125
Parliament House (P)	65	115	205	–	225
Royal Palace (R)	160	275	125	225	–

Mark is to start his tour from the Grand Hotel, visiting each place once before returning to the Grand Hotel. Mark wishes to keep his walking time to a minimum.

(i) Use the nearest neighbour algorithm, starting from the Grand Hotel, to find an upper bound for the walking time for Mark's tour. *(4 marks)*

(ii) By deleting the Grand Hotel, find a lower bound for the walking time for Mark's tour. *(5 marks)*

(iii) The walking time for an optimal tour is T seconds. Use your answers to parts (a)(i) and (a)(ii) to write down a conclusion about T. *(1 mark)*

(b) Mark then intends to start from the Grand Hotel (G), visit three museums, Ibsen (I), Munch (M) and Viking (V), and return to the Grand Hotel. He uses public transport. The table shows the minimum travelling times, in minutes, between the places.

From \ To	**Grand Hotel (G)**	**Ibsen (I)**	**Munch (M)**	**Viking (V)**
Grand Hotel (G)	–	20	17	30
Ibsen (I)	15	–	32	16
Munch (M)	26	18	–	21
Viking (V)	19	27	24	–

(i) Find the length of the tour $GIMVG$. *(1 mark)*

(ii) Find the length of the tour $GVMIG$. *(1 mark)*

(iii) Find the number of different possible tours for Mark. *(1 mark)*

(iv) Write down the number of different possible tours for Mark if he were to visit n museums, starting and finishing at the Grand Hotel. *(1 mark)*

END OF QUESTIONS

Figure 1 (for use in Question 3)

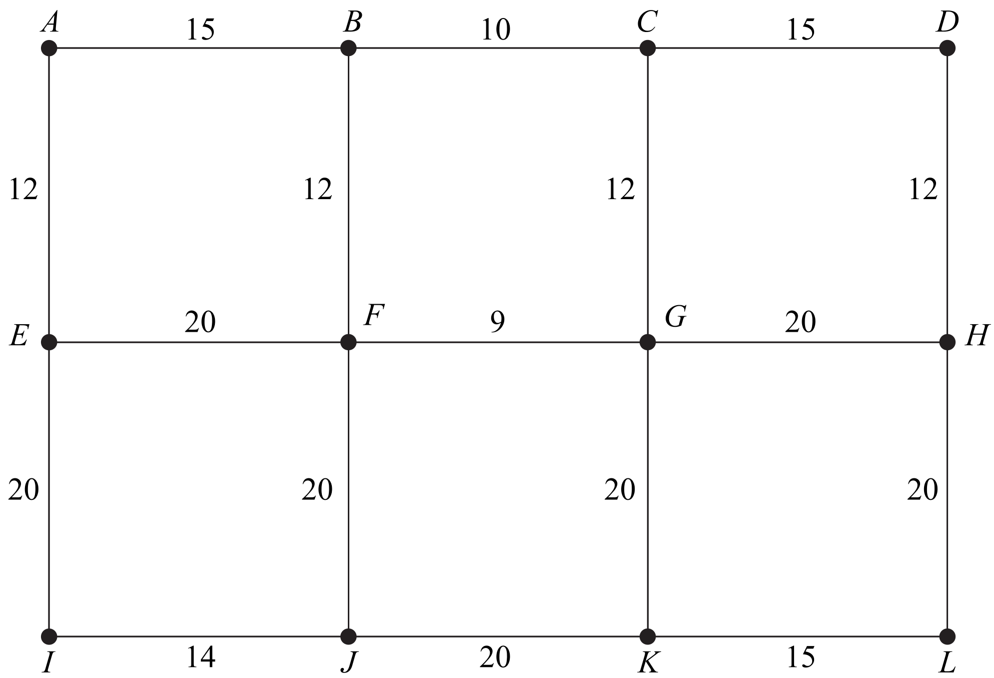
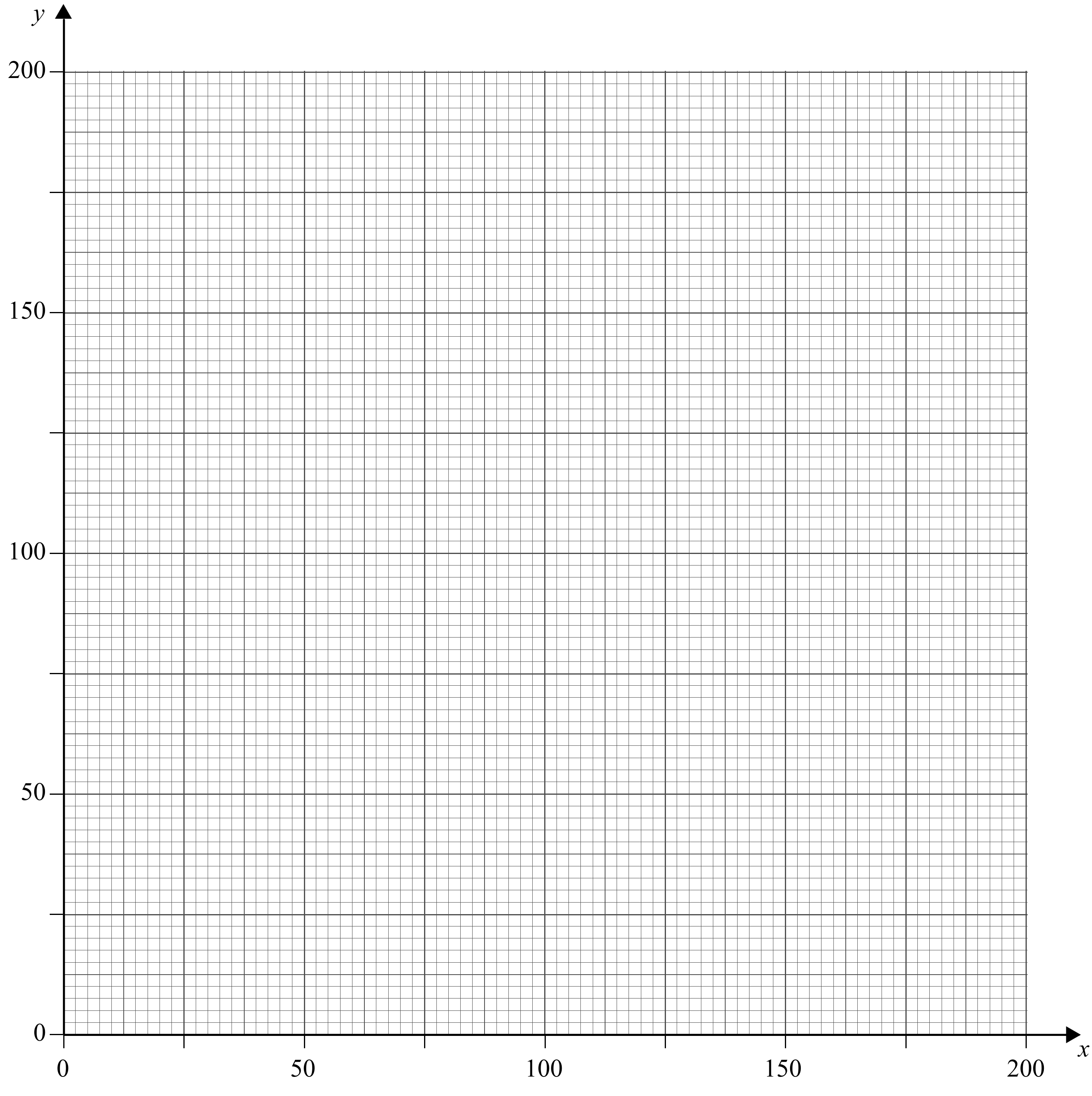


Figure 2 (for use in Question 5)



Answer **all** questions.

1 Five people, A, B, C, D and E, are to be matched to five tasks, J, K, L, M and N. The table shows the tasks that each person is able to undertake.

Person	**Task**
A	J, N
B	J, L
C	L, N
D	M, N
E	K, M

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task N, B to task J, C to task L, and E to task M.

Complete the alternating path $D-M$ ..., from this initial matching, to demonstrate how each person can be matched to a task. *(3 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by

$$x + y \leqslant 30$$
$$2x + y \leqslant 40$$
$$y \geqslant 5$$
$$x \geqslant 4$$
$$y \geqslant \tfrac{1}{2}x$$

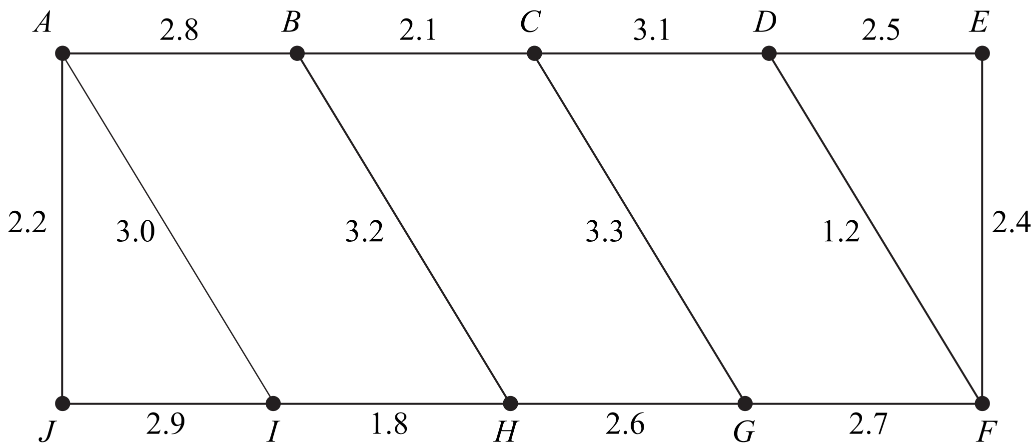
(a) On **Figure 1**, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) Use your diagram to find the maximum value of F, on the feasible region, in the case where:

(i) $F = 3x + y$; *(2 marks)*

(ii) $F = x + 3y$. *(2 marks)*

3 The diagram shows 10 bus stops, A, B, C, ..., J, in Geneva. The number on each edge represents the distance, in kilometres, between adjacent bus stops.



The city council is to connect these bus stops to a computer system which will display waiting times for buses at each of the 10 stops. Cabling is to be laid between some of the bus stops.

(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 bus stops. *(5 marks)*

(b) State the minimum length of cabling needed. *(1 mark)*

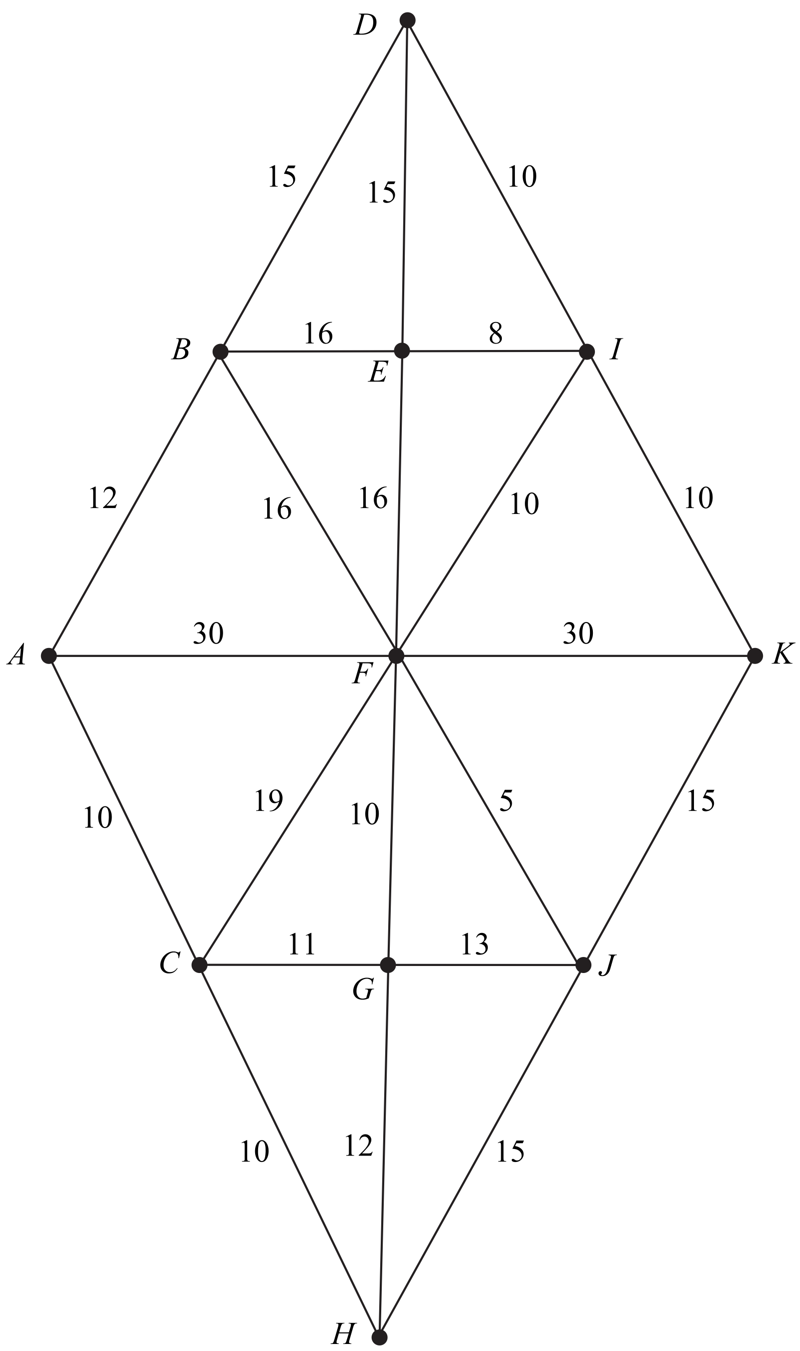
(c) Draw your minimum spanning tree. *(2 marks)*

(d) If Prim's algorithm, starting from A, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(2 marks)*

Turn over for the next question

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows 11 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



The total of all of the times is 308 minutes.

(a) (i) Use Dijkstra's algorithm on **Figure 2** to find the minimum time to travel from A to K. *(6 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) Find the length of an optimum Chinese postman route around the network, starting and finishing at A. (The minimum time to travel from D to H is 40 minutes.) *(5 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

(a) James is solving a travelling salesperson problem.

(i) He finds the following upper bounds: 43, 40, 43, 41, 55, 43, 43.

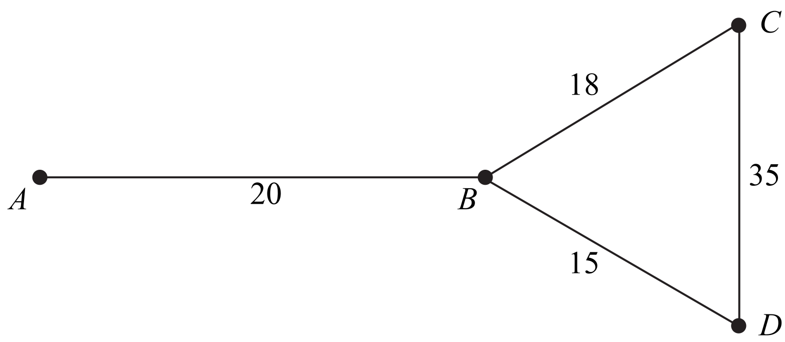
Write down the best upper bound. *(1 mark)*

(ii) James finds the following lower bounds: 33, 40, 33, 38, 33, 38, 38.

Write down the best lower bound. *(1 mark)*

(b) Karen is solving a different travelling salesperson problem and finds an upper bound of 55 and a lower bound of 45. Write down an interpretation of these results. *(1 mark)*

(c) The diagram below shows roads connecting 4 towns, A, B, C and D. The numbers on the edges represent the lengths of the roads, in kilometres, between adjacent towns.



Xiong lives at town A and is to visit each of the other three towns before returning to town A. She wishes to find a route that will minimise her travelling distance.

(i) Complete **Figure 3**, on the insert, to show the shortest distances, in kilometres, between **all** pairs of towns. *(2 marks)*

(ii) Use the nearest neighbour algorithm on **Figure 3** to find an upper bound for the minimum length of a tour of this network that starts and finishes at A. *(3 marks)*

(iii) Hence find the actual route that Xiong would take in order to achieve a tour of the same length as that found in part (c)(ii). *(2 marks)*

6 A student is solving cubic equations that have three different positive integer solutions.

The algorithm that the student is using is as follows:

Line 10	Input A, B, C, D
Line 20	Let $K = 1$
Line 30	Let $N = 0$
Line 40	Let $X = K$
Line 50	Let $Y = AX^3 + BX^2 + CX + D$
Line 60	If $Y \neq 0$ then go to Line 100
Line 70	Print X, "is a solution"
Line 80	Let $N = N + 1$
Line 90	If $N = 3$ then go to Line 120
Line 100	Let $K = K + 1$
Line 110	Go to Line 40
Line 120	End

(a) Trace the algorithm in the case where the input values are:

(i) $A = 1$, $B = -6$, $C = 11$ and $D = -6$; *(4 marks)*

(ii) $A = 1$, $B = -10$, $C = 29$ and $D = -20$. *(4 marks)*

(b) Explain where and why this algorithm will fail if $A = 0$. *(2 marks)*

7 The numbers 17, 3, 16 and 4 are to be sorted into ascending order.

The following four methods are to be compared: bubble sort, shuttle sort, Shell sort and quick sort (with the first number used as the pivot).

A student uses each of the four methods and produces the correct solutions below. Each solution shows the order of the numbers after each pass.

Solution 1	17	3	16	4
	3	17	16	4
	3	16	17	4
	3	4	16	17
Solution 2	17	3	16	4
	16	3	17	4
	3	4	16	17
Solution 3	17	3	16	4
	3	16	4	17
	3	16	4	17
	3	4	16	17
Solution 4	17	3	16	4
	3	16	4	17
	3	4	16	17
	3	4	16	17

(a) Write down which of the four solutions is the bubble sort, the shuttle sort, the Shell sort and the quick sort. *(3 marks)*

(b) For each of the four solutions, write down the number of comparisons and swaps (exchanges) on the first pass. *(8 marks)*

Turn over for the next question

8 Each day, a factory makes three types of hinge: basic, standard and luxury. The hinges produced need three different components: type A, type B and type C.

Basic hinges need 2 components of type A, 3 components of type B and 1 component of type C.

Standard hinges need 4 components of type A, 2 components of type B and 3 components of type C.

Luxury hinges need 3 components of type A, 4 components of type B and 5 components of type C.

Each day, there are 360 components of type A available, 270 of type B and 450 of type C.

Each day, the factory must use at least 720 components in total.

Each day, the factory must use at least 40% of the total components as type A.

Each day, the factory makes x basic hinges, y standard hinges and z luxury hinges.

In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, find five inequalities, each involving x, y and z, which must be satisfied. Simplify each inequality where possible. *(8 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 2)

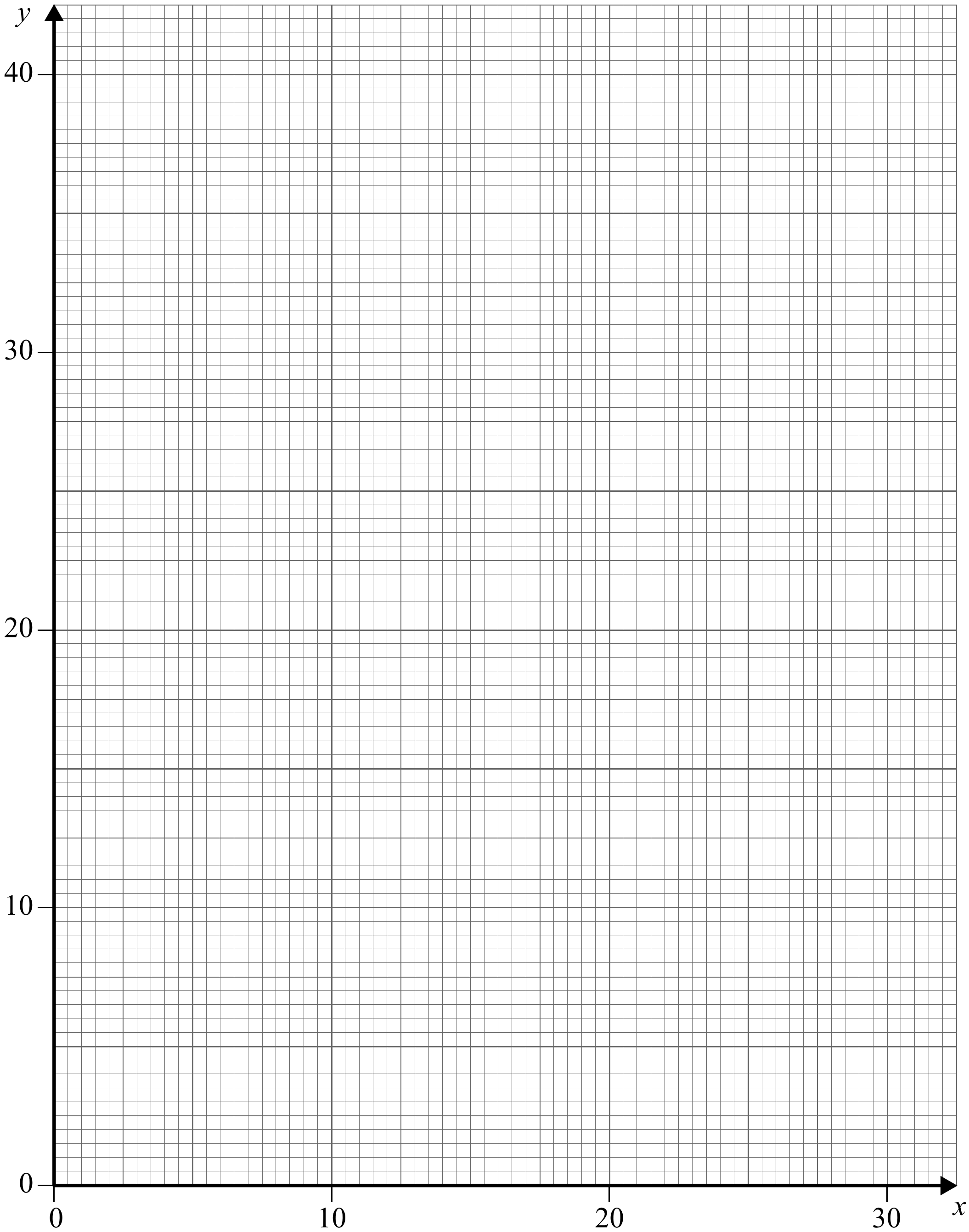


Figure 2 (for use in Question 4)

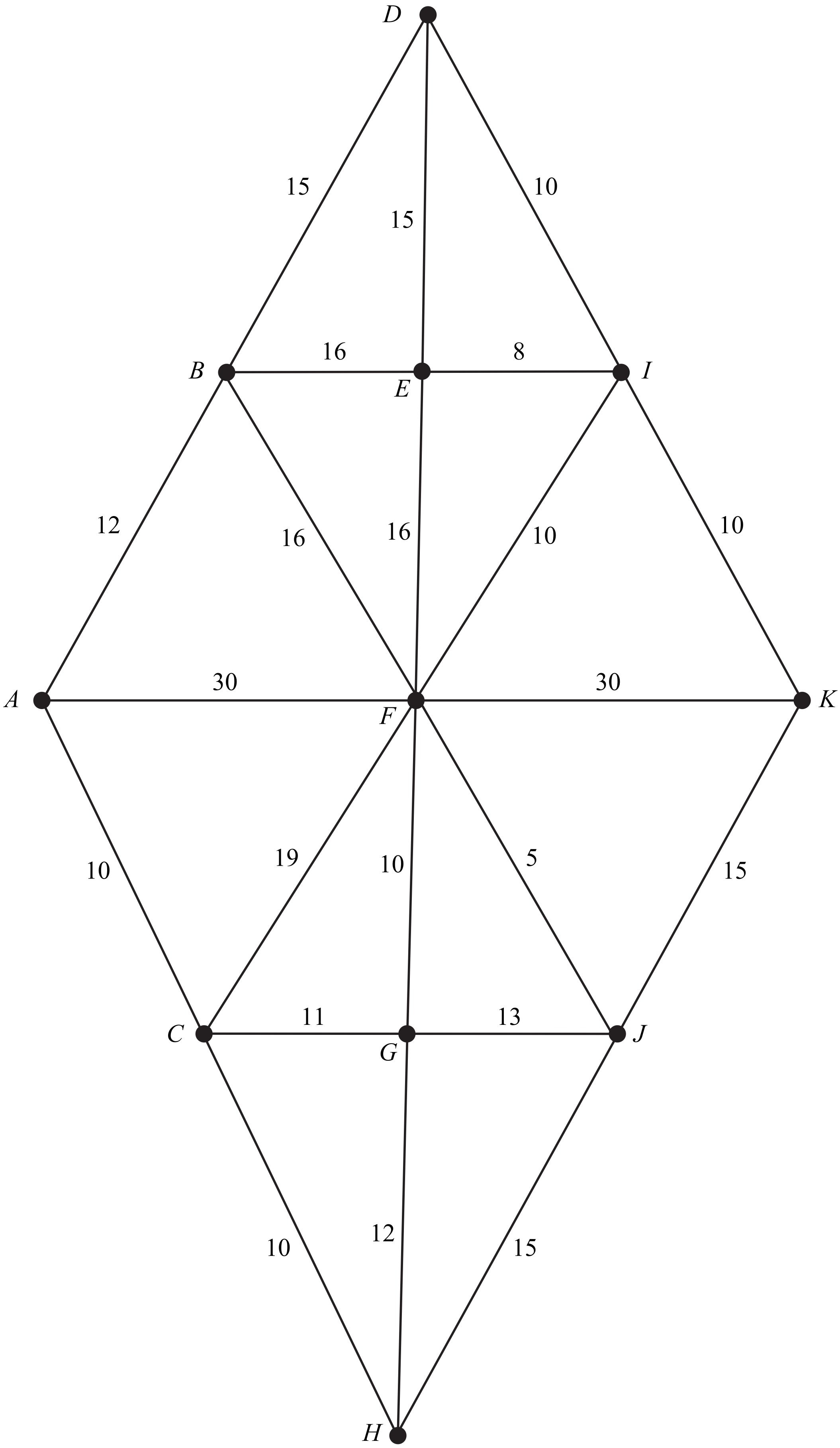


Figure 3 (for use in Question 5)

	A	*B*	*C*	*D*
A	—		38	
B		—		
C	38		—	
D				—