1 Find the average power exerted by a climber of mass 75 kg when climbing a vertical distance of 40 m in 2 minutes. **[3]**

2 A small sphere of mass 0.2 kg is dropped from rest at a height of 3 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.8 m above the ground.

(i) Calculate the magnitude of the impulse which the ground exerts on the sphere. **[5]**

(ii) Calculate the coefficient of restitution between the sphere and the ground. **[2]**

3

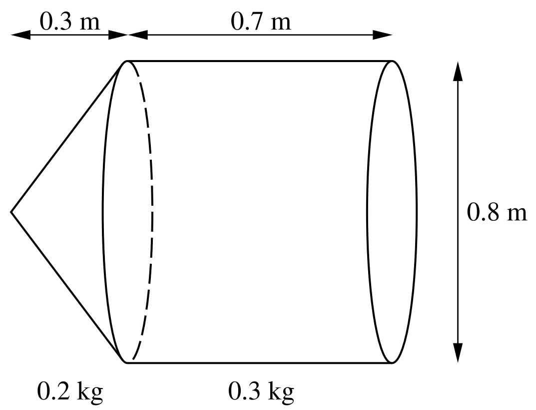


Fig. 1

A uniform conical shell has mass 0.2 kg, height 0.3 m and base diameter 0.8 m. A uniform hollow cylinder has mass 0.3 kg, length 0.7 m and diameter 0.8 m. The conical shell is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

(i) Show that the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m. **[4]**

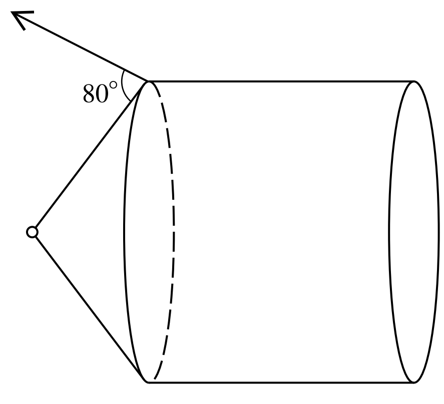


Fig. 2

The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of 80° with the slant edge of the conical shell (see Fig. 2).

(ii) Calculate the tension in the wire. **[4]**

4 A car of mass 700 kg is moving along a horizontal road against a constant resistance to motion of 400 N. At an instant when the car is travelling at $12\,\mathrm{m\,s^{-1}}$ its acceleration is $0.5\,\mathrm{m\,s^{-2}}$.

(i) Find the driving force of the car at this instant. **[2]**

(ii) Find the power at this instant. **[2]**

The maximum steady speed of the car on a horizontal road is $35\,\mathrm{m\,s^{-1}}$.

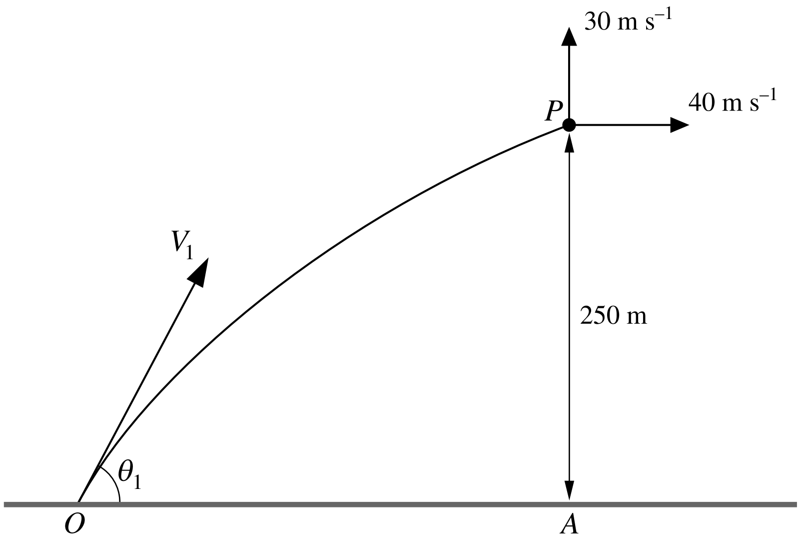
(iii) Find the maximum power of the car. **[2]**

The car now moves at maximum power against the same resistance up a slope of constant angle $\theta°$ to the horizontal. The maximum steady speed up the slope is $12\,\mathrm{m\,s^{-1}}$.

(iv) Find θ. **[4]**

5 Two spheres of the same radius with masses 2 kg and 3 kg are moving directly towards each other on a smooth horizontal plane with speeds $8\,\mathrm{m\,s^{-1}}$ and $4\,\mathrm{m\,s^{-1}}$ respectively. The spheres collide and the kinetic energy lost is 81 J. Calculate the speed and direction of motion of each sphere after the collision. **[12]**

6



A particle P is projected with speed $V_1\,\mathrm{m\,s^{-1}}$ at an angle of elevation θ_1 from a point O on horizontal ground. When P is vertically above a point A on the ground its height is 250 m and its velocity components are $40\,\mathrm{m\,s^{-1}}$ horizontally and $30\,\mathrm{m\,s^{-1}}$ vertically upwards (see diagram).

(i) Show that $V_1 = 86.0$ and $\theta_1 = 62.3°$, correct to 3 significant figures. **[5]**

At the instant when P is vertically above A, a second particle Q is projected from O with speed $V_2\,\mathrm{m\,s^{-1}}$ at an angle of elevation θ_2. P and Q hit the ground at the same time and at the same place.

(ii) Calculate the total time of flight of P and the total time of flight of Q. **[4]**

(iii) Calculate the range of the particles and hence calculate V_2 and θ_2. **[8]**

7

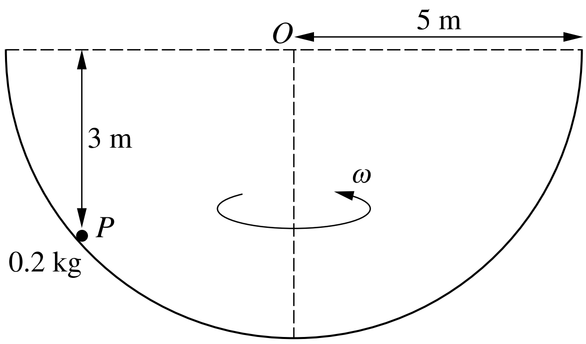


Fig. 1

A particle P of mass 0.2 kg is moving on the smooth inner surface of a fixed hollow hemisphere which has centre O and radius 5 m. P moves with constant angular speed ω in a horizontal circle at a vertical distance of 3 m below the level of O (see Fig. 1).

(i) Calculate the magnitude of the force exerted by the hemisphere on P. **[3]**

(ii) Calculate ω. **[4]**

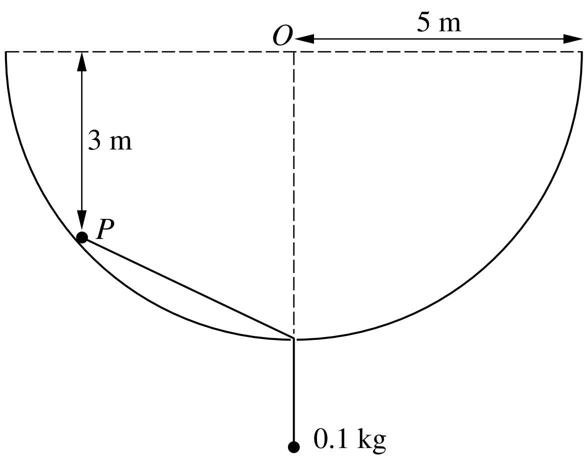


Fig. 2

A light inextensible string is now attached to P. The string passes through a small smooth hole at the lowest point of the hemisphere and a particle of mass 0.1 kg hangs in equilibrium at the end of the string. P moves in the same horizontal circle as before (see Fig. 2).

(iii) Calculate the new angular speed of P. **[8]**

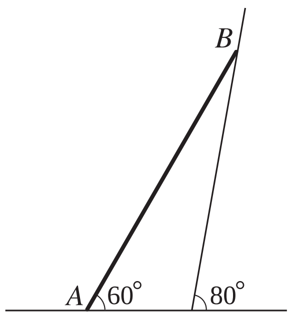
1 A ball is projected with speed $12\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of 55° above the horizontal. At the instant when the ball reaches its greatest height, it hits a vertical wall, which is perpendicular to the ball's path. The coefficient of restitution between the ball and the wall is 0.65. Calculate the speed of the ball

(i) immediately before its impact with the wall, [2]

(ii) immediately after its impact with the wall. [2]

2 A particle of mass m kg is projected directly up a rough plane with a speed of $5\,\text{m}\,\text{s}^{-1}$. The plane makes an angle of 30° with the horizontal and the coefficient of friction is 0.2. Calculate the distance the particle travels up the plane before coming instantaneously to rest. [6]

3



A uniform rod AB, of weight 25 N and length 1.6 m, rests in equilibrium in a vertical plane with the end A in contact with rough horizontal ground and the end B resting against a smooth wall which is inclined at 80° to the horizontal. The rod is inclined at 60° to the horizontal (see diagram). Calculate the magnitude of the force acting on the rod at B. [6]

4 A car of mass 1200 kg has a maximum speed of $30\,\text{m}\,\text{s}^{-1}$ when travelling on a horizontal road. The car experiences a resistance of kv N, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of the car and k is a constant. The maximum power of the car's engine is 45 000 W.

(i) Show that $k = 50$. [2]

(ii) Find the maximum possible acceleration of the car when it is travelling at $20\,\text{m}\,\text{s}^{-1}$ on a horizontal road. [3]

(iii) The car climbs a hill, which is inclined at an angle of 10° to the horizontal, at a constant speed of $15\,\text{m}\,\text{s}^{-1}$. Calculate the power of the car's engine. [3]

5 A particle P of mass $2m$ is moving on a smooth horizontal surface with speed u when it collides directly with a particle Q of mass km whose speed is $3u$ in the opposite direction. As a result of the collision, the directions of motion of both particles are reversed and the speed of P is halved.

(i) Find, in terms of u and k, the speed of Q after the collision. Hence write down the range of possible values of k. [4]

(ii) Calculate the magnitude of the impulse which Q exerts on P. [2]

(iii) Given that $k = \frac{1}{2}$, calculate the coefficient of restitution between P and Q. [3]

6 **(i)**

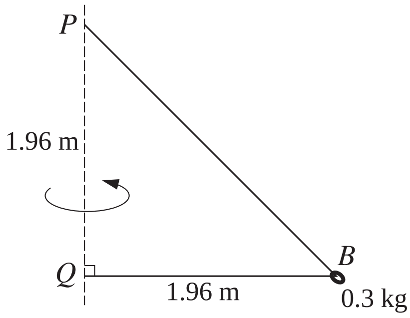


Fig. 1

One end of a light inextensible string is attached to a point P. The other end is attached to a point Q, 1.96 m vertically below P. A small smooth bead B, of mass 0.3 kg, is threaded on the string and moves in a horizontal circle with centre Q and radius 1.96 m. B rotates about Q with constant angular speed ω rad s^{-1} (see Fig. 1).

(a) Show that the tension in the string is 4.16 N, correct to 3 significant figures. [2]

(b) Calculate ω. [3]

(ii)

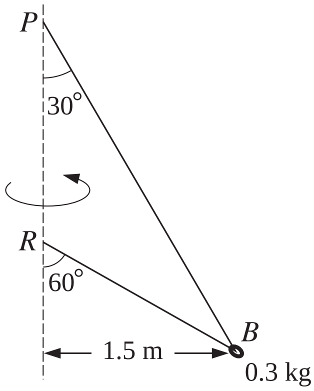


Fig. 2

The lower part of the string is now attached to a point R, vertically below P. PB makes an angle 30° with the vertical and RB makes an angle 60° with the vertical. The bead B now moves in a horizontal circle of radius 1.5 m with constant speed v m s^{-1} (see Fig. 2).

(a) Calculate the tension in the string. [3]

(b) Calculate v. [3]

7 A missile is projected from a point O on horizontal ground with speed 175 m s^{-1} at an angle of elevation θ. The horizontal lower surface of a cloud is 650 m above the ground.

(i) Find the value of θ for which the missile just reaches the cloud. [3]

It is given that $\theta = 55°$.

(ii) Find the length of time for which the missile is above the lower surface of the cloud. [5]

(iii) Find the speed of the missile at the instant it enters the cloud. [4]

8 **(i)** A uniform semicircular lamina has radius 4 cm. Show that the distance from its centre to its centre of mass is 1.70 cm, correct to 3 significant figures. [2]

(ii)

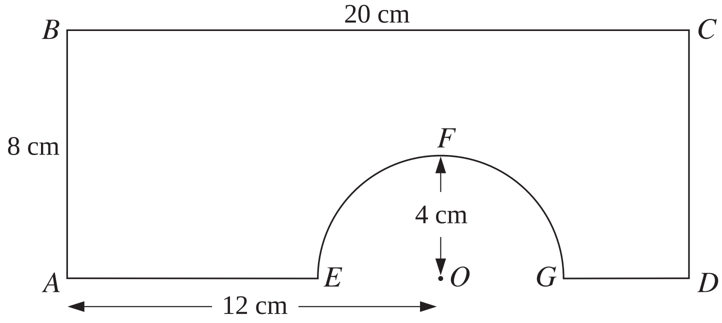


Fig. 1

A model bridge is made from a uniform rectangular board, $ABCD$, with a semicircular section, EFG, removed. O is the mid-point of EG. $AB = 8$ cm, $BC = 20$ cm, $AO = 12$ cm and the radius of the semicircle is 4 cm (see Fig. 1).

(a) Show that the distance from AB to the centre of mass of the model is 9.63 cm, correct to 3 significant figures. [5]

(b) Calculate the distance from AD to the centre of mass of the model. [4]

(iii)

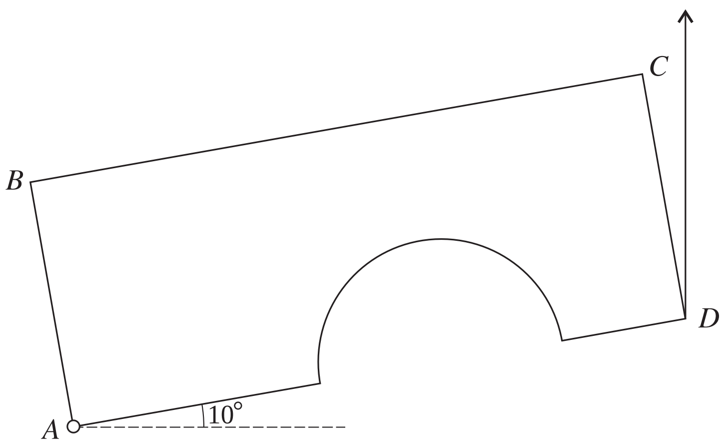


Fig. 2

The model bridge is smoothly pivoted at A and is supported in equilibrium by a vertical wire attached to D. The weight of the model is 15 N and AD makes an angle of $10°$ with the horizontal (see Fig. 2). Calculate the tension in the wire. [5]

1 A car is pulled at constant speed along a horizontal straight road by a force of 200 N inclined at 35° to the horizontal. Given that the work done by the force is 5000 J, calculate the distance moved by the car. [3]

2 A bullet of mass 9 grams passes horizontally through a fixed vertical board of thickness 3 cm. The speed of the bullet is reduced from $250\,\mathrm{m\,s^{-1}}$ to $150\,\mathrm{m\,s^{-1}}$ as it passes through the board. The board exerts a constant resistive force on the bullet. Calculate the magnitude of this resistive force. [4]

3 The resistance to the motion of a car of mass 600 kg is kv N, where $v\,\mathrm{m\,s^{-1}}$ is the car's speed and k is a constant. The car ascends a hill of inclination α, where $\sin\alpha = \frac{1}{10}$. The power exerted by the car's engine is 12 000 W and the car has constant speed $20\,\mathrm{m\,s^{-1}}$.

(i) Show that $k = 0.6$. [3]

The power exerted by the car's engine is increased to 16 000 W.

(ii) Calculate the maximum speed of the car while ascending the hill. [3]

The car now travels on horizontal ground and the power remains 16 000 W.

(iii) Calculate the acceleration of the car at an instant when its speed is $32\,\mathrm{m\,s^{-1}}$. [3]

4 A golfer hits a ball from a point O on horizontal ground with a velocity of $35\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal. The horizontal range of the ball is R metres and the time of flight is t seconds.

(i) Express t in terms of θ, and hence show that $R = 125\sin 2\theta$. [5]

The golfer hits the ball so that it lands 110 m from O.

(ii) Calculate the two possible values of t. [5]

5

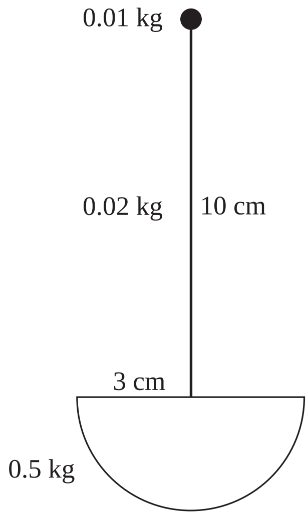


Fig. 1

A toy is constructed by attaching a small ball of mass 0.01 kg to one end of a uniform rod of length 10 cm whose other end is attached to the centre of the plane face of a uniform solid hemisphere with radius 3 cm. The rod has mass 0.02 kg, the hemisphere has mass 0.5 kg and the rod is perpendicular to the plane face of the hemisphere (see Fig. 1).

(i) Show that the distance from the ball to the centre of mass of the toy is 10.7 cm, correct to 1 decimal place. [4]

(ii)

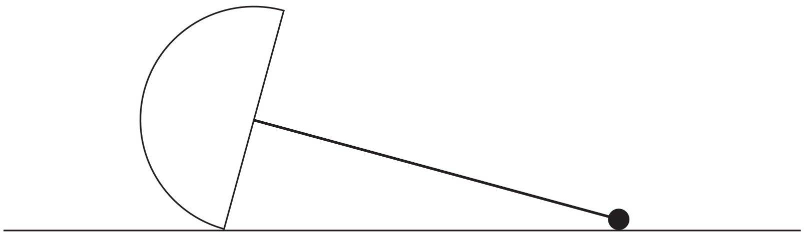
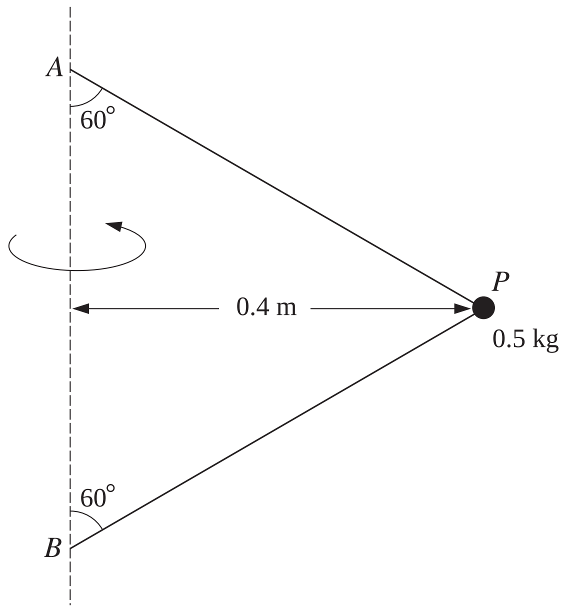


Fig. 2

The toy lies on horizontal ground in a position such that the ball is touching the ground (see Fig. 2). Determine whether the toy is lying in equilibrium or whether it will move to a position where the rod is vertical. [4]

6



A particle P of mass 0.5 kg is attached to points A and B on a fixed vertical axis by two light inextensible strings of equal length. Both strings are taut and each is inclined at 60° to the vertical (see diagram). The particle moves with constant speed $3\,\mathrm{m\,s^{-1}}$ in a horizontal circle of radius 0.4 m.

(i) Calculate the tensions in the two strings. [7]

The particle now moves with constant angular speed $\omega\,\mathrm{rad\,s^{-1}}$ and the string BP is on the point of becoming slack.

(ii) Calculate ω. [5]

7

Two small spheres A and B of masses 2 kg and 3 kg respectively lie at rest on a smooth horizontal platform which is fixed at a height of 4 m above horizontal ground (see diagram). Sphere A is given an impulse of 6 N s towards B, and A then strikes B directly. The coefficient of restitution between A and B is $\frac{2}{3}$.

(i) Show that the speed of B after it has been hit by A is $2\,\mathrm{m\,s^{-1}}$. [6]

Sphere B leaves the platform and follows the path of a projectile.

(ii) Calculate the speed and direction of motion of B at the instant when it hits the ground. [7]

8 **(i)**

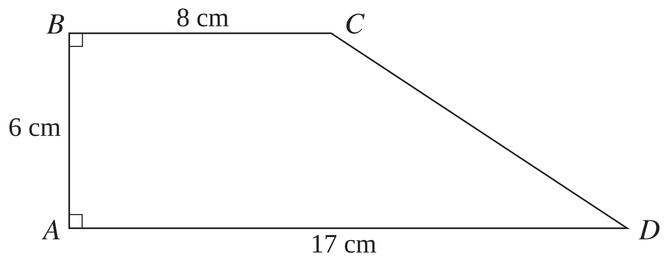


Fig. 1

A uniform lamina $ABCD$ is in the form of a right-angled trapezium. $AB = 6$ cm, $BC = 8$ cm and $AD = 17$ cm (see Fig. 1). Taking x- and y-axes along AD and AB respectively, find the coordinates of the centre of mass of the lamina. [8]

(ii)

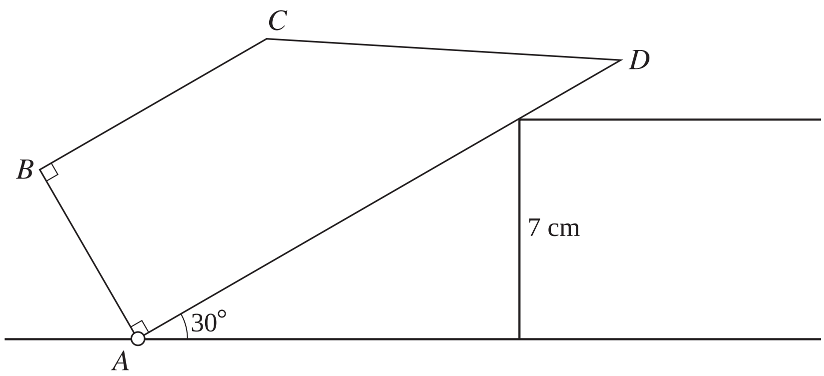


Fig. 2

The lamina is smoothly pivoted at A and it rests in a vertical plane in equilibrium against a fixed smooth block of height 7 cm. The mass of the lamina is 3 kg. AD makes an angle of 30° with the horizontal (see Fig. 2). Calculate the magnitude of the force which the block exerts on the lamina. [5]

1 A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of $35°$ with the horizontal and the tension in the rope is 40 N. Calculate the work done in moving the sack 100 m. [3]

2 Calculate the range on a horizontal plane of a small stone projected from a point on the plane with speed $12\,\mathrm{m\,s^{-1}}$ at an angle of elevation of $27°$. [4]

3 A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from $100\,\mathrm{m\,s^{-1}}$ to $150\,\mathrm{m\,s^{-1}}$ in a time t seconds.

(i) Calculate the value of t. [4]

(ii) Calculate the acceleration of the rocket at the instant when its speed is $120\,\mathrm{m\,s^{-1}}$. [4]

4 A ball is projected from a point O on the edge of a vertical cliff. The horizontal and vertically upward components of the initial velocity are $7\,\mathrm{m\,s^{-1}}$ and $21\,\mathrm{m\,s^{-1}}$ respectively. At time t seconds after projection the ball is at the point (x, y) referred to horizontal and vertically upward axes through O. Air resistance may be neglected.

(i) Express x and y in terms of t, and hence show that $y = 3x - \frac{1}{10}x^2$. [5]

The ball hits the sea at a point which is 25 m below the level of O.

(ii) Find the horizontal distance between the cliff and the point where the ball hits the sea. [3]

5 A cyclist and her bicycle have a combined mass of 70 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of $4\,\mathrm{m\,s^{-1}}$ at B. The level of B is 6 m above the level of A. For the cyclist's motion from A to B, find

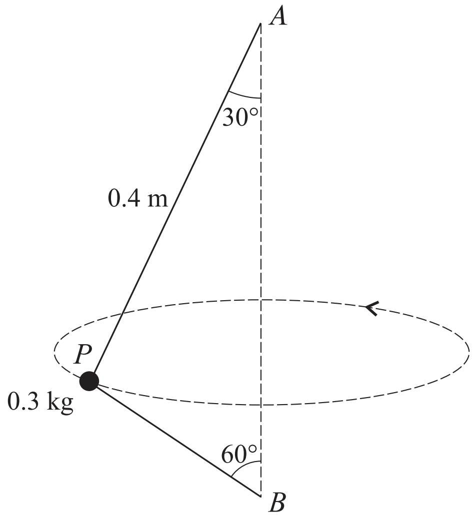
(i) the increase in kinetic energy, [2]

(ii) the increase in gravitational potential energy. [2]

During the ascent the resistance to motion is constant and has magnitude 60 N. The work done by the cyclist in moving from A to B is 8000 J.

(iii) Calculate the distance AB. [4]

6



A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings. The other end of the longer string is attached to a fixed point A and the other end of the shorter string is attached to a fixed point B, which is vertically below A. AP makes an angle of 30° with the vertical and is 0.4 m long. PB makes an angle of 60° with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut (see diagram). The tension in the string AP is 5 N.

Calculate

(i) the tension in the string PB, [3]

(ii) the angular speed of P, [3]

(iii) the kinetic energy of P. [3]

7 Two small spheres A and B, with masses 0.3 kg and m kg respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed $6\,\text{m s}^{-1}$ and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are $1\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively. The coefficient of restitution between A and B is e.

(i) Show that $m = 0.7$. [2]

(ii) Find e. [2]

B continues to move at $3\,\text{m s}^{-1}$ and strikes a vertical wall at right angles. The coefficient of restitution between B and the wall is f.

(iii) Find the range of values of f for which there will be a second collision between A and B. [2]

(iv) Find, in terms of f, the magnitude of the impulse that the wall exerts on B. [3]

(v) Given that $f = \frac{3}{4}$, calculate the final speeds of A and B, correct to 1 decimal place. [7]

[Question 8 is printed overleaf.]

[Turn over

8

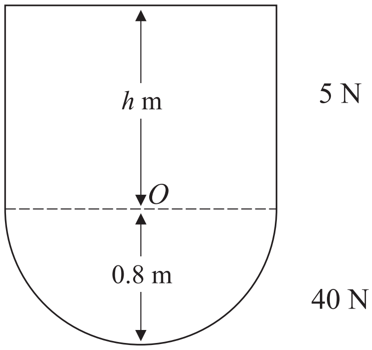


Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height h m. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point O (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from O.

(i) Show that $h = 1.2$. [6]

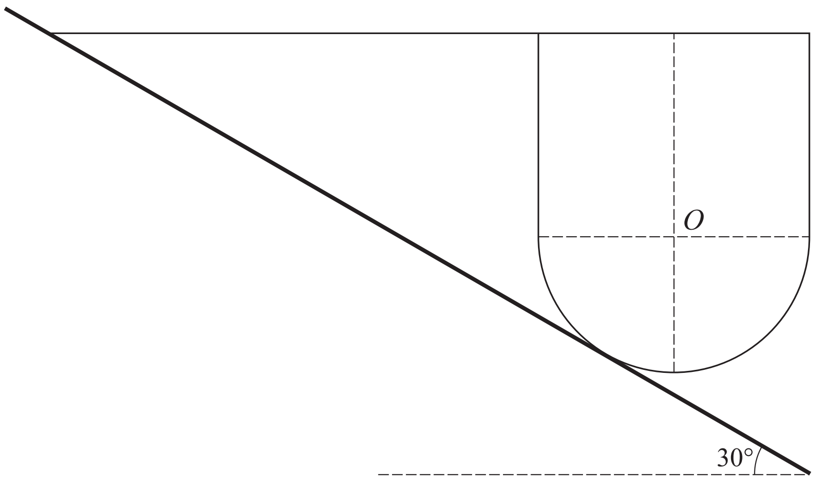


Fig. 2

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of 30° with the horizontal (see Fig. 2). The tension in the string is T N and the frictional force acting on the object is F N.

(ii) By taking moments about O, express F in terms of T. [4]

(iii) Find another equation connecting T and F. Hence calculate the tension and the frictional force. [6]

1 A particle is projected horizontally with a speed of 7 m s^{-1} from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. **[6]**

2 **(i)**

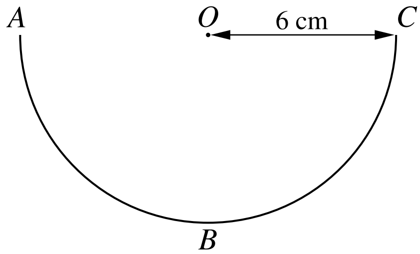


Fig. 1

A uniform piece of wire, ABC, forms a semicircular arc of radius 6 cm. O is the mid-point of AC (see Fig. 1). Show that the distance from O to the centre of mass of the wire is 3.82 cm, correct to 3 significant figures. **[2]**

(ii)

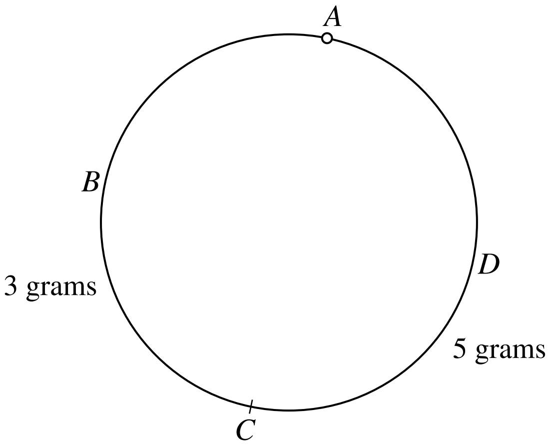


Fig. 2

Two semicircular pieces of wire, ABC and ADC, are joined together at their ends to form a circular hoop of radius 6 cm. The mass of ABC is 3 grams and the mass of ADC is 5 grams. The hoop is freely suspended from A (see Fig. 2). Calculate the angle which the diameter AC makes with the vertical, giving your answer correct to the nearest degree. **[5]**

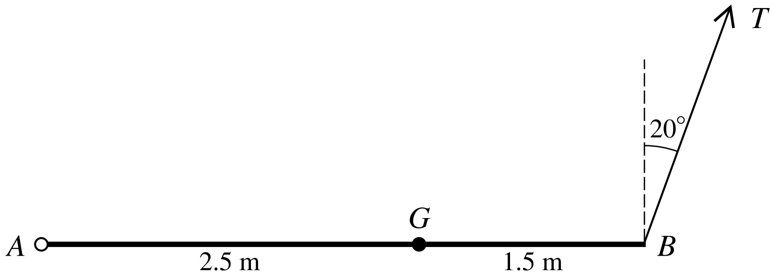
3 The maximum power produced by the engine of a small aeroplane of mass 2 tonnes is 128 kW. Air resistance opposes the motion directly and the lift force is perpendicular to the direction of motion. The magnitude of the air resistance is proportional to the square of the speed and the maximum steady speed in level flight is 80 m s^{-1}.

(i) Calculate the magnitude of the air resistance when the speed is 60 m s^{-1}. **[5]**

The aeroplane is climbing at a constant angle of $2°$ to the horizontal.

(ii) Find the maximum acceleration at an instant when the speed of the aeroplane is 60 m s^{-1}. **[4]**

4

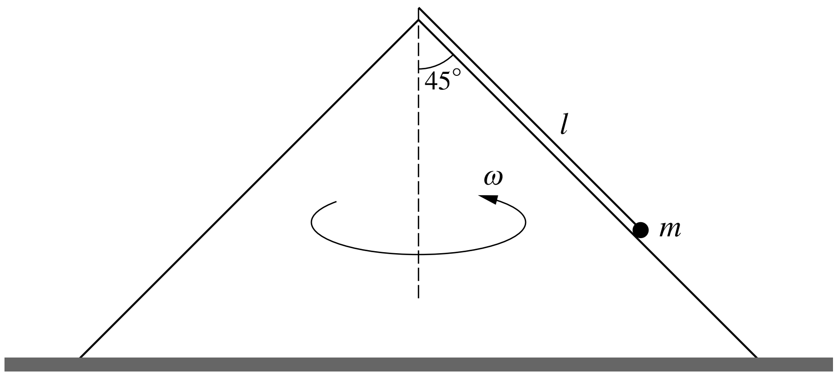


A non-uniform beam AB of length 4 m and mass 5 kg has its centre of mass at the point G of the beam where $AG = 2.5$ m. The beam is freely suspended from its end A and is held in a horizontal position by means of a wire attached to the end B. The wire makes an angle of 20° with the vertical and the tension is T N (see diagram).

(i) Calculate T. **[3]**

(ii) Calculate the magnitude and the direction of the force acting on the beam at A. **[7]**

5



One end of a light inextensible string of length l is attached to the vertex of a smooth cone of semi-vertical angle 45°. The cone is fixed to the ground with its axis vertical. The other end of the string is attached to a particle of mass m which rotates in a horizontal circle in contact with the outer surface of the cone. The angular speed of the particle is ω (see diagram). The tension in the string is T and the contact force between the cone and the particle is R.

(i) By resolving horizontally and vertically, find two equations involving T and R and hence show that $T = \frac{1}{2}m(\sqrt{2}g + l\omega^2)$. **[6]**

(ii) When the string has length 0.8 m, calculate the greatest value of ω for which the particle remains in contact with the cone. **[4]**

[Questions 6 and 7 are printed overleaf.]

6 A particle A of mass $2m$ is moving with speed u on a smooth horizontal surface when it collides with a stationary particle B of mass m. After the collision the speed of A is v, the speed of B is $3v$ and the particles move in the same direction.

(i) Find v in terms of u. **[3]**

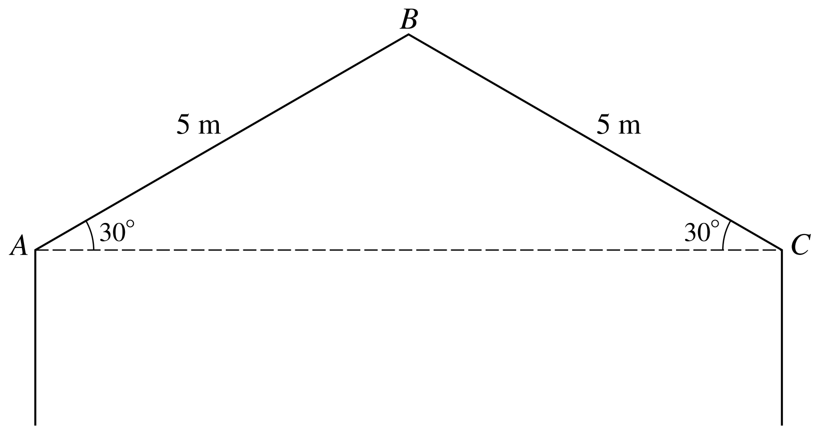
(ii) Show that the coefficient of restitution between A and B is $\frac{4}{5}$. **[2]**

B subsequently hits a vertical wall which is perpendicular to the direction of motion. As a result of the impact, B loses $\frac{3}{4}$ of its kinetic energy.

(iii) Show that the speed of B after hitting the wall is $\frac{3}{5}u$. **[4]**

(iv) B then hits A. Calculate the speeds of A and B, in terms of u, after this collision and state their directions of motion. **[8]**

7



A small ball of mass 0.2 kg is projected with speed $11\,\mathrm{m\,s^{-1}}$ up a line of greatest slope of a roof from a point A at the bottom of the roof. The ball remains in contact with the roof and moves up the line of greatest slope to the top of the roof at B. The roof is rough and the coefficient of friction is $\frac{1}{2}$. The distance AB is 5 m and AB is inclined at $30°$ to the horizontal (see diagram).

(i) Show that the speed of the ball when it reaches B is $5.44\,\mathrm{m\,s^{-1}}$, correct to 2 decimal places. **[6]**

The ball leaves the roof at B and moves freely under gravity. The point C is at the lower edge of the roof. The distance BC is 5 m and BC is inclined at $30°$ to the horizontal.

(ii) Determine whether or not the ball hits the roof between B and C. **[7]**