1 A particle P is moving with simple harmonic motion in a straight line. The period is 6.1 s and the amplitude is 3 m. Calculate, in either order,

(i) the maximum speed of P, [3]

(ii) the distance of P from the centre of motion when P has speed $2.5\,\text{m s}^{-1}$. [3]

2 A tennis ball of mass 0.057 kg has speed $10\,\text{m s}^{-1}$. The ball receives an impulse of magnitude 0.6 N s which reduces the speed of the ball to $7\,\text{m s}^{-1}$. Using an impulse-momentum triangle, or otherwise, find the angle the impulse makes with the original direction of motion of the ball. [7]

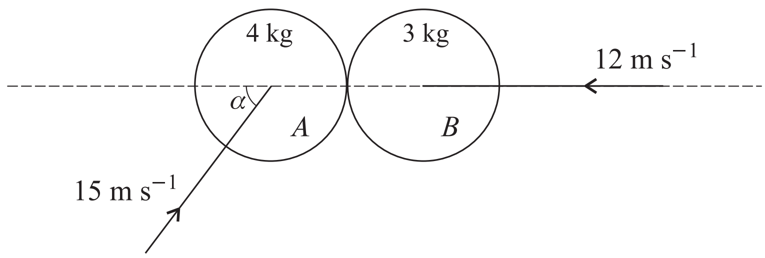
3 A particle P of mass 0.2 kg is projected horizontally with speed $u\,\text{m s}^{-1}$ from a fixed point O on a smooth horizontal surface. P moves in a straight line and, at time t s after projection, P has speed $v\,\text{m s}^{-1}$ and is x m from O. The only force acting on P has magnitude $0.4v^2$ N and is directed towards O.

(i) Show that $\frac{1}{v}\frac{dv}{dx} = -2$. [2]

(ii) Hence show that $v = ue^{-2x}$. [4]

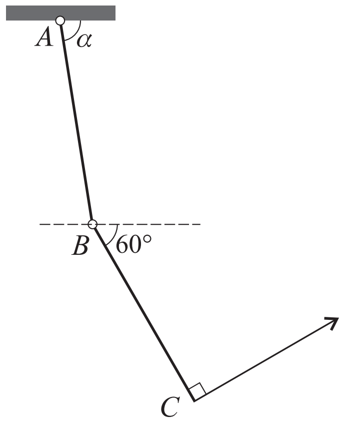
(iii) Find u, given that $x = 2$ when $t = 4$. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 3 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, A is moving with speed $15\,\text{m s}^{-1}$ at an angle α to the line of centres, where $\sin\alpha = 0.8$, and B is moving along the line of centres with speed $12\,\text{m s}^{-1}$ (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [10]

5

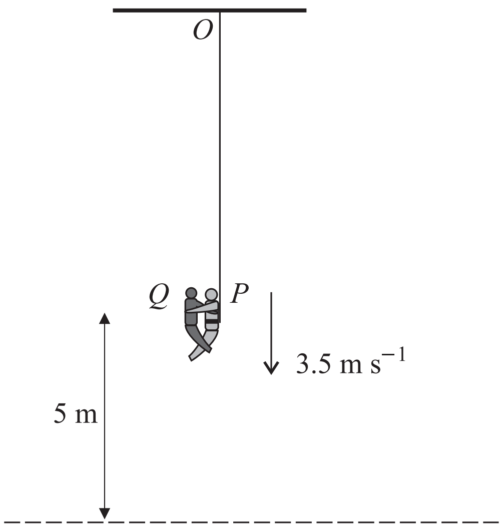


Two uniform rods AB and BC, each of length 1.4 m and weight 80 N, are freely jointed to each other at B, and AB is freely jointed to a fixed point at A. They are held in equilibrium with AB at an angle α to the horizontal, and BC at an angle of 60° to the horizontal, by a light string, perpendicular to BC, attached to C (see diagram).

(i) By taking moments about B for BC, calculate the tension in the string. Hence find the horizontal and vertical components of the force acting on BC at B. [7]

(ii) Find α. [4]

6



A circus performer P of mass 80 kg is suspended from a fixed point O by an elastic rope of natural length 5.25 m and modulus of elasticity 2058 N. P is in equilibrium at a point 5 m above a safety net. A second performer Q, also of mass 80 kg, falls freely under gravity from a point above P. P catches Q and together they begin to descend vertically with initial speed 3.5 m s^{-1} (see diagram). The performers are modelled as particles.

(i) Show that, when P is in equilibrium, $OP = 7.25$ m. [3]

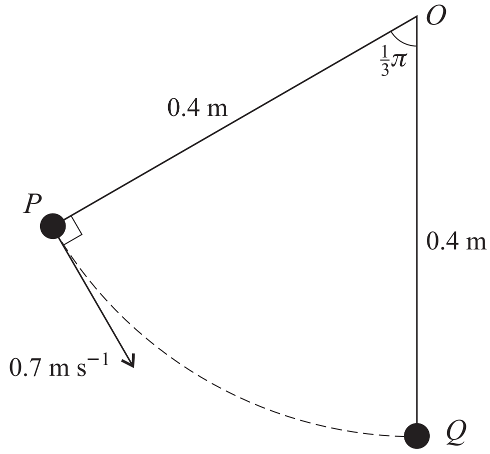
(ii) Verify that P and Q together just reach the safety net. [5]

(iii) At the lowest point of their motion P releases Q. Prove that P subsequently just reaches O. [3]

(iv) State two additional modelling assumptions made when answering this question. [2]

[Turn over

7



A particle P of mass 0.8 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. A particle Q is suspended from O by an identical string. With the string OP taut and inclined at $\frac{1}{3}\pi$ radians to the vertical, P is projected with speed 0.7 m s^{-1} in a direction perpendicular to the string so as to strike Q directly (see diagram). The coefficient of restitution between P and Q is $\frac{1}{7}$.

(i) Calculate the tension in the string immediately after P is set in motion. [4]

(ii) Immediately after P and Q collide they have equal speeds and are moving in opposite directions. Show that Q starts to move with speed 0.15 m s^{-1}. [4]

(iii) Prove that before the second collision between P and Q, Q is moving with approximate simple harmonic motion. [5]

(iv) Hence find the time interval between the first and second collisions of P and Q. [2]

1 A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.8 m and modulus of elasticity $1.35mg$ N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P is held at rest at a point on the surface 3 m from O. The particle is then released. Find

(i) the initial acceleration of P, [3]

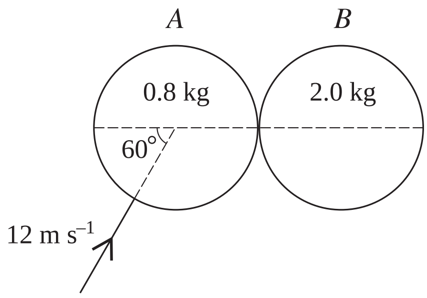
(ii) the speed of P at the instant the string becomes slack. [3]

2 A particle P of mass 0.2 kg is moving with speed 8 m s^{-1} when it hits a horizontal smooth surface. The direction of motion of P immediately before impact makes an angle of 27° with the surface. Given that the coefficient of restitution between the particle and the surface is 0.6, find

(i) the vertical component of the velocity of P immediately after impact, [3]

(ii) the magnitude of the impulse exerted on P. [3]

3



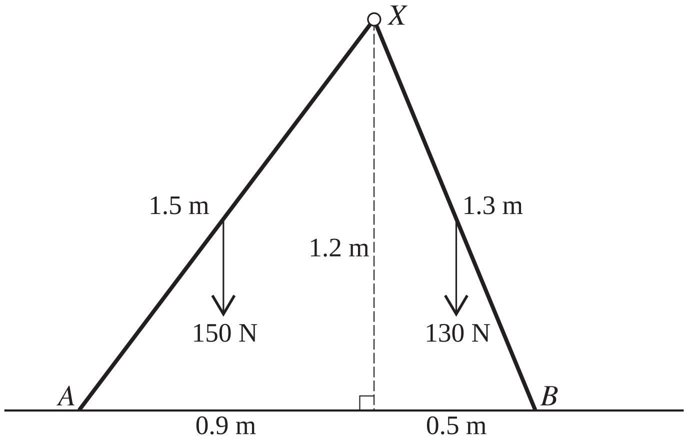
Two uniform smooth spheres A and B, of equal radius, have masses 0.8 kg and 2.0 kg respectively. The spheres are on a horizontal surface. A is moving with speed 12 m s^{-1} at 60° to the line of centres when it collides with B, which is stationary (see diagram). The coefficient of restitution between the spheres is 0.75. Find the speed and direction of motion of A immediately after the collision. [10]

4 A particle P of mass m kg is held at rest at a point O on a fixed plane inclined at an angle $\sin^{-1}\left(\frac{4}{7}\right)$ to the horizontal. P is released and moves down the plane. The total resistance acting on P is $0.2mv$ N, where v m s^{-1} is the velocity of P at time t s after leaving O.

(i) Show that $5\frac{dv}{dt} = 28 - v$ and hence find an expression for v in terms of t. [8]

(ii) Find the acceleration of P when $t = 10$. [2]

5



Two uniform rods XA and XB are freely jointed at X. The lengths of the rods are 1.5 m and 1.3 m respectively, and their weights are 150 N and 130 N respectively. The rods are in equilibrium in a vertical plane with A and B in contact with a rough horizontal surface. A and B are at distances horizontally from X of 0.9 m and 0.5 m respectively, and X is 1.2 m above the surface (see diagram).

(i) The normal components of the contact forces acting on the rods at A and B are R_A N and R_B N respectively. Show that $R_A = 125$ and find R_B. [4]

(ii) Find the frictional components of the contact forces acting on the rods at A and B. [4]

(iii) Find the horizontal and vertical components of the force exerted on XA at X, stating their directions. [3]

6 A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal surface. A force of $(0.36 - 0.144x)$ N acts on P in the direction from O to P, where x m is the displacement of P from a point O on the surface at time t s.

(i) By using the substitution $x = y + 2.5$, or otherwise, show that P moves with simple harmonic motion of period 5.24 s, correct to 3 significant figures. [5]

The maximum value of x during the motion is 3.

(ii) Write down the amplitude of P's motion and find the two possible values of x for which P's speed is $0.48\,\text{m s}^{-1}$. [4]

(iii) On each of the first two occasions when P has speed $0.48\,\text{m s}^{-1}$, P is moving towards O. Find the time interval between

(a) these first two occasions,

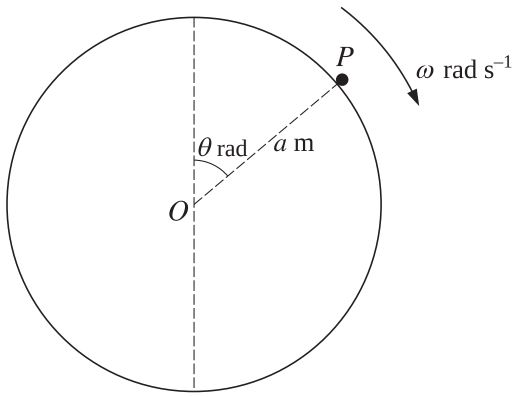
(b) the second and third occasions when P has speed $0.48\,\text{m s}^{-1}$.

[5]

[Question 7 is printed overleaf.]

[Turn over

7



A particle P of mass m kg is slightly disturbed from rest at the highest point on the surface of a smooth fixed sphere of radius a m and centre O. The particle starts to move downwards on the surface. While P remains on the surface OP makes an angle of θ radians with the upward vertical and has angular speed $\omega\,\mathrm{rad\,s^{-1}}$ (see diagram). The sphere exerts a force of magnitude R N on P.

(i) Show that $a\omega^2 = 2g(1 - \cos\theta)$. [3]

(ii) Find an expression for R in terms of m, g and θ. [4]

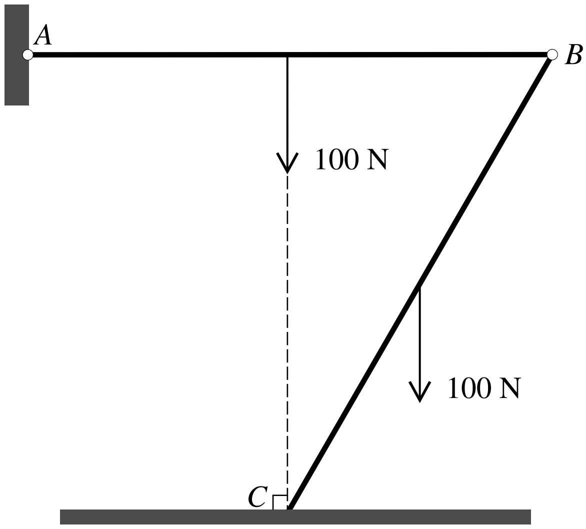
At the instant that P loses contact with the surface of the sphere, find

(iii) the transverse component of the acceleration of P, [4]

(iv) the rate of change of R with respect to time t, in terms of m, g and a. [4]

1 A small ball of mass 0.8 kg is moving with speed $10.5\,\mathrm{m\,s^{-1}}$ when it receives an impulse of magnitude 4 N s. The speed of the ball immediately afterwards is $8.5\,\mathrm{m\,s^{-1}}$. The angle between the directions of motion before and after the impulse acts is α. Using an impulse-momentum triangle, or otherwise, find α. **[6]**

2



Two uniform rods *AB* and *BC* are of equal length and each has weight 100 N. The rods are freely jointed to each other at *B*, and *A* is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with *AB* horizontal and *C* resting on a rough horizontal surface. *C* is vertically below the mid-point of *AB* (see diagram).

(i) By taking moments about *A* for *AB*, find the vertical component of the force on *AB* at *B*. Hence find the vertical component of the contact force on *BC* at *C*. **[3]**

(ii) Calculate the magnitude of the frictional force on *BC* at *C* and state its direction. **[4]**

3

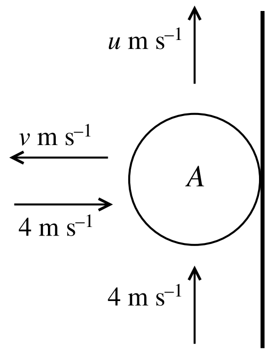


Fig. 1

A uniform smooth sphere A moves on a smooth horizontal surface towards a smooth vertical wall. Immediately before the sphere hits the wall it has components of velocity parallel and perpendicular to the wall each of magnitude $4\,\mathrm{m\,s^{-1}}$. Immediately after hitting the wall the components have magnitudes $u\,\mathrm{m\,s^{-1}}$ and $v\,\mathrm{m\,s^{-1}}$, respectively (see Fig. 1).

(i) Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, state the values of u and v. **[2]**

Shortly after hitting the wall the sphere A comes into contact with another uniform smooth sphere B, which has the same mass and radius as A. The sphere B is stationary and at the instant of contact the line of centres of the spheres is parallel to the wall (see Fig. 2). The contact between the spheres is perfectly elastic.

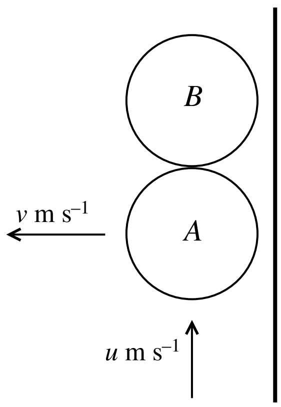


Fig. 2

(ii) Find, for each sphere, its speed and its direction of motion immediately after the contact. **[6]**

4 O is a fixed point on a horizontal plane. A particle P of mass 0.25 kg is released from rest at O and moves in a straight line on the plane. At time t s after release the only horizontal force acting on P has magnitude

$$\tfrac{1}{2400}\left(144 - t^2\right)\text{ N} \quad \text{for } 0 \leqslant t \leqslant 12$$

and

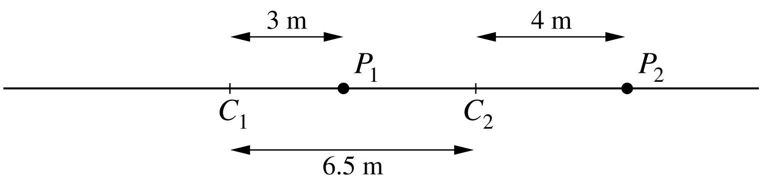
$$\tfrac{1}{2400}\left(t^2 - 144\right)\text{ N} \quad \text{for } t \geqslant 12.$$

The force acts in the direction of P's motion. P's velocity at time t s is $v\,\text{m s}^{-1}$.

(i) Find an expression for v in terms of t, valid for $t \geqslant 12$, and hence show that v is three times greater when $t = 24$ than it is when $t = 12$. **[8]**

(ii) Sketch the (t, v) graph for $0 \leqslant t \leqslant 24$. **[3]**

5



Particles P_1 and P_2 are each moving with simple harmonic motion along the same straight line. P_1's motion has centre C_1, period 2π s and amplitude 3 m; P_2's motion has centre C_2, period $\frac{4}{3}\pi$ s and amplitude 4 m. The points C_1 and C_2 are 6.5 m apart. The displacements of P_1 and P_2 from their centres of oscillation at time t s are denoted by x_1 m and x_2 m respectively. The diagram shows the positions of the particles at time $t = 0$, when $x_1 = 3$ and $x_2 = 4$.

(i) State expressions for x_1 and x_2 in terms of t, which are valid until the particles collide. **[3]**

The particles collide when $t = 5.99$, correct to 3 significant figures.

(ii) Find the distance travelled by P_2 before the collision takes place. **[4]**

(iii) Find the velocities of P_1 and P_2 immediately before the collision, and state whether the particles are travelling in the same direction or in opposite directions. **[4]**

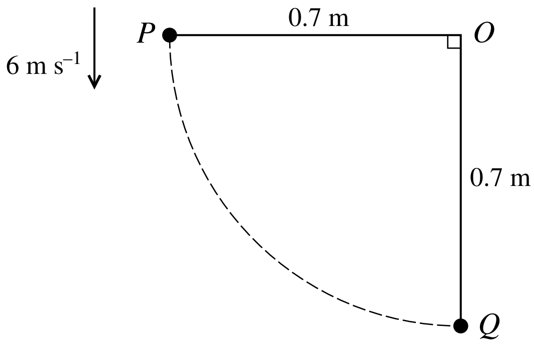
6 A bungee jumper of weight W N is joined to a fixed point O by a light elastic rope of natural length 20 m and modulus of elasticity 32 000 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Given that the jumper just reaches a point 25 m below O, find the value of W. **[5]**

(ii) Find the maximum speed reached by the jumper. **[4]**

(iii) Find the maximum value of the deceleration of the jumper during the downward motion. **[3]**

7



A particle P is attached to a fixed point O by a light inextensible string of length 0.7 m. A particle Q is in equilibrium suspended from O by an identical string. With the string OP taut and horizontal, P is projected vertically downwards with speed $6\,\mathrm{m\,s^{-1}}$ so that it strikes Q directly (see diagram). P is brought to rest by the collision and Q starts to move with speed $4.9\,\mathrm{m\,s^{-1}}$.

(i) Find the speed of P immediately before the collision. Hence find the coefficient of restitution between P and Q. **[3]**

(ii) Given that the speed of Q is $v\,\mathrm{m\,s^{-1}}$ when OQ makes an angle θ with the downward vertical, find an expression for v^2 in terms of θ, and show that the tension in the string OQ is $14.7m(1 + 2\cos\theta)$ N, where m kg is the mass of Q. **[6]**

(iii) Find the radial and transverse components of the acceleration of Q at the instant that the string OQ becomes slack. **[4]**

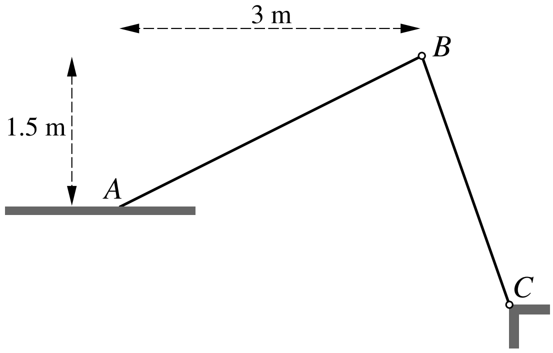
(iv) Show that $V^2 = 0.8575$, where $V\,\mathrm{m\,s^{-1}}$ is the speed of Q when it reaches its greatest height (after the string OQ becomes slack). Hence find the greatest height reached by Q above its initial position. **[4]**

1 A smooth sphere of mass 0.3 kg bounces on a fixed horizontal surface. Immediately before the sphere bounces the components of its velocity horizontally and vertically downwards are $4\,\text{m s}^{-1}$ and $6\,\text{m s}^{-1}$ respectively. The speed of the sphere immediately after it bounces is $5\,\text{m s}^{-1}$.

(i) Show that the vertical component of the velocity of the sphere immediately after impact is $3\,\text{m s}^{-1}$, and hence find the coefficient of restitution between the surface and the sphere. **[3]**

(ii) State the direction of the impulse on the sphere and find its magnitude. **[3]**

2

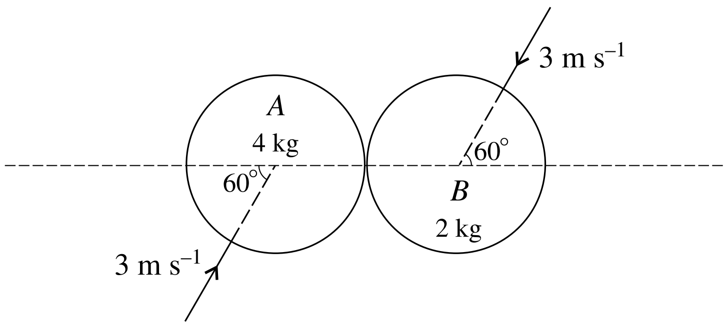


Two uniform rods, *AB* and *BC*, are freely jointed to each other at *B*, and *C* is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with *A* resting on a rough horizontal surface. This surface is 1.5 m below the level of *B* and the horizontal distance between *A* and *B* is 3 m (see diagram). The weight of *AB* is 80 N and the frictional force acting on *AB* at *A* is 14 N.

(i) Write down the horizontal component of the force acting on *AB* at *B* and show that the vertical component of this force is 33 N upwards. **[4]**

(ii) Given that the force acting on *BC* at *C* has magnitude 50 N, find the weight of *BC*. **[4]**

3



Two uniform smooth spheres *A* and *B*, of equal radius, have masses 4 kg and 2 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision both spheres have speed $3\,\text{m s}^{-1}$. The spheres are moving in opposite directions, each at 60° to the line of centres (see diagram). After the collision *A* moves in a direction perpendicular to the line of centres.

(i) Show that the speed of *B* is unchanged as a result of the collision, and find the angle that the new direction of motion of *B* makes with the line of centres. **[8]**

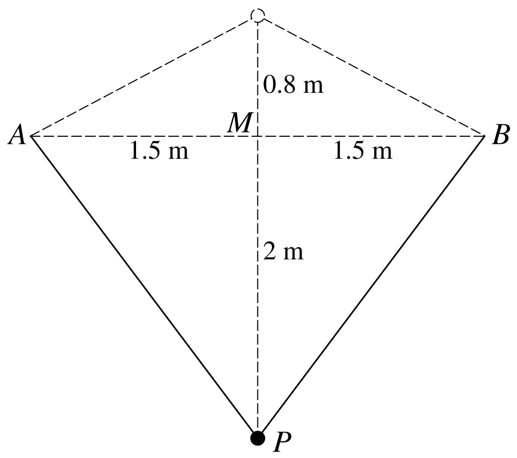
(ii) Find the coefficient of restitution between the spheres. **[2]**

4 A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point A with speed $40\,\mathrm{m\,s^{-1}}$ and when it has travelled a distance of x m beyond A its speed is $v\,\mathrm{m\,s^{-1}}$. The engine develops a constant power of 8 kW and resistances are modelled by a force of $0.25v^2$ N opposing the motion.

(i) Show that $\dfrac{480v^2}{v^3 - 32\,000}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[5]**

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond A. **[6]**

5



Each of two identical strings has natural length 1.5 m and modulus of elasticity 18 N. One end of one of the strings is attached to A and one end of the other string is attached to B, where A and B are fixed points which are 3 m apart and at the same horizontal level. M is the mid-point of AB. A particle P of mass m kg is attached to the other end of each of the strings. P is held at rest at the point 0.8 m vertically above M, and then released. The lowest point reached by P in the subsequent motion is 2 m below M (see diagram).

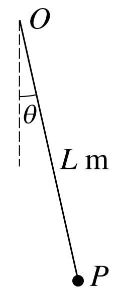
(i) Find the maximum tension in each of the strings during P's motion. **[3]**

(ii) By considering energy,

(a) show that the value of m is 0.42, correct to 2 significant figures, **[5]**

(b) find the speed of P at M. **[3]**

6



A particle P of mass m kg is attached to one end of a light inextensible string of length L m. The other end of the string is attached to a fixed point O. The particle is held at rest with the string taut and then released. P starts to move and in the subsequent motion the angular displacement of OP, at time t s, is θ radians from the downward vertical (see diagram). The initial value of θ is 0.05.

(i) Show that $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$. **[2]**

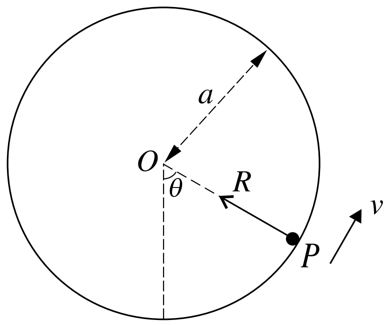
(ii) Hence show that the motion of P is approximately simple harmonic. **[2]**

(iii) Given that the period of the approximate simple harmonic motion is $\frac{4}{7}\pi$ s, find the value of L. **[2]**

(iv) Find the value of θ when $t = 0.7$ s, and the value of t when θ next takes this value. **[4]**

(v) Find the speed of P when $t = 0.7$ s. **[3]**

7



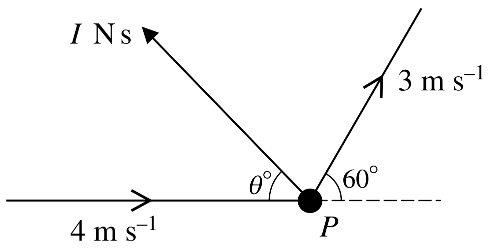
A hollow cylinder has internal radius a. The cylinder is fixed with its axis horizontal. A particle P of mass m is at rest in contact with the smooth inner surface of the cylinder. P is given a horizontal velocity u, in a vertical plane perpendicular to the axis of the cylinder, and begins to move in a vertical circle. While P remains in contact with the surface, OP makes an angle θ with the downward vertical, where O is the centre of the circle. The speed of P is v and the magnitude of the force exerted on P by the surface is R (see diagram).

(i) Find v^2 in terms of u, a, g and θ and show that $R = \frac{mu^2}{a} + mg(3\cos\theta - 2)$. **[7]**

(ii) Given that P just reaches the highest point of the circle, find u^2 in terms of a and g, and show that in this case the least value of v^2 is ag. **[4]**

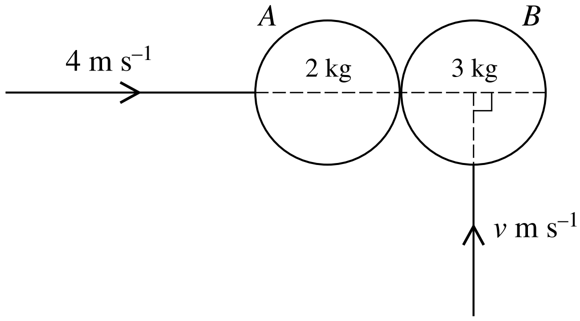
(iii) Given instead that P oscillates between $\theta = \pm\frac{1}{6}\pi$ radians, find u^2 in terms of a and g. **[2]**

1



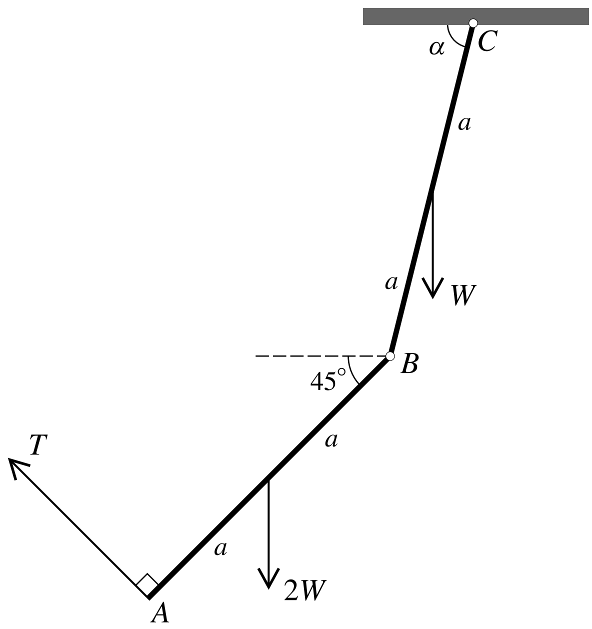
A particle P of mass 0.4 kg is moving horizontally with speed $4\,\text{m}\,\text{s}^{-1}$ when it receives an impulse of magnitude I N s, in a direction which makes an angle $(180 - \theta)^\circ$ with the direction of motion of P. Immediately after the impulse acts P moves horizontally with speed $3\,\text{m}\,\text{s}^{-1}$. The direction of motion of P is turned through an angle of 60° by the impulse (see diagram). Find I and θ. **[7]**

2



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed $4\,\text{m}\,\text{s}^{-1}$ and is moving along the line of centres, and B has speed $v\,\text{m}\,\text{s}^{-1}$ and is moving perpendicular to the line of centres (see diagram). The coefficient of restitution is 0.6. The direction of motion of B after the collision makes an angle of 45° with the line of centres. Find the value of v. **[7]**

3



Two uniform rods AB and BC, each of length $2a$, have weights $2W$ and W respectively. The rods are freely jointed to each other at B, and BC is freely jointed to a fixed point at C. The rods are held in equilibrium in a vertical plane by a light string attached to A and perpendicular to AB. The rods AB and BC make angles $45°$ and α, respectively, with the horizontal. The tension in the string is T (see diagram).

(i) By taking moments about B for AB, show that $W = \sqrt{2}T$. **[3]**

(ii) Find the value of $\tan\alpha$. **[6]**

4 A particle P of mass 0.2 kg travels in a straight line on a horizontal surface. It passes through a point O on the surface with speed $2\,\text{m s}^{-1}$. A resistive force of magnitude $0.2(v + v^2)$ N acts on P in the direction opposite to its motion, where $v\,\text{m s}^{-1}$ is the speed of P when it is at a distance x m from O.

(i) Show that $\frac{1}{1+v}\frac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[3]**

(ii) By solving the differential equation in part **(i)** show that $\frac{-\mathrm{e}^x}{3-\mathrm{e}^x}\frac{\mathrm{d}x}{\mathrm{d}t} = -1$, where t s is the time taken for P to travel x m from O. **[5]**

(iii) Hence find the value of t when $x = 1$. **[3]**

5 A light elastic string of natural length 1.6 m has modulus of elasticity 120 N. One end of the string is attached to a fixed point O and the other end is attached to a particle P of weight 1.5 N. The particle is released from rest at the point A, which is 2.1 m vertically below O. It comes instantaneously to rest at B, which is vertically above O.

(i) Verify that the distance AB is 4 m. **[4]**

(ii) Find the maximum speed of P during its upward motion from A to B. **[7]**

Turn over

6

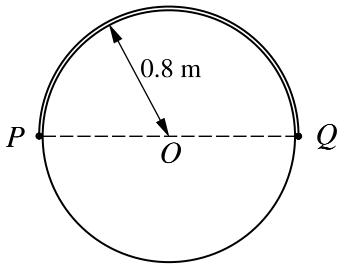


Fig. 1

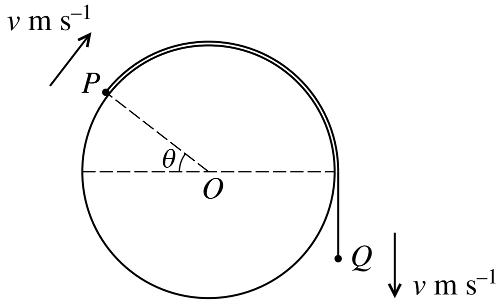


Fig. 2

A light inextensible string of length 0.8π m has particles P and Q, of masses 0.4 kg and 0.58 kg respectively, attached to its ends. The string passes over a smooth horizontal cylinder of radius 0.8 m, which is fixed with its axis horizontal and passing through a fixed point O. The string is held at rest in a vertical plane perpendicular to the axis of the cylinder, with P and Q at opposite ends of the horizontal diameter of the cylinder through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is $v\,\mathrm{m\,s^{-1}}$ (see Fig. 2).

(i) By considering the total energy of the system, obtain an expression for v^2 in terms of θ. **[5]**

(ii) Show that the magnitude of the force exerted on P by the cylinder is $(7.12\sin\theta - 4.64\theta)$ N. **[4]**

(iii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $1.53 < \alpha < 1.54$. **[4]**

7 A particle P of mass 0.5 kg is attached to one end of each of two identical light elastic strings of natural length 1.6 m and modulus of elasticity 19.6 N. The other ends of the strings are attached to fixed points A and B on a line of greatest slope of a smooth plane inclined at 30° to the horizontal. The distance AB is 4.8 m and A is higher than B.

(i) Find the distance AP for which P is in equilibrium on the line AB. **[5]**

P is released from rest at a point on AB where both strings are taut. The strings remain taut during the subsequent motion of P and t seconds after release the distance AP is $(2.5 + x)$ m.

(ii) Use Newton's second law to obtain an equation of the form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = kx$. State the property of the constant k for which the equation indicates that P's motion is simple harmonic, and find the period of this motion. **[5]**

(iii) Given that $x = 0.5$ when $t = 0$, find the values of x for which the speed of P is $2.8\,\mathrm{m\,s^{-1}}$. **[4]**