Answer **all** questions.

1 A study undertaken by Goodhealth Hospital found that the number of patients each month, X, contracting a particular superbug can be modelled by a Poisson distribution with a mean of 1.5 .

(a) (i) Calculate $\mathrm{P}(X = 2)$. *(2 marks)*

(ii) Hence determine the probability that exactly 2 patients will contract this superbug in each of three consecutive months. *(2 marks)*

(b) (i) Write down the distribution of Y, the number of patients contracting this superbug in a given 6-month period. *(1 mark)*

(ii) Find the probability that at least 12 patients will contract this superbug during a given 6-month period. *(2 marks)*

(c) State **two** assumptions implied by the use of a Poisson model for the number of patients contracting this superbug. *(2 marks)*

2 Year 12 students at Newstatus School choose to participate in one of four sports during the Spring term.

The students' choices are summarised in the table.

	Squash	**Badminton**	**Archery**	**Hockey**	**Total**
Male	5	16	30	19	70
Female	4	20	33	53	110
Total	9	36	63	72	180

(a) Use a χ^2 test, at the 5% level of significance, to determine whether the choice of sport is independent of gender. *(10 marks)*

(b) Interpret your result in part (a) as it relates to students choosing hockey. *(2 marks)*

3 The time, T minutes, that parents have to wait before seeing a mathematics teacher at a school parents' evening can be modelled by a normal distribution with mean μ and standard deviation σ.

At a recent parents' evening, a random sample of 9 parents was asked to record the times that they waited before seeing a mathematics teacher.

The times, in minutes, are

5 12 10 8 7 6 9 7 8

(a) Construct a 90% confidence interval for μ. *(7 marks)*

(b) Comment on the headteacher's claim that the mean time that parents have to wait before seeing a mathematics teacher is 5 minutes. *(2 marks)*

4 (a) A random variable X has probability density function defined by

$$f(x) = \begin{cases} k & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that $k = \dfrac{1}{b-a}$. *(1 mark)*

(ii) Prove, using integration, that $E(X) = \frac{1}{2}(a+b)$. *(4 marks)*

(b) The error, X grams, made when a shopkeeper weighs out loose sweets can be modelled by a rectangular distribution with the following probability density function:

$$f(x) = \begin{cases} k & -2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(i) Write down the value of the mean, μ, of X. *(1 mark)*

(ii) Evaluate the standard deviation, σ, of X. *(2 marks)*

(iii) Hence find $P\left(X < \dfrac{2-\mu}{\sigma}\right)$. *(3 marks)*

5 The Globe Express agency organises trips to the theatre. The cost, £X, of these trips can be modelled by the following probability distribution:

x	40	45	55	74
$P(X = x)$	0.30	0.24	0.36	0.10

(a) Calculate the mean and standard deviation of X. *(4 marks)*

(b) For special celebrity charity performances, Globe Express increases the cost of the trips to £Y, where

$$Y = 10X + 250$$

Determine the mean and standard deviation of Y. *(2 marks)*

6 In previous years, the marks obtained in a French test by students attending Topnotch College have been modelled satisfactorily by a normal distribution with a mean of 65 and a standard deviation of 9.

Teachers in the French department at Topnotch College suspect that this year their students are, on average, underachieving.

In order to investigate this suspicion, the teachers selected a random sample of 35 students to take the French test and found that their mean score was 61.5 .

(a) Investigate, at the 5% level of significance, the teachers' suspicion. *(6 marks)*

(b) Explain, in the context of this question, the meaning of a Type I error. *(2 marks)*

7 Engineering work on the railway network causes an increase in the journey time of commuters travelling into work each morning.

The increase in journey time, T hours, is modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} 4t(1-t^2) & 0 \leqslant t \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $E(T) = \frac{8}{15}$. *(3 marks)*

(b) (i) Find the cumulative distribution function, $F(t)$, for $0 \leqslant t \leqslant 1$. *(2 marks)*

(ii) Hence, or otherwise, for a commuter selected at random, find

$$P(\text{mean} < T < \text{median})$$ *(5 marks)*

8 Bottles of sherry nominally contain 1000 millilitres. After the introduction of a new method of filling the bottles, there is a suspicion that the mean volume of sherry in a bottle has changed.

In order to investigate this suspicion, a random sample of 12 bottles of sherry is taken and the volume of sherry in each bottle is measured.

The volumes, in millilitres, of sherry in these bottles are found to be

996	1006	1009	999	1007	1003
998	1010	997	996	1008	1007

Assuming that the volume of sherry in a bottle is normally distributed, investigate, at the 5% level of significance, whether the mean volume of sherry in a bottle differs from 1000 millilitres. *(10 marks)*

END OF QUESTIONS

Practice 2

Leave blank

1. (a) Explain what you understand by a census.

(1)

Each cooker produced at GT Engineering is stamped with a unique serial number.
GT Engineering produces cookers in batches of 2000. Before selling them, they test a random sample of 5 to see what electric current overload they will take before breaking down.

(b) Give one reason, other than to save time and cost, why a sample is taken rather than a census.

(1)

(c) Suggest a suitable sampling frame from which to obtain this sample.

(1)

(d) Identify the sampling units.

(1)

Q1

(Total 4 marks)

Turn over

Leave blank

2. The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are

(a) exactly 2 faulty bolts,
(2)

(b) more than 3 faulty bolts.
(2)

These bolts are sold in bags of 20. John buys 10 bags.

(c) Find the probability that exactly 6 of these bags contain more than 3 faulty bolts.
(3)

Leave blank

Question 2 continued

(Total 7 marks)

Q2

Leave blank

3. (a) State two conditions under which a Poisson distribution is a suitable model to use in statistical work.

(2)

The number of cars passing an observation point in a 10 minute interval is modelled by a Poisson distribution with mean 1.

(b) Find the probability that in a randomly chosen 60 minute period there will be

(i) exactly 4 cars passing the observation point,

(ii) at least 5 cars passing the observation point.

(5)

The number of other vehicles, other than cars, passing the observation point in a 60 minute interval is modelled by a Poisson distribution with mean 12.

(c) Find the probability that exactly 1 vehicle, of any type, passes the observation point in a 10 minute period.

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

(Total 11 marks)

Q3

Leave blank

4. The continuous random variable Y has cumulative distribution function F(y) given by

$$\mathrm{F}(y) = \begin{cases} 0 & y < 1 \\ k(y^4 + y^2 - 2) & 1 \leqslant y \leqslant 2 \\ 1 & y > 2 \end{cases}$$

(a) Show that $k = \frac{1}{18}$.

(2)

(b) Find P(Y > 1.5).

(2)

(c) Specify fully the probability density function f(y).

(3)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Turn over

Leave blank

5. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly.

(7)

Leave blank

Question 5 continued

(Total 7 marks)

Q5

Turn over

Leave blank

6. The probability that a sunflower plant grows over 1.5 metres high is 0.25. A random sample of 40 sunflower plants is taken and each sunflower plant is measured and its height recorded.

(a) Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using

(i) a Poisson approximation,

(ii) a Normal approximation.

(10)

(b) Write down which of the approximations used in part (a) is the most accurate estimate of the probability. You must give a reason for your answer.

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 12 marks)

Q6

Turn over

Leave blank

7. (a) Explain what you understand by

(i) a hypothesis test,

(ii) a critical region.

(3)

During term time, incoming calls to a school are thought to occur at a rate of 0.45 per minute. To test this, the number of calls during a random 20 minute interval, is recorded.

(b) Find the critical region for a two-tailed test of the hypothesis that the number of incoming calls occurs at a rate of 0.45 per 1 minute interval. The probability in each tail should be as close to 2.5% as possible.

(5)

(c) Write down the actual significance level of the above test.

(1)

In the school holidays, 1 call occurs in a 10 minute interval.

(d) Test, at the 5% level of significance, whether or not there is evidence that the rate of incoming calls is less during the school holidays than in term time.

(5)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 14 marks)

Q7

Turn over

Leave blank

8. The continuous random variable X has probability density function f(x) given by

$$\text{f}(x)=\begin{cases}2(x-2) & 2\leqslant x\leqslant 3\\ 0 & \text{otherwise}\end{cases}$$

(a) Sketch f(x) for all values of x.
(3)

(b) Write down the mode of X.
(1)

Find

(c) E(X),
(3)

(d) the median of X.
(4)

(e) Comment on the skewness of this distribution. Give a reason for your answer.
(2)

Leave blank

Question 8 continued

Turn over

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Turn over

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

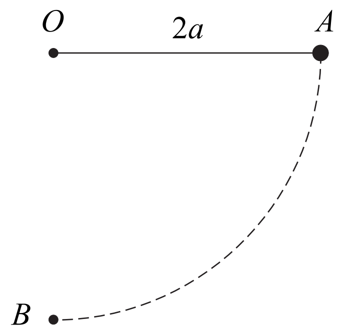
Answer **all** questions.

1 A lift containing miners moves vertically from rest at one level to rest at a higher level.

The total mass of the lift and the miners is 800 kg. The vertical distance between the two levels is 200 metres.

Find the work done in raising the lift and the miners from the lower level to the higher level. *(3 marks)*

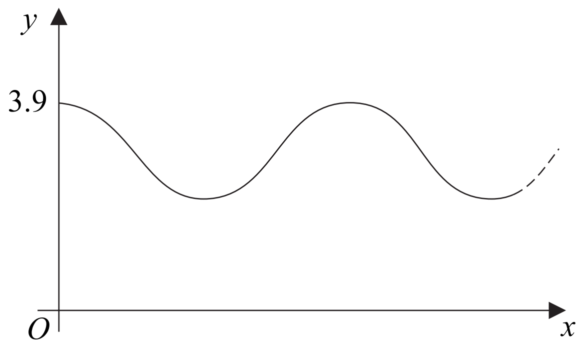
2 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.

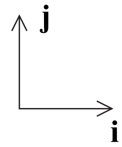


(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

3 Jane is on a ride in a theme park. Part of the curved path of the ride is shown in the diagram.





Jane's position vector, **r** metres, at time t seconds is given by

$$\mathbf{r} = 1.2t\,\mathbf{i} + (3 + 0.9\cos t)\mathbf{j}$$

where the perpendicular unit vectors **i** and **j** are directed horizontally and vertically upwards respectively.

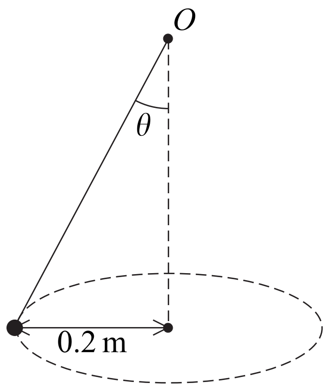
(a) Find an expression for Jane's velocity at time t. *(2 marks)*

(b) (i) Find an expression for Jane's speed at time t. *(2 marks)*

(ii) Find Jane's maximum speed during the ride. *(2 marks)*

Turn over for the next question

4 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

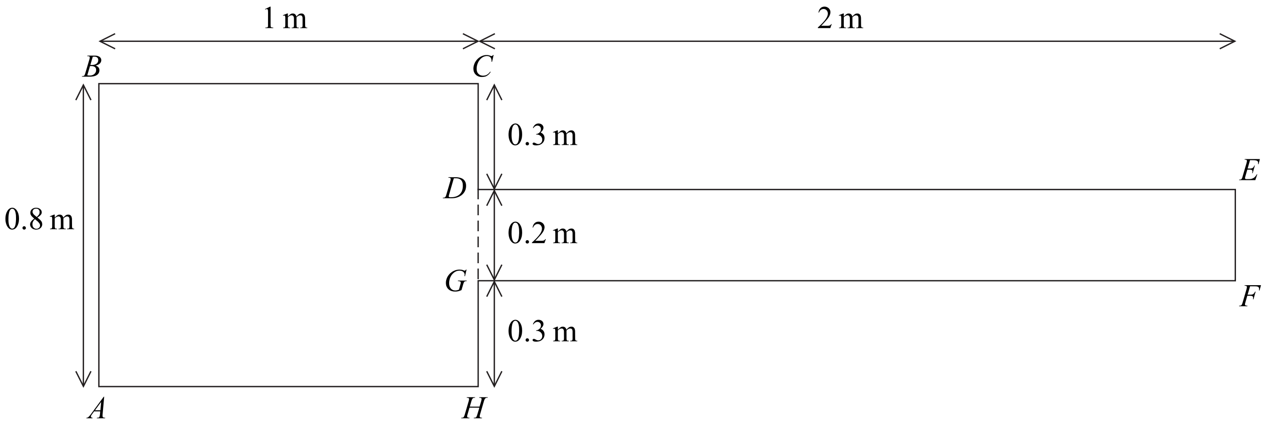
(c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

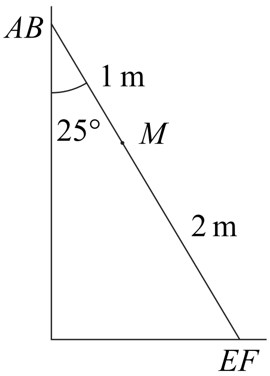
(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

5 A sign advertising a gym consists of two rectangles *ABCH* and *DEFG* fixed rigidly together. The sign can be modelled as a uniform lamina, as shown in the diagram.



(a) The centre of mass of the sign is at the point M. Show that M lies on the line CH. *(4 marks)*

(b) The sign is placed with its side *EF* on rough horizontal ground and its side *AB* against a smooth vertical wall. The sign rests in equilibrium at an angle of 25° with the **vertical**, as shown in the diagram.



The weight of the sign is 90 newtons.

(i) By taking moments, show that the normal reaction force between the sign and the wall is 60 tan 25° newtons. *(3 marks)*

(ii) The coefficient of friction between the sign and the ground is μ.

Show that $\mu \geqslant \frac{2}{3} \tan 25°$. *(4 marks)*

6 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V \text{ m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is 60 m s^{-1}.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at 20 m s^{-1}, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v \text{ m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

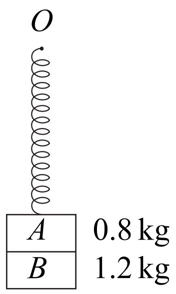
The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\dfrac{dv}{dt} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from 20 m s^{-1} to 10 m s^{-1}. *(6 marks)*

7 Two small blocks, A and B, of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O.

(a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. *(3 marks)*

(b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. *(2 marks)*

(c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

(i) Show that x satisfies the equation

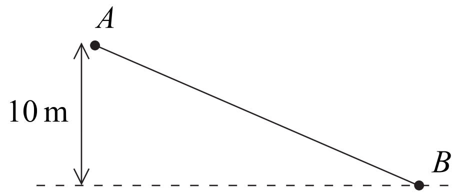
$$x^2 + 0.16x - 0.008 = 0$$ *(5 marks)*

(ii) Find the value of x. *(2 marks)*

END OF QUESTIONS

Answer **all** questions.

1 A child, of mass 35 kg, slides down a slide in a water park. The child, starting from rest, slides from the point A to the point B, which is 10 metres vertically below the level of A, as shown in the diagram.



(a) In a simple model, all resistance forces are ignored.

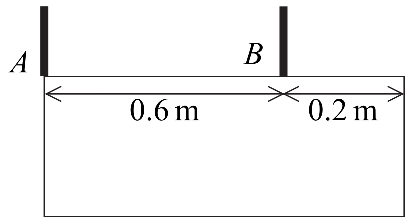
Use an energy method to find the speed of the child at B. *(3 marks)*

(b) State one resistance force that has been ignored in answering part (a). *(1 mark)*

(c) In fact, when the child slides down the slide, she reaches B with a speed of $12\,\mathrm{m\,s^{-1}}$.

Given that the slide is 20 metres long and the sum of the resistance forces has a constant magnitude of F newtons, use an energy method to find the value of F. *(4 marks)*

2 A hotel sign consists of a uniform rectangular lamina of weight W. The sign is suspended in equilibrium in a vertical plane by two vertical light chains attached to the sign at the points A and B, as shown in the diagram. The edge containing A and B is horizontal.



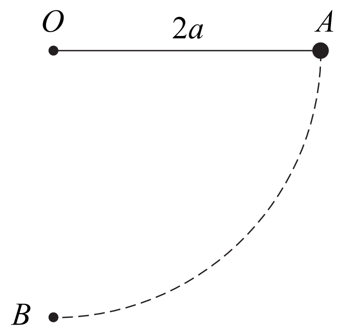
The tensions in the chains attached at A and B are T_A and T_B respectively.

(a) Draw a diagram to show the forces acting on the sign. *(1 mark)*

(b) Find T_A and T_B in terms of W. *(4 marks)*

(c) Explain how you have used the fact that the lamina is uniform in answering part (b). *(1 mark)*

3 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.

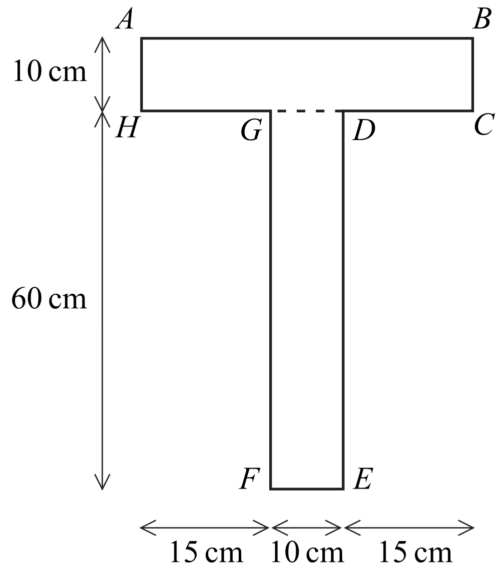


(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

Turn over for the next question

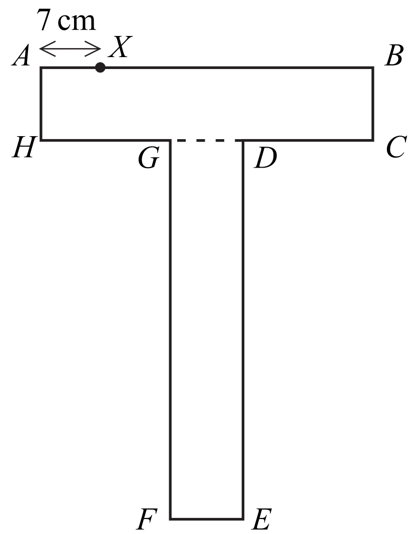
4 A uniform T-shaped lamina is formed by rigidly joining two rectangles *ABCH* and *DEFG*, as shown in the diagram.



(a) Show that the centre of mass of the lamina is 26 cm from the edge *AB*. *(4 marks)*

(b) Explain why the centre of mass of the lamina is 5 cm from the edge *GF*. *(1 mark)*

(c) The point *X* is on the edge *AB* and is 7 cm from *A*, as shown in the diagram below.



The lamina is freely suspended from *X* and hangs in equilibrium.

Find the angle between the edge *AB* and the vertical, giving your answer to the nearest degree. *(4 marks)*

5 Tom is on a fairground ride.

Tom's position vector, $\mathbf{r}$ metres, at time t seconds is given by

$$\mathbf{r} = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (10 - 0.4t)\mathbf{k}$$

The perpendicular unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the horizontal plane and the unit vector $\mathbf{k}$ is directed vertically upwards.

(a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*

(ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*

(iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*

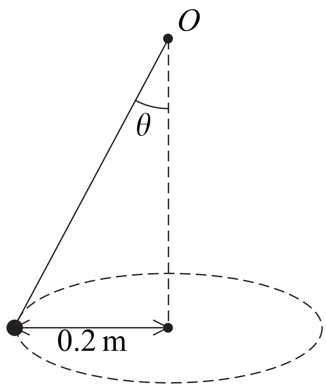
(b) Find an expression for Tom's velocity at time t. *(3 marks)*

(c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude. State the magnitude of the resultant force. *(5 marks)*

Turn over for the next question

6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

(c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\ \text{m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\ \text{m s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\ \text{m s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\ \text{m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

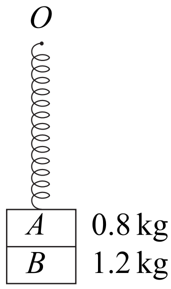
(i) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\ \text{m s}^{-1}$ to $10\ \text{m s}^{-1}$. *(6 marks)*

Turn over for the next question

8 Two small blocks, A and B, of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O.

(a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. *(3 marks)*

(b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. *(2 marks)*

(c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

(i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0$$ *(5 marks)*

(ii) Find the value of x. *(2 marks)*

END OF QUESTIONS

Practice 5

Leave blank

1. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

(a) more than 2 daisies,

(3)

(b) either 5 or 6 daisies.

(2)

The botanist decides to count the number of daisies, x, in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \qquad \sum x^2 = 1386$$

(c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.

(3)

(d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model.

(1)

(e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square.

(2)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 11 marks)

Turn over

Leave blank

2. The continuous random variable X is uniformly distributed over the interval [–2, 7].

(a) Write down fully the probability density function f(x) of X.
(2)

(b) Sketch the probability density function f(x) of X.
(2)

Find

(c) E(X^2),
(3)

(d) P($-0.2 < X < 0.6$).
(2)

Leave blank

Question 2 continued

(Total 9 marks)

Q2

Leave blank

3. A single observation x is to be taken from a Binomial distribution B(20, p).

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test.
The probability of rejecting either tail should be as close as possible to 2.5%.
(3)

(b) State the actual significance level of this test.
(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.
(2)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Turn over

Leave blank

4. The length of a telephone call made to a company is denoted by the continuous random variable T. It is modelled by the probability density function

$$f(t) = \begin{cases} kt & 0 \leqslant t \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{1}{50}$.
(3)

(b) Find $P(T > 6)$.
(2)

(c) Calculate an exact value for $E(T)$ and for $Var(T)$.
(5)

(d) Write down the mode of the distribution of T.
(1)

It is suggested that the probability density function, $f(t)$, is not a good model for T.

(e) Sketch the graph of a more suitable probability density function for T.
(1)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 12 marks)

Turn over

Leave blank

5. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.
(2)

(b) Find the probability that there are at least 2 defective components in the box.
(3)

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components.
(4)

Leave blank

Question 5 continued

(Total 9 marks)

Q5

Leave blank

6. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(7)

(b) State an assumption that has been made about the visits to the server.

(1)

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday.

(6)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 14 marks)

Q6

Leave blank

7. A random variable X has probability density function given by

$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the cumulative distribution function F(x) can be written in the form $ax^2 + bx + c$, for $1 \leqslant x \leqslant 4$ where a, b and c are constants.
(3)

(b) Define fully the cumulative distribution function F(x).
(2)

(c) Show that the upper quartile of X is 2.5 and find the lower quartile.
(6)

Given that the median of X is 1.88

(d) describe the skewness of the distribution. Give a reason for your answer.
(2)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 A ball is thrown vertically upwards from ground level with an initial speed of $15\,\mathrm{m\,s^{-1}}$. The ball has a mass of 0.6 kg. Assume that the only force acting on the ball after it is thrown is its weight.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) By using conservation of energy, find the maximum height above ground level reached by the ball. *(3 marks)*

(c) By using conservation of energy, find the kinetic energy and the speed of the ball when it is at a height of 3 m above ground level. *(4 marks)*

2 A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time t seconds, the position vector, **r** metres, of the particle is given by

$$\mathbf{r} = (t^3 - 3t^2 + 4)\mathbf{i} + (4t + t^2)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

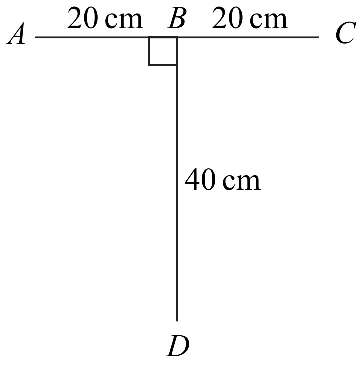
(b) The mass of the particle is 3 kg.

(i) Find an expression for **F** at time t. *(3 marks)*

(ii) Find the magnitude of **F** when $t = 3$. *(2 marks)*

(c) Find the value of t when **F** acts due north. *(2 marks)*

3 Two identical uniform rods, AC and BD, are rigidly joined together to form a letter ⊤, as shown in the diagram. The two rods are perpendicular.



(a) Explain why the centre of mass of the letter ⊤ lies on BD. *(1 mark)*

(b) Find the distance of the centre of mass of the letter ⊤ from AC. *(3 marks)*

(c) The letter ⊤ is freely suspended from A.

Find, to the nearest degree, the angle between AC and the horizontal when the letter ⊤ hangs in equilibrium. *(3 marks)*

4 A light elastic string has natural length 6 metres and modulus of elasticity 300 newtons. It has one end attached to a fixed point, A, on a rough horizontal plane. The other end of the string is attached to a particle of mass 4 kilograms. The particle is pulled along the plane until it is 8 metres from the point A. The particle is then released from rest.

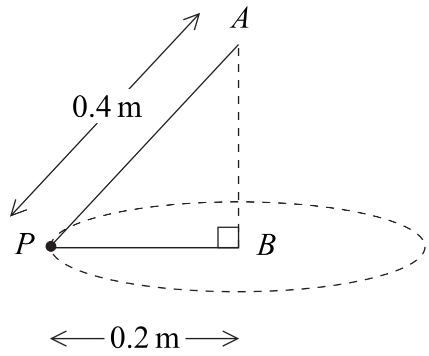
(a) Calculate the elastic potential energy of the string when the particle is 8 metres from the point A. *(2 marks)*

(b) The coefficient of friction between the particle and the plane is 0.3 .

Show that the speed of the particle when the string becomes slack is $6.18\,\text{m s}^{-1}$, correct to three significant figures. *(6 marks)*

Turn over for the next question

5 Two light inextensible strings, of lengths 0.4 m and 0.2 m, each have one end attached to a particle, *P*, of mass 4 kg. The other ends of the strings are attached to the points *A* and *B* respectively. The point *A* is vertically above the point *B*. The particle moves in a horizontal circle, centre *B* and radius 0.2 m, at a speed of $2\,\mathrm{m\,s^{-1}}$. The particle and strings are shown in the diagram.

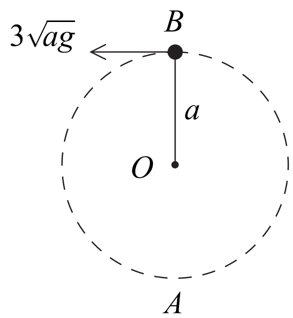


(a) Calculate the magnitude of the acceleration of the particle. *(2 marks)*

(b) Show that the tension in string *PA* is 45.3 N, correct to three significant figures. *(4 marks)*

(c) Find the tension in string *PB*. *(3 marks)*

6 A light inextensible string, of length *a*, has one end attached to a fixed point *O*. A particle, of mass *m*, is attached to the other end. The particle is moving in a vertical circle, centre *O*. When the particle is at *B*, vertically above *O*, the string is taut and the particle is moving with speed $3\sqrt{ag}$.



(a) Find, in terms of *g* and *a*, the speed of the particle at the lowest point, *A*, of its path. *(4 marks)*

(b) Find, in terms of *g* and *m*, the tension in the string when the particle is at *A*. *(4 marks)*

7 A car of mass 600 kg is driven along a straight horizontal road. The resistance to motion of the car is kv^2 newtons, where $v \text{ m s}^{-1}$ is the velocity of the car at time t seconds and k is a constant.

(a) When the engine of the car has power 8 kW, show that the equation of motion of the car is

$$600\frac{dv}{dt} - \frac{8000}{v} + kv^2 = 0$$ *(4 marks)*

(b) When the velocity of the car is 20 m s^{-1}, the engine is turned off.

(i) Show that the equation of motion of the car now becomes

$$600\frac{dv}{dt} = -kv^2$$ *(1 mark)*

(ii) Find, in terms of k, the time taken for the velocity of the car to drop to 10 m s^{-1}. *(5 marks)*

END OF QUESTIONS