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Answer **all** questions.

1 Three forces $2\mathbf{i}$, $3\mathbf{i} - 5\mathbf{j} + a\mathbf{k}$ and $b\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ act at the points with coordinates $(1, 1, 0)$, $(0, 0, 0)$ and $(-1, 2, 1)$ respectively, where a and b are constants.

Given that the three forces form a couple, find:

(a) the values of a and b; *(2 marks)*

(b) the moment of the couple. *(4 marks)*

2 Forces $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$ act at the points with coordinates $(0, 0)$, $(0, 3)$, $(3, 4)$ and $(4, 0)$ respectively.

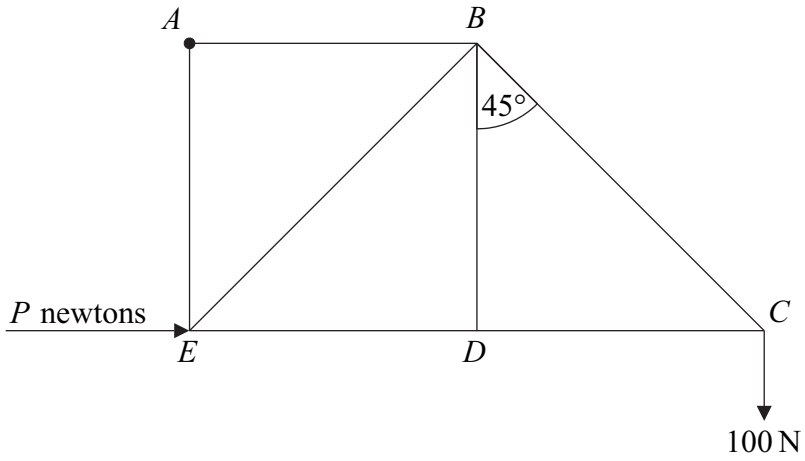
(a) (i) Find the magnitude of the resultant $\mathbf{F}$ of this system of forces. *(3 marks)*

(ii) Show that the line of action of $\mathbf{F}$ cuts the y-axis at the point $(0, 2)$. *(4 marks)*

(b) The system of forces is equivalent to a force acting at the origin together with a couple C. Write down the magnitude of C and indicate its sense on a diagram. *(2 marks)*

3 A framework is composed of seven light smoothly-jointed rods AB, AE, BE, BD, ED, BC and DC, so that $ABDE$ is a square and BDC is a right-angled triangle. The rod AB has length l and angle $CBD = 45°$.

The framework is in a vertical plane and is freely hinged at A to a fixed support. A vertical force of 100 N acts at C. The rod AE is kept vertical by a horizontal force of magnitude P newtons applied at E, as shown in the diagram.



(a) Show that $P = 200$. *(2 marks)*

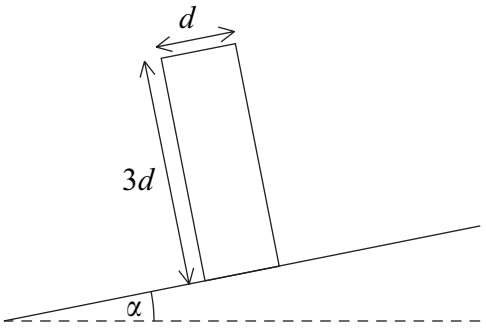
(b) (i) Find the magnitude of the reaction force on the framework at A. *(2 marks)*

(ii) Find the angle between this reaction force and the horizontal, giving your answer to the nearest degree. *(1 mark)*

(c) Find the magnitudes of the forces in each of the rods AB, AE and BE, stating whether they are in tension or compression. *(5 marks)*

Turn over for the next question

4 A uniform solid circular cylinder is in equilibrium with one plane face on a rough inclined plane. The plane is inclined to the horizontal at an angle α degrees, which can be varied. The cylinder has weight W, diameter d and height $3d$.



(a) Draw a diagram showing the forces acting on the cylinder. *(2 marks)*

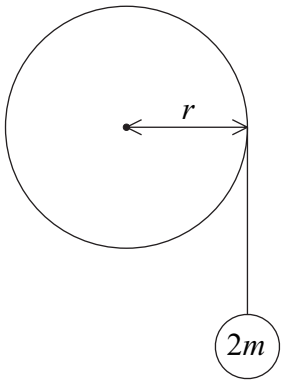
(b) If the plane is sufficiently rough to prevent sliding, find the maximum value of α for the cylinder to remain in equilibrium. *(3 marks)*

(c) The coefficient of friction between the cylinder and the plane is $\frac{2}{9}$. If the value of α is gradually increased from zero, show that the cylinder will slide before it topples. *(5 marks)*

5 A light inextensible string is wrapped several times around a uniform cylinder and a particle of mass $2m$ is attached to the free end of the string.

The cylinder, of radius r, is free to rotate about a smooth fixed horizontal axis through its centre, perpendicular to its plane face. The moment of inertia of the cylinder about this axis is $4mr^2$.

The system is released from rest with the particle hanging freely. After time t, the cylinder has turned through an angle θ radians. Assume that during this subsequent motion no slipping of the string occurs.

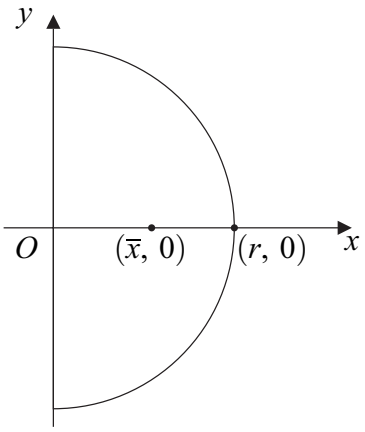


(a) Show that $\ddot{\theta} = \dfrac{g}{3r}$. *(6 marks)*

(b) Hence find an expression for the tension in the string in terms of m and g. *(1 mark)*

Turn over for the next question

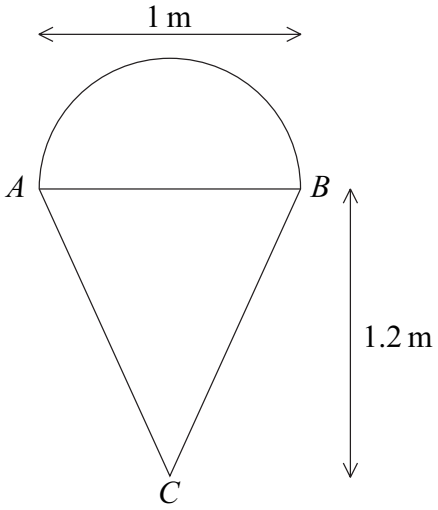
6 (a) A uniform semicircular lamina of radius r has its centre at the origin and its axis of symmetry along Ox. The position of its centre of mass has coordinates $(\overline{x}, 0)$.



(i) Show that $\frac{1}{2}\pi r^2\overline{x} = \int_0^r 2x\sqrt{r^2 - x^2}\,\mathrm{d}x$. *(4 marks)*

(ii) Hence prove that $\overline{x} = \dfrac{4r}{3\pi}$. *(3 marks)*

(b) The diagram below shows a uniform lamina, used as an advertising feature in a local supermarket to promote ice cream. It consists of a semicircle, of diameter $AB = 1$m, and an isosceles triangle ABC, where C is at a distance 1.2 m from AB.



(i) State the distance of the centre of mass of the **triangle** from C. *(1 mark)*

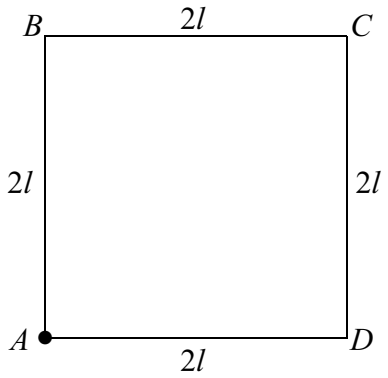
(ii) Show that the distance of the centre of mass of the **semicircle** from C is approximately 1.41 m. *(1 mark)*

(iii) Find the distance of the centre of mass of the complete lamina from C. *(4 marks)*

(c) The lamina is freely suspended from A. Find the angle that AB makes with the vertical through A, giving your answer to the nearest degree. *(3 marks)*

7 A rigid square framework $ABCD$ is formed from four identical uniform rods. Each rod has length $2l$ and mass m.

The framework can rotate freely in a vertical plane about a horizontal axis through A perpendicular to the plane of the square $ABCD$.



(a) Show that the moment of inertia of the **rod *BC*** about the axis is $\frac{16ml^2}{3}$. *(4 marks)*

(b) Particles of masses $4m$, $3m$, $2m$ and m are fixed at the vertices A, B, C and D respectively.

Show that the moment of inertia of the whole system about the axis through A is $\frac{136ml^2}{3}$. *(6 marks)*

(c) The system is released from rest with AD horizontal and B vertically **above** A. Find, in terms of g and l, the angular velocity of the system when B is vertically **below** A. *(7 marks)*

END OF QUESTIONS

Answer **all** questions.

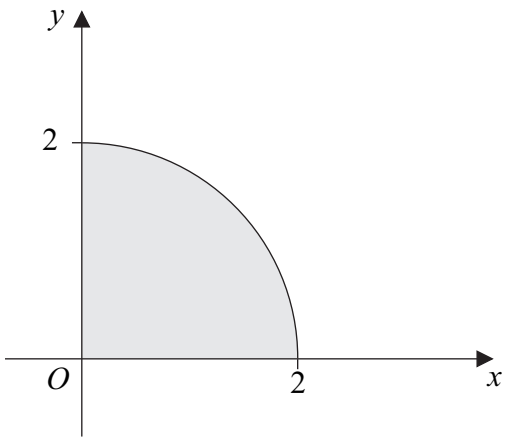
1 Two forces, $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$, act at the points $P(1, -1, 6)$ and $Q(0, 3, -2)$ respectively. These two forces together with a force $\mathbf{F}$, acting through the point Q, form a couple.

(a) (i) Find the force $\mathbf{F}$. *(3 marks)*

(ii) Show that the magnitude of $\mathbf{F}$ is $3\sqrt{10}$. *(2 marks)*

(b) Find the moment of the couple. *(5 marks)*

2 The region bounded by the positive x-axis, the positive y-axis and the curve with equation $y = \sqrt{4 - x^2}$ is shown in the diagram.



The region is rotated through 2π radians around the x-axis to form a uniform solid.

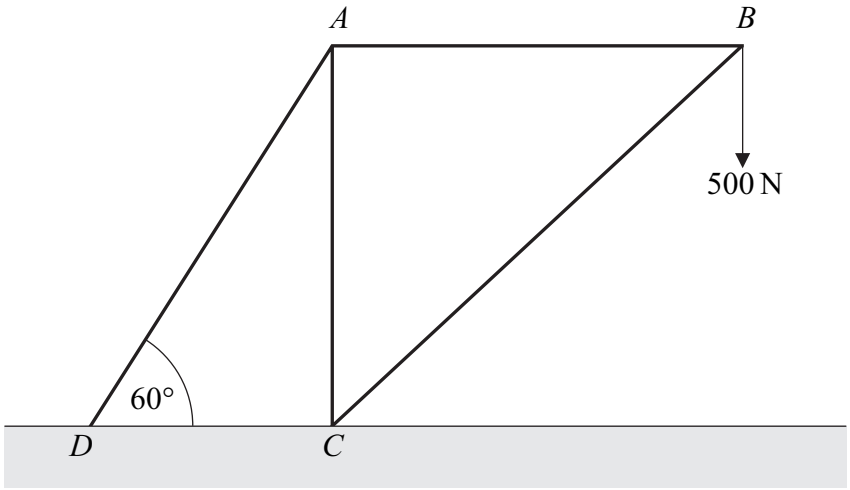
(a) Use integration to show that the volume of the solid is $\frac{16\pi}{3}$. *(3 marks)*

(b) Use integration to find the distance of the centre of mass of the solid from the y-axis. *(4 marks)*

(c) The solid is suspended from a point on the edge of its circular face and hangs in equilibrium.

Find the angle between the circular face and the vertical. *(3 marks)*

3 Dominic designs a crane. He models the crane by four light smoothly-jointed rods AB, AC, AD and BC. The crane is smoothly jointed to the horizontal ground at C and D. The framework is at rest in a vertical plane, with AB horizontal and AC vertical. The framework is such that $AC = AB$ and the angle CDA is 60°. A force of magnitude 500 N acts vertically downwards on the framework at B, as shown in the diagram.



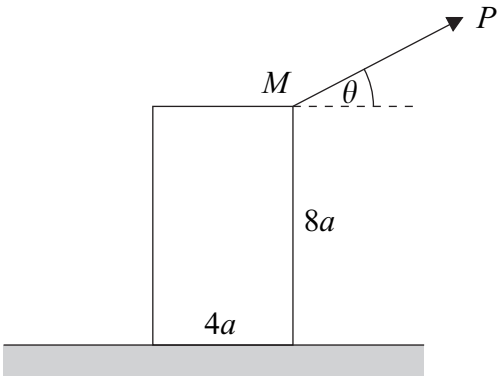
(a) (i) Find the magnitudes of the forces in the rods BC, AB and AD. *(9 marks)*

(ii) State which of the rods BC, AB and AD could be replaced by ropes, giving a reason for your answer. *(3 marks)*

(b) Find the magnitude of the force acting on the framework at D. *(1 mark)*

Turn over for the next question

4 A uniform block in the shape of a cuboid has weight W, a square base of side $4a$, a height of $8a$ and stands on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A rope is attached to the point M, the midpoint of a top edge of the block.



The rope is pulled with a force P, which acts at an angle of θ above the horizontal and is perpendicular to the top edge.

(a) Find P, in terms of W and θ, if the block is on the point of toppling. *(4 marks)*

(b) Show that $P = \dfrac{\mu W}{\cos\theta + \mu \sin\theta}$ if the block is on the point of sliding. *(7 marks)*

(c) Given that $\tan\theta = 1$, find an inequality that μ must satisfy if the block slides before it topples. *(5 marks)*

5 A thin uniform rectangular plate, of mass m and with sides of length $4a$ and $6a$, is hanging freely from a horizontal axis along a side of length $4a$. Initially the plate is stationary and hangs in a vertical plane.

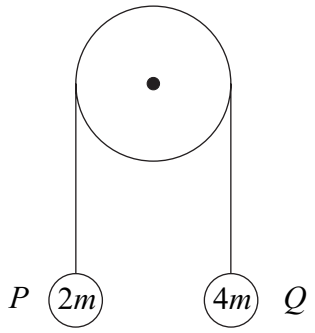
(a) Use integration to show that the moment of inertia of the plate about the horizontal axis is $12ma^2$. *(5 marks)*

(b) The plate is struck at its centre of mass by a small lump of clay of mass $\frac{1}{2}m$ moving with speed u horizontally at right angles to the plate.

Assume that the clay sticks to the plate throughout the subsequent motion. Find, in terms of u and a, the angular speed of the plate immediately after impact. *(7 marks)*

6 A uniform circular disc, of radius a, can rotate freely in a vertical plane about a fixed horizontal axis through its centre perpendicular to its plane face. The moment of inertia of the disc about this axis is $10ma^2$.

A light inextensible string passes over the rough rim of the disc, and two particles P and Q, of masses $2m$ and $4m$ respectively, are attached to its ends.



Initially the system is held at rest with the particles hanging freely in equilibrium. The system is then released. In the subsequent motion, no slipping occurs between the string and the disc. When the disc has turned through an angle θ, the particle P has **not** reached the disc.

(a) (i) Show that the gain in kinetic energy of the system is $8ma^2\dot{\theta}^2$. *(4 marks)*

(ii) Hence show that $a\dot{\theta}^2 = \frac{1}{4}g\theta$. *(3 marks)*

(b) Find the force exerted by the string:

(i) on the particle P;

(ii) on the particle Q. *(7 marks)*

END OF QUESTIONS

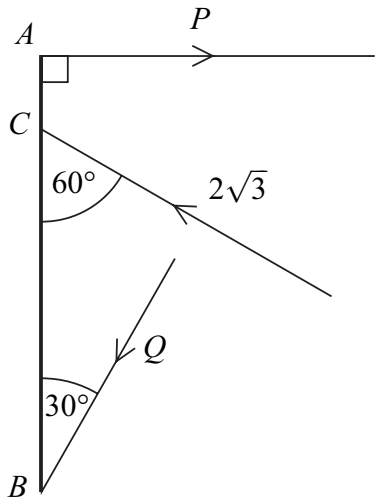
Answer **all** questions.

1 A light rod AB has length 5 metres and the point C on the rod is 1 metre from A. The rod is on a smooth horizontal table and is acted upon by three horizontal forces of magnitude P, Q and $2\sqrt{3}$ newtons.

The force of magnitude P acts at A, at right angles to the rod.

The force of magnitude $2\sqrt{3}$ acts at C, at an angle of 60° to the rod.

The force of magnitude Q acts at B, at an angle of 30° to the rod, as shown in the diagram.



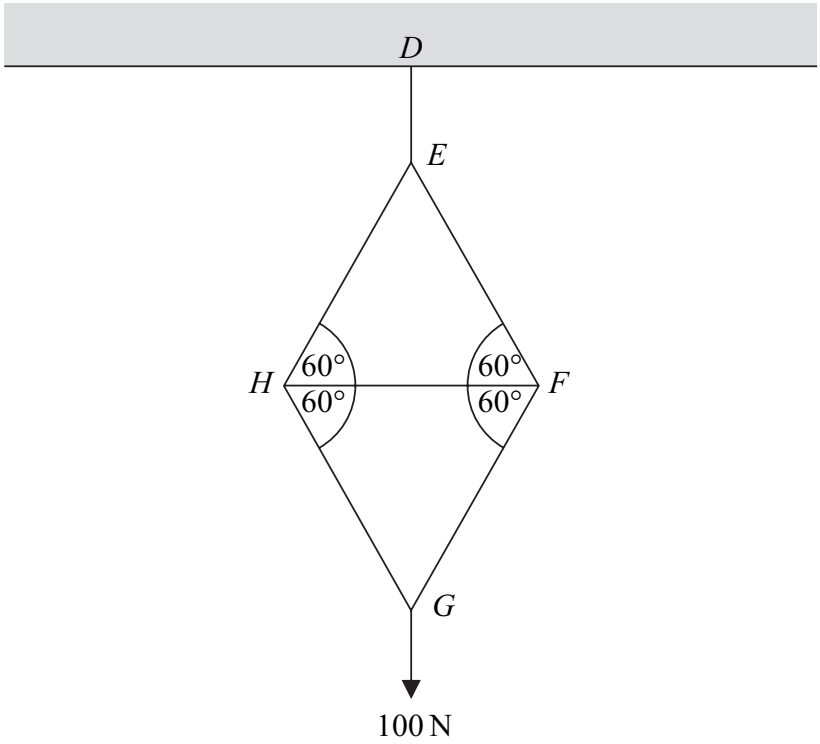
The three forces are equivalent to a couple.

(a) Show that $Q = 2$ and find the value of P. *(5 marks)*

(b) Determine the magnitude of the couple. *(3 marks)*

(c) State the sense of the couple. *(1 mark)*

2 A framework *EFGH* consists of five identical light rods, *EF*, *EH*, *FG*, *GH* and *FH*, which are smoothly jointed at *E*, *F*, *G* and *H*. Each of the rods *EF*, *EH*, *FG* and *GH* makes an angle of 60° with the rod *FH*. The framework is suspended from a fixed point *D* by a string *DE*. The rod *FH* is horizontal, and *G* is vertically below *D*. A force of 100 N is applied vertically at *G*. The system, as shown in the diagram, is in equilibrium.



(a) State the magnitude of the force in the string *DE*, giving a reason for your answer. *(2 marks)*

(b) Explain why the forces in the rods *EF*, *EH*, *FG* and *GH* must be of equal magnitude. *(2 marks)*

(c) Find the magnitude of the forces in each of the rods *EF*, *EH*, *FG* and *GH*. *(2 marks)*

(d) Find the magnitude of the force in the rod *FH*. *(3 marks)*

(e) State which of the five rods could be replaced by ropes, giving reasons for your answers. *(2 marks)*

3 A light rod has its ends at the points $A(2, 3, 5)$ and $B(4, 6, -1)$. A force $\mathbf{F}$ acts at B, where

$$\mathbf{F} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$$

(a) Find $\overrightarrow{AB}$. *(1 mark)*

(b) Find the moment of $\mathbf{F}$ about the point A. *(3 marks)*

(c) Show that the magnitude of this moment is $10\sqrt{5}$. *(2 marks)*

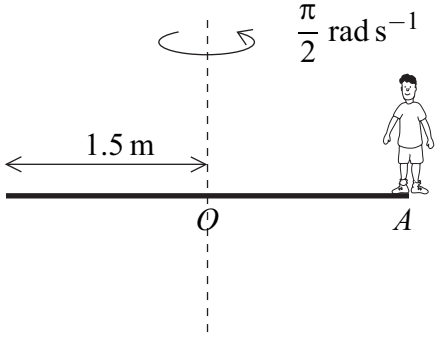
(d) Hence, or otherwise, find the acute angle between $\mathbf{F}$ and the rod, giving your answer to the nearest degree. *(4 marks)*

4 (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}mr^2$. *(5 marks)*

(b) A roundabout in a playground can be modelled as a uniform circular disc of mass 200 kg and radius 1.5 m. The roundabout can rotate freely in a horizontal plane about a vertical axis through its centre O.

The roundabout is rotating at $\frac{\pi}{2}$ radians per second, with Dominic, a child of mass 25 kg, standing at a point A on the edge, as shown in **Figure 1**.

Figure 1

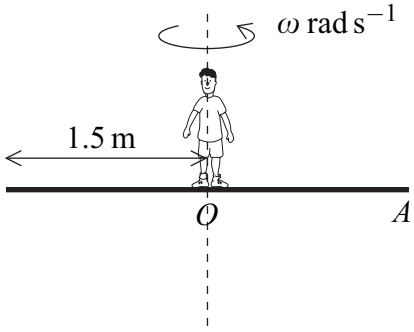


Assume that Dominic can be modelled as a particle.

(i) Show that the moment of inertia of the system about the vertical axis through O shown in **Figure 1** is 281.25 kg m^2. *(3 marks)*

(ii) Dominic then walks to the centre O, as shown in **Figure 2**. The angular speed of the roundabout changes from $\frac{\pi}{2}$ radians per second to ω radians per second.

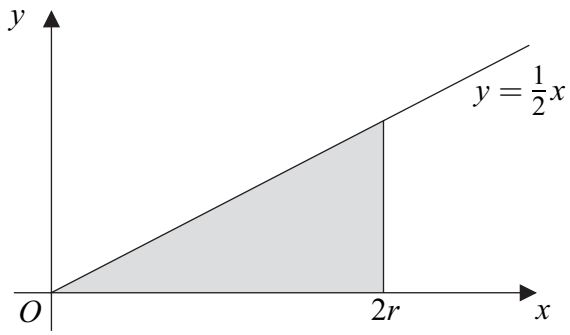
Figure 2



Explain why the total angular momentum of the system remains constant as Dominic walks from A to O. *(1 mark)*

(iii) Find the value of ω. *(4 marks)*

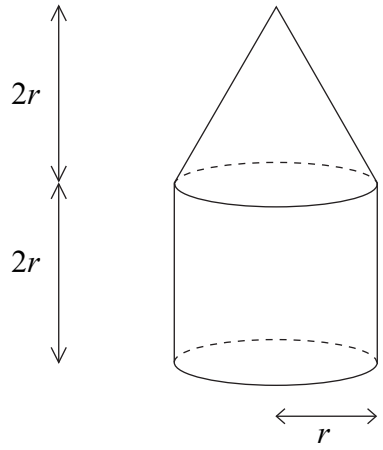
5 The region bounded by the line $y = \frac{1}{2}x$, the x-axis and the line $x = 2r$ is shown in the diagram.



This region is rotated about the x-axis to form a uniform solid cone of height $2r$ and radius r.

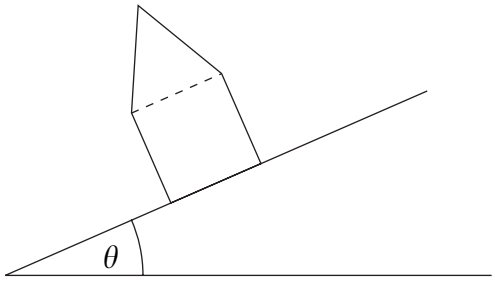
(a) Show, using integration, that the centre of mass of the cone is at a distance of $\frac{3r}{2}$ from the origin. *(5 marks)*

(b) A rocket consists of two parts. The lower part of the rocket may be modelled as a uniform solid cylinder with radius r, height $2r$ and density ρ. The upper part of the rocket may be modelled as a uniform solid cone of radius r, height $2r$ and density $k\rho$, as shown in the diagram.



(i) Show that the centre of mass of the rocket is at a distance of $\left(\frac{6+5k}{6+2k}\right)r$ from the base of the rocket. *(5 marks)*

(ii) The rocket is now placed on a rough plane, which is inclined at an angle of θ to the horizontal, where $\tan\theta = \frac{2}{3}$.

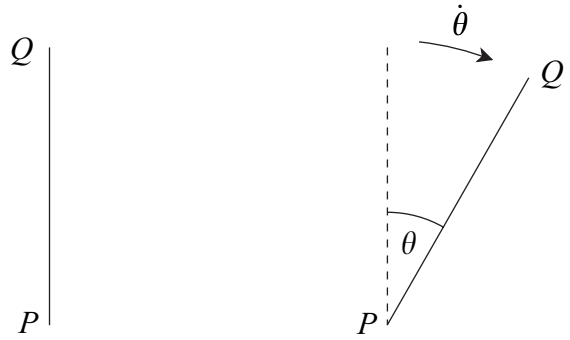


Given that the rocket does **not** slide and is just on the point of toppling, find the value of k. *(5 marks)*

Turn over for the next question

6 A uniform rod PQ, of mass m and length $6a$, is free to rotate in a vertical plane about a fixed horizontal axis through P. Initially, the rod is at rest with Q vertically above P.

The rod is slightly disturbed from its initial position. In the subsequent motion, it makes an angle θ with the upward vertical at time t.



Initial position During motion

(a) (i) Show that the moment of inertia of the rod about the axis through P is $12ma^2$. *(1 mark)*

(ii) Show that $\dot{\theta}^2 = \frac{g}{2a}(1 - \cos\theta)$. *(4 marks)*

(iii) Hence, or otherwise, determine an expression for $\ddot{\theta}$ in terms of a, g and θ. *(2 marks)*

(b) Find, in terms of m, g and θ, the force at P which the axis exerts on the rod:

(i) in the direction PQ; *(4 marks)*

(ii) perpendicular to PQ. *(3 marks)*

(c) Determine the magnitude of the force exerted by the axis on the rod when Q is vertically below P. *(3 marks)*

END OF QUESTIONS

Answer **all** questions.

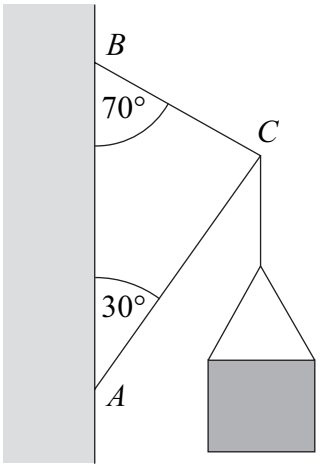
1 The cylindrical drum in a spin dryer rotates about its vertical axis. Initially, the drum is at rest. It then rotates with a constant acceleration and reaches its maximum angular speed of 1200 revolutions per minute in 10 seconds.

(a) Show that the magnitude of the angular acceleration is $4\pi \text{ rad s}^{-2}$. *(4 marks)*

(b) A couple of constant magnitude 100π N m causes the drum to rotate with this angular acceleration. Find the moment of inertia of the drum about the axis of rotation. *(2 marks)*

2 Two light smoothly-jointed rods, *AC* and *BC*, support a shop sign of mass 20 kg.

The two rods are smoothly hinged to a vertical wall at *A* and *B*, with *B* directly above *A*. Angle *BAC* is 30° and angle *ABC* is 70°. The shop sign hangs in equilibrium from *C*, as shown in the diagram.



Find the magnitudes of the forces in rods *AC* and *BC*, stating whether the rods are in tension or compression. *(7 marks)*

3 The forces $\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, $2\mathbf{i} - 7\mathbf{j} - \mathbf{k}$ and $4\mathbf{j} - 2\mathbf{k}$ act at the points with coordinates $(2, 1, 0)$, $(1, 13, -2)$ and $(6, 4, -7)$ respectively. The resultant of the three forces is a single force $\mathbf{F}$.

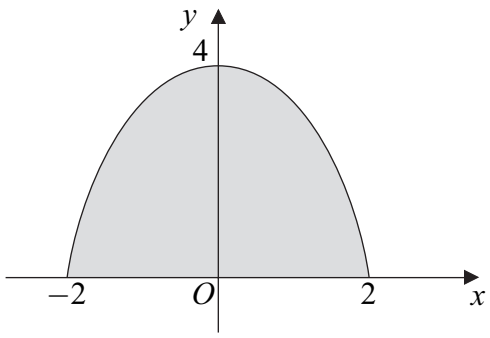
(a) Show that the magnitude of $\mathbf{F}$ is 7. *(3 marks)*

(b) The point P has coordinates $(3, 6, -3)$.

(i) Find the moment of the force $4\mathbf{j} - 2\mathbf{k}$ about P. *(4 marks)*

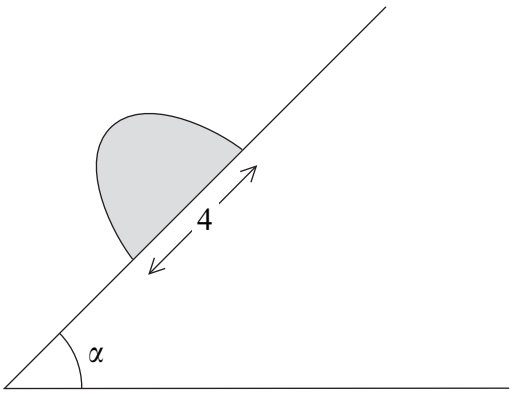
(ii) Given that the resultant of the two forces $\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ and $2\mathbf{i} - 7\mathbf{j} - \mathbf{k}$ acts through P, state the moment of $\mathbf{F}$ about P, giving a reason for your answer. *(2 marks)*

4 (a) A uniform lamina is bounded by the curve $y = 4 - x^2$ and the x-axis, as shown in the diagram.



Given that the area of the lamina is $\frac{32}{3}$ square units, find the y-coordinate of the centre of mass of the lamina. *(5 marks)*

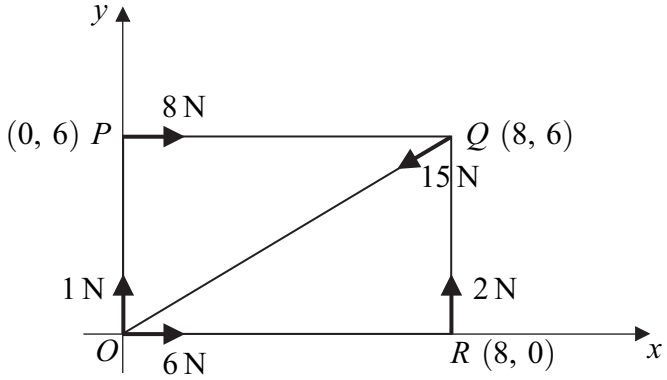
(b) The cross-section of a uniform prism is the same shape as the lamina in part (a). The prism is placed on a plane inclined at an angle α to the horizontal with the rectangular base of the prism in contact with the inclined plane, as shown in the diagram.



Given that the prism is just about to topple and that no slipping occurs, find the value of α, giving your answer to the nearest degree. *(4 marks)*

5 The points O, P, Q and R have coordinates $(0, 0)$, $(0, 6)$, $(8, 6)$ and $(8, 0)$ respectively. The units of length are metres.

A force of 1 N acts at O along OP.
A force of 2 N acts at R along RQ.
A force of 6 N acts at O along OR.
A force of 8 N acts at P along PQ.
A force of 15 N acts at Q along QO.



(a) Show that the resultant of the five forces has magnitude $2\sqrt{10}$ N. *(5 marks)*

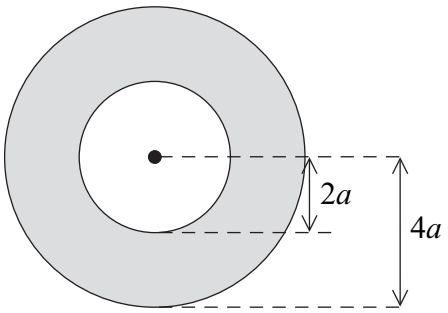
(b) An anticlockwise couple of magnitude 20 N m together with these five forces is equivalent to a single force of magnitude $2\sqrt{10}$ which has a line of action passing through the point $(0, d)$.

(i) Find d. *(5 marks)*

(ii) Determine the equation of the line of action of the resultant, giving your answer in vector form. *(3 marks)*

6 (a) Show, by integration, that the moment of inertia of a uniform disc, of mass m and radius r, about an axis through its centre and perpendicular to the plane of the disc is $\frac{mr^2}{2}$. *(5 marks)*

(b) A disc, of radius $2a$, is removed from the centre of a uniform disc, of radius $4a$. The resulting ring has mass M and is shown in the diagram.

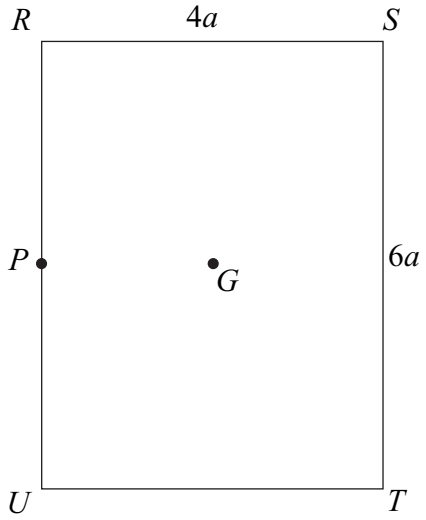


Using the result from part (a), or otherwise, show that the moment of inertia of the ring about an axis through its centre and perpendicular to the plane of the ring is $10Ma^2$. *(5 marks)*

(c) Determine the moment of inertia of the ring about an axis along a diameter, stating any theorem that you use. *(3 marks)*

Turn over for the next question

7 A uniform rectangular lamina, $RSTU$, has mass M, with $RS = 4a$ and $ST = 6a$. The centre of mass of the lamina is G, and the mid-point of RU is P.



(a) (i) Show that the moment of inertia of the rectangular lamina about an axis perpendicular to its plane and passing through G is $\frac{13Ma^2}{3}$. *(2 marks)*

(ii) Hence find the moment of inertia of the lamina about an axis perpendicular to its plane and passing through P. *(2 marks)*

(b) The lamina is smoothly hinged at P. It is free to rotate in a vertical plane about a fixed horizontal axis which is perpendicular to its plane and passes through P. Initially, the lamina is held with PG horizontal and then released. At time t after release, PG makes an angle θ with the horizontal.

(i) Show that $\dot{\theta}^2 = \frac{12g\sin\theta}{25a}$. *(5 marks)*

(ii) Hence, or otherwise, determine an expression for $\ddot{\theta}$, in terms of a, g and θ. *(3 marks)*

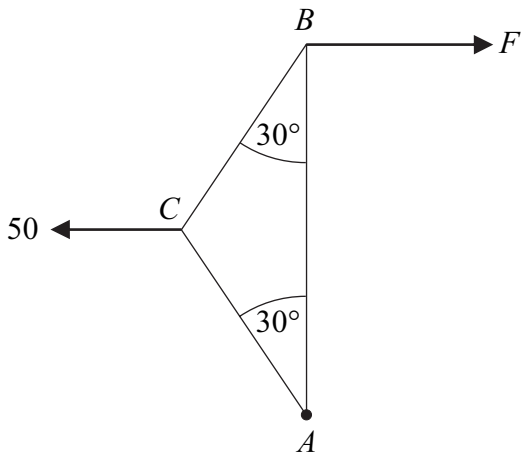
(iii) Show that the magnitude of the component in the direction GP of the force at P which the hinge exerts on the lamina is $\frac{49Mg\sin\theta}{25}$. *(3 marks)*

(iv) Find, in terms of M, g and θ, the magnitude of the component in the direction PR of the force at P which the hinge exerts on the lamina. *(3 marks)*

END OF QUESTIONS

Answer **all** questions in the spaces provided.

1 A framework consists of three light inextensible smoothly jointed rods AB, BC and CA. Rods BC and CA each have length 2 metres and angle BAC = angle $ABC = 30°$. The framework is freely pivoted to a fixed support at A. Two horizontal forces, of magnitudes 50 newtons and F newtons, act on the framework. The system is in equilibrium in a vertical plane with AB vertical, as shown in the diagram.



(a) By taking moments about A, find F. *(2 marks)*

(b) State the magnitude and direction of the reaction force acting on the framework at A. *(2 marks)*

(c) (i) Find the magnitude of the force in the rod BC. *(2 marks)*

(ii) Find the magnitude of the force in the rod AB. *(2 marks)*

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2 Stephanie is practising a ballet dancing routine. As part of the routine, she rotates about a vertical axis through her centre of mass.

(a) When both her arms are fully extended, her moment of inertia about her axis of rotation is $0.6\,\text{kg}\,\text{m}^2$ and her angular speed is $3\,\text{rad}\,\text{s}^{-1}$. Find her angular momentum. *(2 marks)*

(b) Stephanie now lowers her arms until they are vertical. Her moment of inertia in this position is $0.45\,\text{kg}\,\text{m}^2$. Find her angular speed when her arms are vertical. *(2 marks)*

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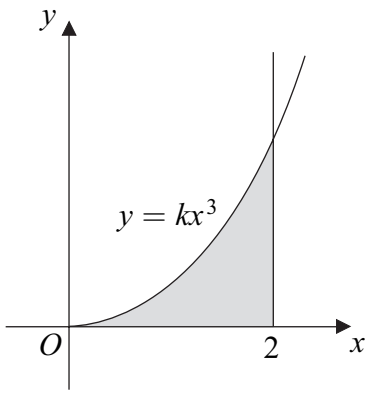
3 A uniform lamina is bounded by the curve $y = kx^3$, the line $x = 2$ and the x-axis, as shown in the diagram.



(a) Find an expression for the area of the lamina in terms of k. *(2 marks)*

(b) Find the x-coordinate of the centre of mass of the lamina. *(4 marks)*

(c) The y-coordinate of the centre of mass of the lamina is 8.

(i) Determine the value of k. *(4 marks)*

(ii) The lamina is freely suspended from the corner at the origin O. Find the acute angle between the straight edge at the point of suspension and the vertical. *(3 marks)*

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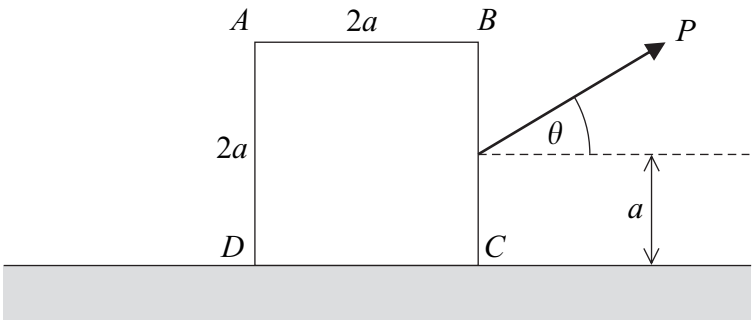
4 A uniform cube, of side $2a$ and mass m, rests on a rough horizontal plane. The diagram shows a vertical cross-section $ABCD$ through the centre of mass of the cube.



A force, of magnitude P, is applied at the mid-point of BC. This force acts in the plane $ABCD$ and makes an angle θ with the horizontal. The coefficient of friction between the cube and the plane is μ.

(a) In the case where the cube does not slide but is on the point of toppling about the edge through C, find an expression for P in terms of m, g and θ. *(3 marks)*

(b) In the case where the cube remains upright but is on the point of sliding along the plane, show that $P = \dfrac{\mu mg}{\cos\theta + \mu \sin\theta}$. *(4 marks)*

(c) Find an inequality that μ must satisfy if the cube slides before it topples. *(3 marks)*

(d) Would your answer in part **(c)** change if the mass of the cube were doubled? Explain why. *(2 marks)*

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5 A pulley Q is fixed to the edge of a smooth horizontal table. The pulley can rotate freely in a vertical plane about a horizontal axis through its centre.

A light inextensible string runs over the pulley, connecting a block R, of mass m, to a block P, of mass $3m$. The block R is held at rest on the table with block P hanging freely, as shown in the diagram.

Model the pulley as a uniform disc of mass $12m$ and radius r. Model the blocks as particles.

(a) Write down the moment of inertia of the pulley about the horizontal axis through its centre and perpendicular to its plane. *(1 mark)*

(b) Block R is released. In the subsequent motion, R moves on the table. The string between P and Q is vertical and has tension T_1. The string between Q and R is horizontal and has tension T_2. The pulley has angular acceleration $\ddot{\theta}$. Assume that the string does not slip and that R does not reach the pulley.

(i) Show that $T_1 - T_2 = 6mr\ddot{\theta}$. *(3 marks)*

(ii) Show that $\ddot{\theta} = \dfrac{3g}{10r}$. *(6 marks)*

(iii) Find T_1 and T_2 in terms of m and g. *(3 marks)*

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6 Two forces, $2\mathbf{i} + a\mathbf{k}$ and $-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, act at the points whose coordinates are $(1, 0, 3)$ and $(–1, 2, 0)$ respectively.

(a) Show that the resultant moment of these forces about the origin is $6\mathbf{i} + (9 - a)\mathbf{j} + 3\mathbf{k}$. *(5 marks)*

(b) This system is equivalent to a force **F** that acts at the origin together with a couple of magnitude 7.

(i) Show that one possible value of a is 7 and find the other possible value of a. *(4 marks)*

(ii) In the case where $a = 7$, find **F**. *(2 marks)*

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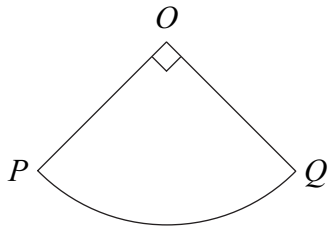
7 (a) Prove by integration that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis through one end of the rod and perpendicular to the rod is $\frac{4}{3}ma^2$. *(4 marks)*

(b) The diagram shows a simple model of a theme park swingboat ride.



The model consists of two uniform rods, OP and OQ, and a seat in the form of a circular arc PQ with centre O. Each rod has mass m and length $2a$. The seat is of mass $4m$ and angle $POQ = 90°$. The rods and the seat are rigidly fixed together and the model is free to rotate about a horizontal axis through O. The axis is perpendicular to the plane of OPQ.

(i) Show that the moment of inertia of the model about this axis is $\frac{56ma^2}{3}$. *(4 marks)*

(ii) The centre of mass of the model is at a distance of approximately $1.44a$ from the point O. The model is rotated until OQ is horizontal, with P vertically below O, and is then released from rest.

In the case where $a = 1.5$, find the greatest angular speed during the subsequent motion. *(6 marks)*

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