edexcel

International GCSE

Further Pure Mathematics (4PM0)

Specification

First examination 2011

Introduction

The Edexcel International General Certificate of Secondary Education (International GCSE) in Further Pure Mathematics is designed for use in schools and colleges. It is part of a suite of International GCSE qualifications offered by Edexcel.

This specification emphasises the importance of a common core of Pure Mathematics at International GCSE Level.

This specification has been constructed to broadly extend knowledge of the pure mathematics topics in the specifications for the International GCSE Mathematics (Specification A) (Higher Tier) and the International GCSE Mathematics (Specification B).

This specification has been designed for students who have a high ability in or are motivated by mathematics.

The range of grades available for this qualification is for the International GCSE Higher Tier, A* – D with a 'safety net' grade E available.

Key subject aims

The Edexcel International GCSE in Further Pure Mathematics encourages students to:

- develop knowledge and understanding of mathematical concepts and techniques
- develop mathematical skills for further study in the subject or related areas
- enjoy using and applying mathematical techniques and concepts, and become confident to use mathematics to solve problems
- appreciate the importance of mathematics in society, employment and study.

About this specification

Key features and benefits of the specification

The Edexcel International GCSE in Further Pure Mathematics has been developed to:

- provide a broad overview of mathematical techniques for those who may not study mathematics beyond this level or for those whose course of study requires a knowledge of mathematical techniques beyond International GCSE Mathematics content
- provide a course of study for those whose mathematical competence may have developed early
- enable students to acquire knowledge and skills with confidence, satisfaction and enjoyment
- develop an understanding of mathematical reasoning and processes, and the ability to relate different areas of mathematics
- develop resourcefulness when solving problems
- provide a solid basis for students wishing to progress to Edexcel's AS and Advanced GCE in Mathematics, or equivalent qualifications
- provide papers that are balanced for topics and difficulty.

Contents

Specification at a glance

The Edexcel International GCSE in Further Pure Mathematics comprises two externally assessed examination papers.

The qualification is offered through a single tier.

Questions are targeted at grades in the range A*– D.

There is a 'safety net' grade E for students who narrowly fail to achieve grade D.

Students who fail to achieve grade E will be awarded 'Ungraded'.

Paper 1	**Paper code: 4PM0/01**
Paper 2	**Paper code: 4PM0/02**
• Externally assessed • Availability: January and June series • First assessment: June 2011	**Each paper is 50% of the total International GCSE marks**
Overview of content in both papers: • Number • Algebra and calculus • Geometry and trigonometry	
Overview of assessment • Each examination paper will be 2 hours. • Each paper will carry a total of 100 marks. • Each paper will consist of about 11 questions with varying mark allocations per question which will be stated on the paper. • Each paper will address all of the Assessment Objectives. • Each paper will have approximately equal marks available for each of the targeted grades. • Calculators are allowed. • Questions will be set in SI units. • Each paper will contain questions from more than one section of the specification content and the solution of any question may require knowledge of more than one section of the specification content.	

Notation

The following notation will be used.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers $\{x: x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers $\{x: x \in \mathbb{R}, x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero $\{x: x \in \mathbb{R}, x \geq 0\}$
$\lvert x \rvert$	the modulus of x, $\lvert x \rvert = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$
$\approx$	is approximately equal to
$\sum_{r=1}^{n} \mathrm{f}(r)$	$\mathrm{f}(1) + \mathrm{f}(2) + \ldots + \mathrm{f}(n)$
$\binom{n}{r}$	the binominal coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{R}^+$ $\frac{n(n-1)...(n-r+1)}{r!}$ for $n \in \mathbb{R}$
$\ln x$	the natural logarithm of x, $\log_e x$
$\lg x$	the common logarithm of x, $\log_{10} x$
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\mathrm{f}'''(x)$,	the first, second and third derivatives of $\mathrm{f}(x)$ with respect to x
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$

Calculators

Students are expected to have a calculator available with at least the following keys:

+, -, ×, ÷, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, ln x, e^x, sine, cosine and tangent and their inverses in degrees and decimals of a degree or radians.

Calculators with any of the following facilities are prohibited in any examination:

- databanks, retrieval of text or formulae, QWERTY keyboards, built-in symbolic algebra manipulations, symbolic differentiation or integration.

Qualification content

Knowledge, skills and understanding

This Edexcel International GCSE in Further Pure Mathematics requires students to demonstrate application and understanding of the following.

Number

Students should:

- be able to apply their numerical skills in a purely mathematical way and to real-life situations.

Algebra and calculus

Students should:

- use algebra and calculus to set up and solve problems
- develop competence and confidence when manipulating mathematical expressions
- construct and use graphs in a range of situations.

Geometry and trigonometry

Students should use:

- properties of shapes, angles and transformations
- vectors and rates of change to model situations
- coordinate geometry
- trigonometry.

Summary of the specification content

Students will be expected to have a thorough knowledge of the content common to either of these Edexcel International GCSE Mathematics specifications:

- International GCSE in Mathematics (Specification A) (Higher Tier)
- International GCSE in Mathematics (Specification B).

This specification does not require knowledge of the statistics or matrices content of these specifications.

Questions may be set which assumes knowledge of some topics covered in these specifications.

Students will be expected to carry out arithmetic and algebraic manipulation, such as being able to:

- change the subject of a formula
- evaluate numerically the value of any variable in a formula, given the values of the other variables.

The use and notation of set theory will be adopted wherever appropriate.

Specification content

1	Logarithmic functions and indices	Notes
	The functions a^x and $\log_b x$ (where b is a natural number greater than one).	A knowledge of the shape of the graphs of a^x and $\log_b x$ is expected, but not a formal expression for the gradient.
	Use and properties of indices and logarithms including change of base.	To include: $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^k = k \log_a x$, $\log_a \frac{1}{x} = -\log_a x$, $\log_a a = 1$ $\log_a 1 = 0$ The solution of equations of the form $a^x = b$. Students may use the change of base formulae: $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_a b = \frac{1}{\log_b a}$
	Simple manipulation of surds.	Students should understand what surds represent and their use for exact answers. Manipulation will be very simple. For example: $5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$ $\sqrt{48} = 4\sqrt{3}$
	Rationalising the denominator where the denominator is a pure surd.	$10 \times \frac{1}{\sqrt{5}} = 2\sqrt{5}$

2	The quadratic function	Notes
	The manipulation of quadratic expressions.	Students should be able to factorise quadratic expressions and complete the square.
	The roots of a quadratic equation.	Students should be able to use the discriminant to identify whether the roots are equal real, unequal real or not real.
	Simple examples involving functions of the roots of a quadratic equation.	Students are expected to understand and use: $ax^2 + bx + c = 0$ has roots $\alpha, \beta = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and forming an equation with given roots, which are expressed in terms of α and β: $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

3	Identities and inequalities	Notes
	Simple algebraic division.	Division by $(x + a)$, $(x - a)$, $(ax + b)$ or $(ax - b)$ will be required.
	The factor and remainder theorems.	Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$. Students may be required to factorise cubic expressions such as: $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$, when a factor has been provided. Students should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial $f(x)$ is divided by $(ax + b)$ or $(ax - b)$.
	Solutions of equations, extended to include the simultaneous solution of one linear and one quadratic equation in two variables.	The solution of a cubic equation containing at least one rational root may be set.
	Simple inequalities, linear and quadratic.	For example $ax + b > cx + d$, $px^2 + qx + r < sx^2 + tx + u$.
	The graphical representation of linear inequalities in two variables.	The emphasis will be on simple questions designed to test fundamental principles. Simple problems on linear programming may be set.

4	**Graphs**	**Notes**
	Graphs of polynomials and rational functions with linear denominators. The solution of equations (which may include transcendental functions) by graphical methods.	The concept of asymptotes parallel to the coordinate axes is expected. Non-graphical iterative methods are not required.
5	**Series**	**Notes**
	Use of the $\sum$ notation. Arithmetic and geometric series.	The $\sum$ notation may be employed wherever its use seems desirable. The general term and the sum to n terms of an arithmetic series are required. The general term of a geometric series is required. The sum to n terms of a finite geometric series is required. The sum to infinity of a convergent geometric series, including the use of $\lvert r \rvert < 1$ is required. Proofs of these are not required.
6	**The Binomial series**	**Notes**
	Use of the binomial series $(1 + x)^n$.	Use of the series when: (i) n is a positive integer (ii) n is rational and $\lvert x \rvert < 1$. The validity condition for (ii) is expected.

7	Scalar and vector quantities	Notes
	The addition and subtraction of coplanar vectors and the multiplication of a vector by a scalar.	Knowledge of the fact that if $\alpha_1 \mathbf{a} + \beta_1 \mathbf{b} = \alpha_2 \mathbf{a} + \beta_2 \mathbf{b}$, where **a** and **b** are non-parallel vectors, then $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, is expected.
	Components and resolved parts of a vector.	Use of the vectors **i** and **j** will be expected.
	Magnitude of a vector.	
	Position vector.	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$
	Unit vector.	
	Use of vectors to establish simple properties of geometrical figures.	The 'simple properties' will, in general, involve collinearity, parallel lines and concurrency. Position vector of a point dividing the line *AB* in the ratio *m:n* is expected.

8	**Rectangular cartesian coordinates**	**Notes**
	The distance between two points.	The distance d between two points (x_1, y_1) and (x_2, y_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$.
	The point dividing a line in a given ratio.	The coordinates of the point dividing the line joining (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ are given by $\left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$
	Gradient of a straight line joining two points.	
	The straight line and its equation.	The $y = mx + c$ and $y - y_1 = m(x - x_1)$ forms of the equation of a straight line are expected to be known. The interpretation of $ax + by = c$ as a straight line is expected to be known.
	The condition for two lines to be parallel or to be perpendicular.	

9	**Calculus**	**Notes**
	Differentiation and integration of sums of multiples of powers of x (excluding integration of $\frac{1}{x}$), sin ax, cos ax, e^{ax}.	No formal proofs of the results for ax^n, sin ax, cos ax and e^{ax} will be required.
	Differentiation of a product, quotient and simple cases of a function of a function.	
	Applications to simple linear kinematics and to determination of areas and volumes.	Understanding how displacement, velocity and acceleration are related using calculus. The volumes will be obtained only by revolution about the coordinate axes.
	Stationary points.	
	Maxima and minima.	Maxima and minima problems may be set in the context of a practical problem. Justification of maxima and minima will be expected.
	The equations of tangents and normals to the curve $y = f(x)$.	$f(x)$ may be any function which the students are expected to be able to differentiate.
	Application of calculus to rates of change and connected rates of change.	The emphasis will be on simple examples to test principles. A knowledge of $\mathrm{d}y \approx \frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x$ for small $\mathrm{d}x$ is expected.

10	Trigonometry	Notes
	Radian measure, including use for arc length and area of sector.	The formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for a circle are expected to be known.
	The three basic trigonometric ratios of angles of any magnitude (in degrees or radians) and their graphs.	To include the exact values for sine, cosine and tangent of 30°, 45°, 60° (and the radian equivalents), and the use of these to find the trigonometric ratios of related values such as 120°, 300°.
	Applications to simple problems in two or three dimensions (including angles between a line and a plane and between two planes).	
	Use of the sine and cosine formulae.	General proofs of the sine and cosine formulae will not be required. The formulae are expected to be known. The area of a triangle in the form $\frac{1}{2}ab\sin C$ is expected to be known.
	The identity $\cos^2\theta + \sin^2\theta = 1$.	$\cos^2\theta + \sin^2\theta = 1$ is expected to be known.
	Use of the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ will be provided when needed.
	The use of the basic addition formulae of trigonometry.	Formal proofs of basic formulae will not be required. Questions using the formulae for $\sin(A + B)$, $\cos(A + B)$, $\tan(A + B)$ may be set; the formulae will be provided, for example: $\sin(A + B) = \sin A\cos B + \cos A\sin B$ Long questions, explicitly involving excessive manipulation, will not be set.
	Solution of simple trigonometric equations for a given interval.	Students should be able to solve equations such as: $\sin\left(x - \frac{\pi}{2}\right) = \frac{3}{4}$ for $0 < x < 2\pi$, $\cos(x + 30°) = \frac{1}{2}$ for $-180° < x < 180°$, $\tan 2x = 1$ for $90° < x < 270°$, $6\cos^2 x° + \sin x° - 5 = 0$ for $0 \leq x < 360$, $\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$ for $-\pi \leq x < \pi$.

Assessment

Assessment summary

- Each paper is externally assessed.
- Each paper carries a total of 100 marks.
- The examination duration for each paper is 2 hours.
- Each paper contributes 50% of the final grade.
- Calculators are allowed in each paper.

Assessment Objectives and weightings

		% in International GCSE
AO1	demonstrate a confident knowledge of the techniques of pure mathematics required in the specification	**30–40%**
AO2	apply a knowledge of mathematics to the solutions of problems for which an immediate method of solution is not available and which may involve knowledge of more than one topic in the specification	**20–30%**
AO3	write clear and accurate mathematical solutions	**35–50%**
	TOTAL	**100%**

Relationship of Assessment Objectives to Papers for International GCSE

Paper number	**Assessment Objective**			
	AO1	**AO2**	**AO3**	**Total for AO1, AO2 and AO3**
1	15–20%	10–15%	17.5–25%	50%
2	15–20%	10–15%	17.5–25%	50%
Total for International GCSE	30–40%	20–30%	35–50%	100%

Entering your students for assessment

Student entry

Details of how to enter students for this qualification can be found in Edexcel's *International Information Manual*, copies of which are sent to all active Edexcel centres. The information can also be found on Edexcel's international website.

Combinations of entry

There are no forbidden combinations.

Access arrangements and special requirements

Edexcel's policy on access arrangements and special considerations for GCE, GCSE, International GCSE, and Entry Level qualifications aims to enhance access to the qualifications for students with disabilities and other difficulties without compromising the assessment of skills, knowledge, understanding or competence.

Please see the Edexcel website (www.edexcel.org.uk/sfc) for:

- the Joint Council for Qualifications (JCQ) policy *Access Arrangements and Special Considerations, Regulations and Guidance Relating to Students who are Eligible for Adjustments in Examinations*
- the forms to submit for requests for access arrangements and special considerations
- dates for submission of the forms.

Requests for access arrangements and special considerations must be addressed to:

Special Requirements
Edexcel
One90 High Holborn
London WC1V 7BH

Assessing your students

The first assessment opportunity for this qualification will take place in the June 2011 series and in each following January and June series for the lifetime of the specification.

Your student assessment opportunities

Paper availability	June 2011	January 2012	June 2012	January 2013
Paper 1 and Paper 2	✓	✓	✓	✓

Awarding and reporting

The grading, awarding and certification of this qualification will follow the processes outlined in the current GCSE/GCE Code of Practice for courses starting in September 2009, which is published by the Qualifications and Curriculum Authority (QCA). The International GCSE qualification will be graded and certificated on a six-grade scale from A* to E.

Students whose level of achievement is below the minimum standard for Grade E will receive an unclassified U. Where unclassified is received it will not be recorded on the certificate.

The first certification opportunity for the Edexcel International GCSE in Further Pure Mathematics will be 2011.

Language of assessment

Assessment of this specification will be available in English only. Assessment materials will be published in English only and all work submitted for examination must be produced in English.

Malpractice and plagiarism

For up-to-date advice on malpractice and plagiarism, please refer to the JCQ's *Suspected Malpractice in Examinations: Policies and Procedures* document on the JCQ website www.jcq.org.uk.

Student recruitment

Edexcel's access policy concerning recruitment to our qualifications is that:

- they must be available to anyone who is capable of reaching the required standard
- they must be free from barriers that restrict access and progression
- equal opportunities exist for all students.

Previous knowledge

Students will be expected to have thorough knowledge of the content common to both of these specifications:

- International GCSE in Mathematics (Specification A) (Higher Tier)
- International GCSE in Mathematics (Specification B).

Knowledge of matrices or statistics content in these specifications are not required.

Progression

This qualification supports progression to:

- GCE AS and Advanced Level in Mathematics
- GCE AS and Advanced Level in Further Mathematics
- GCE AS and Advanced Level in Pure Mathematics
- GCE and other Level 3 qualifications in numerate disciplines, such as the sciences, economics or business
- further training or employment where numeracy skills are required.

Grade descriptions

The following grade descriptions indicate the level of attainment characteristic of the given grade at International GCSE. They give a general indication of the required learning outcomes at each specified grade. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the student has met the Assessment Objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A

Candidates can:

- recall or recognise almost all the mathematical facts, concepts and techniques that are needed and can select appropriate ones to use in a wide variety of contexts
- manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill
- use mathematical language correctly and can construct logical and rigorous extended arguments
- when confronted with an unstructured problem, often devise and implement an effective strategy for its solution
- sometimes notice and correct their own errors
- make appropriate and efficient use of contemporary calculator technology and are aware of any limitations to its use
- present answers to the stated degree of accuracy or give an exact form.

Grade C

Candidates can:

- recall or recognise many of the mathematical facts, concepts and techniques that are needed and can usually select appropriate ones to use in a wide variety of contexts
- manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill
- use mathematical language with some skill and can sometimes construct complete logical and rigorous extended arguments
- when confronted with an unstructured problem, sometimes devise and implement an effective strategy for its solution
- occasionally notice and attempt to correct their own errors
- usually make appropriate and efficient use of contemporary calculator technology and are sometimes aware of any limitations to its use
- usually present answers to the stated degree of accuracy.

Grade D

Candidates can:

- recall or recognise some of the mathematical facts, concepts and techniques that are needed and sometimes select appropriate ones to use in some contexts
- manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill
- sometimes use mathematical language correctly and occasionally construct logical and rigorous extended arguments
- often make appropriate and efficient use of contemporary calculator technology
- often present answers to the stated degree of accuracy.

Support and training

Edexcel support services

Edexcel has a wide range of support services to help you implement this qualification successfully.

ResultsPlus — ResultsPlus is an application launched by Edexcel to help subject teachers, senior management teams, and students by providing detailed analysis of examination performance. Reports that compare performance between subjects, classes, your centre and similar centres can be generated in 'one-click'. Skills maps that show performance according to the specification topic being tested are available for some subjects. For further information about which subjects will be analysed through ResultsPlus, and for information on how to access and use the service, please visit www.edexcel.org.uk/resultsplus.

Ask the Expert — Ask the Expert is a new service, launched in 2007, that provides direct email access to senior subject specialists who will be able to answer any questions you might have about this or any other specification. All of our specialists are senior examiners, moderators or verifiers and they will answer your email personally. You can read a biography for all of them and learn more about this unique service on our website at www.edexcel.org.uk/asktheexpert.

Ask Edexcel — Ask Edexcel is Edexcel's online question and answer service. You can access it at www.edexcel.org.uk/ask or by going to the main website and selecting the Ask Edexcel menu item on the left.

The service allows you to search through a database of thousands of questions and answers on everything Edexcel offers. If you don't find an answer to your question, you can choose to submit it straight to us. One of our customer services team will log your query, find an answer and send it to you. They'll also consider adding it to the database if appropriate. This way the volume of helpful information that can be accessed via the service is growing all the time.

Examzone — The Examzone site is aimed at students sitting external examinations and gives information on revision, advice from examiners and guidance on results, including re-marking, re-sitting and progression opportunities. Further services for students — many of which will also be of interest to parents — will be available in the near future. Links to this site can be found on the main homepage at www.examzone.co.uk.

Training

A programme of professional development and training courses, covering various aspects of the specification and examination, will be arranged by Edexcel. Full details can be obtained from our website: www.edexcel.org.uk.

Appendices

Appendix 1: Suggested resources

Textbooks

- For this Edexcel International GCSE, the following titles could be used as teaching aids.
- Although they are designed for AS/A2 in the UK, these course books are equally useful for International GCSE students at this level.
- The list is not exhaustive.
- The books listed are neither recommended by Edexcel nor mandatory for International GCSE qualifications.
- The internet is valuable as a tool for research and learning.

Please note that while resources are checked at the time of publication, materials may be withdrawn from circulation and website locations may change at any time.

Backhouse J K, Houldsworth S T P and Horril P J F – *Pure Mathematics: A First Course* (Longman, 1991) ISBN 0582066581

Bostock L and Chandler S – *Mathematics: The Core Course for Advanced Level* (Nelson Thornes, 2000) ISBN 0748755098

Emanuel R and Wood J – *Longman Advanced Maths AS Core for Edexcel* and *A2 Core for Edexcel* (Longman, 2006)

Pledger K et al – *Edexcel AS and A2 Modular Mathematics Units C1 to C4* (Heinemann, 2008-9)

Sadler A J and Thorning D W S – *Understanding Pure Mathematics* (Oxford University Press, 1987) ISBN 0199142439

Smedley R and Wiseman G – *Introducing Pure Mathematics* (Oxford University Press, 2001) ISBN 0199148035

Websites

www.math.com

www.mathsnet.net

Appendix 2: Formulae

This appendix gives formulae that students are expected to remember and will not be included on the examination paper.

Logarithmic functions and indices

$\log_a xy = \log_a x + \log_a y$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a x^k = k \log_a x$

$\log_a \frac{1}{x} = -\log_a x$

$\log_a a = 1$

$\log_a x = \frac{\log_b x}{\log_b a}$

$\log_a 1 = 0$

$\log_a b = \frac{1}{\log_b a}$

Quadratic equations

$ax^2 + bx + c = 0$ has roots given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

When the roots of $ax^2 + bx + c = 0$ are α and β then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

and the equation can be written $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Series

Arithmetic series: nth term = $l = a + (n-1)d$

Sum to n terms $= \frac{n}{2}\{2a + (n-1)d\}$

Geometric series: nth term $= ar^{n-1}$

Sum to n terms $= \frac{a(r^n - 1)}{r - 1}$

Sum to infinity $= \frac{a}{1-r}$ $\quad |r| < 1$

Binomial series

for $|x| < 1$, $n \in \mathbb{Q}$,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots$$

Coordinate geometry

The gradient of the line joining two points (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by

$$d^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2$$

The coordinates of the point dividing the line joining (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ are $\left(\dfrac{nx_1 + mx_2}{m+n}, \dfrac{ny_1 + my_2}{m+n}\right)$

Calculus

Differentiation:	function	derivative
	x^n	nx^{n-1}
	$\sin ax$	$a\cos ax$
	$\cos ax$	$-a\sin ax$
	e^{ax}	$a\mathrm{e}^{ax}$
	$\mathrm{f}(x)\mathrm{g}(x)$	$\mathrm{f}'(x)\mathrm{g}(x) + \mathrm{f}(x)\mathrm{g}'(x)$
	$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$
	$\mathrm{f}(\mathrm{g}(x))$	$\mathrm{f}'(\mathrm{g}(x))\mathrm{g}'(x)$

Integration:	function	integral
	x^n	$\dfrac{1}{n+1}x^{n+1} + c \qquad n \neq -1$
	$\sin ax$	$-\dfrac{1}{a}\cos ax + c$
	$\cos ax$	$\dfrac{1}{a}\sin ax + c$
	e^{ax}	$\dfrac{1}{a}\mathrm{e}^{ax} + c$

Area and volume:

Area between a curve and the x axis = $\int_a^b y\,dx,\ y \geq 0$

$\left|\int_a^b y\,dx\right|,\ y < 0$

Area between a curve and the y-axis = $\int_c^d x\,dy\ ,\ x \geq 0$

$\left|\int_c^d x\,dy\right|,\ x < 0$

Area between g(x) and f(x) = $\int_a^b \left|g(x) - f(x)\right| dx$

Volume of revolution $= \int_a^b \pi y^2 dx$ or $\int_c^d \pi x^2 dy$

Trigonometry

Radian measure: length of arc $= r\theta$

area of sector $= \frac{1}{2} r^2 \theta$

In triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos^2\theta + \sin^2\theta = 1$

area of a triangle = $\frac{1}{2}ab\sin C$

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International GCSE

Further Pure Mathematics (4PM0)

Sample Assessment Material

First examination 2011

ALWAYS LEARNING

PEARSON

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						4 PM 0 / 0 1	Signature	

Paper Reference(s)

4PM0/01

Edexcel International GCSE

Further Pure Mathematics

Paper 1

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Sample Assessment Material

Time: 2 hours

Materials required for examination
Nil

Items included with question papers
Nil

Candidates are expected to have an electronic calculator when answering this paper.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 100.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

Write your answers neatly and legibly.

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1. Solve the equations

$$x^2 + 4x - xy = 10$$

$$2x - y = 3$$

(6)

Q1

(Total 6 marks)

Leave blank

2.

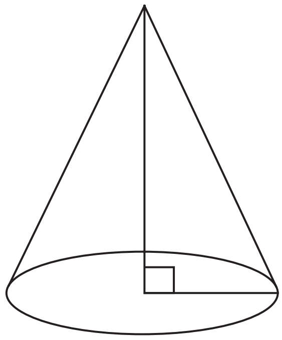


Figure 1

The volume of a right circular cone is increasing at the rate of $45\text{ cm}^3\text{ s}^{-1}$.
The height of the cone is always three times the radius of the base of the cone.
Find the rate of increase of the radius of the base, in cm s^{-1}, when the radius of the cone is 4 cm.
Give your answer correct to 3 significant figures.

(6)

Q2

(Total 6 marks)

Leave blank

3. A curve has equation $y = 2 + \frac{1}{x+1}$, $x \neq -1$

(a) Find an equation of the asymptote to the curve which is parallel to

(i) the x-axis, (ii) the y-axis.

(2)

(b) Find the coordinates of the points where the curve crosses the coordinate axes.

(2)

(c) Sketch the curve, showing clearly the asymptotes and the coordinates of the points where the curve crosses the coordinate axes.

(3)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4. The sum of the first four terms of an arithmetic series is 34
The sum of the first six terms of the series is 69

Find,

(a) the common difference of the series,
(4)

(b) the first term of the series.
(1)

The sum of the first p terms of this series is 650

(c) Find the value of p.
(3)

Another arithmetic series is formed.
The sum of the first four terms of the new series is 54
The sum of the first six terms of this new series is 99

Find, for the new series,

(d) the common difference,
(1)

(e) the first term.
(1)

Leave blank

Question 4 continued

(Total 10 marks)

Q4

Leave blank

5.

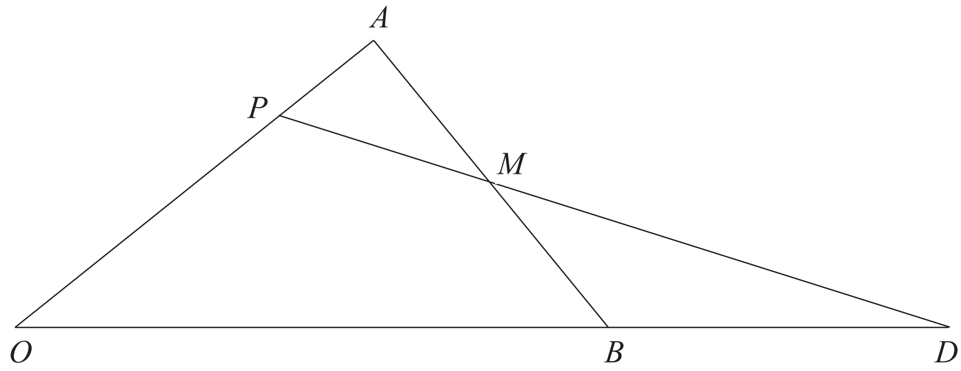


Figure 2

In Figure 2, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and M is the midpoint of AB.
The point P divides OA in the ratio 2:1, and PM produced meets OB produced at D.

(a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(i) $\overrightarrow{AB}$,

(ii) $\overrightarrow{PA}$,

(iii) $\overrightarrow{PM}$.

(4)

Given that $\overrightarrow{PD} = \mu\overrightarrow{PM}$ and $\overrightarrow{OD} = \lambda\overrightarrow{OB}$,

(b) find the value of μ and the value of λ.

(4)

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. (a) Complete the table below of values for $y = e^{-\frac{1}{2}x} + 1$, giving your values of y to 2 decimal places.

x	–1	0	1	2	3	4	5
y		2	1.61		1.22	1.14	

(2)

(b) Draw the graph of $y = e^{-\frac{1}{2}x} + 1$ for $-1 \leqslant x \leqslant 5$

(2)

(c) Use your graph to estimate, to 2 significant figures, the solution of the equation

$$e^{-\frac{1}{2}x} = 0.8$$

showing your method clearly.

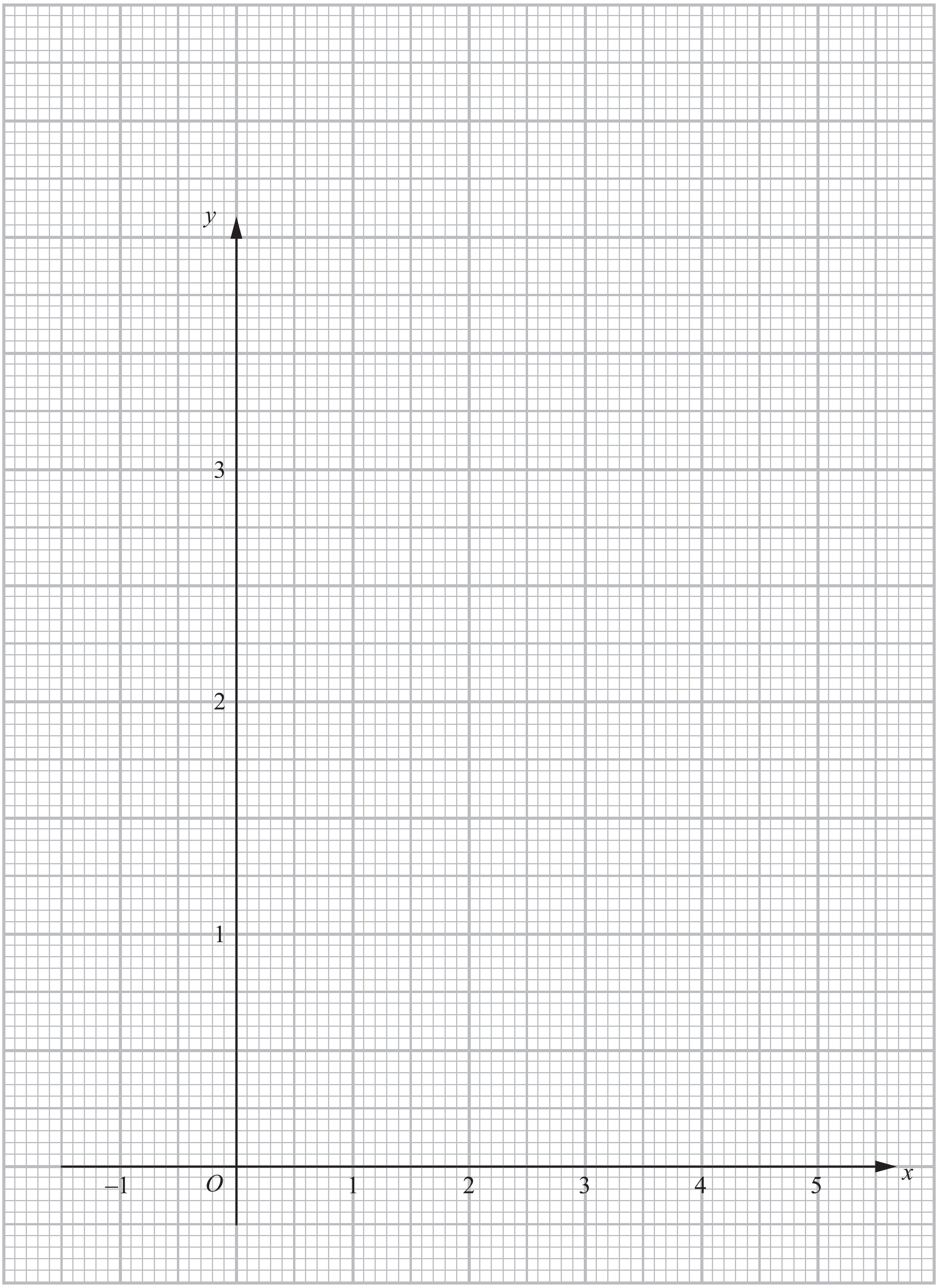
(2)

(d) By drawing a straight line on your graph, estimate, to 2 significant figures, the solution of the equation $x = -2\ln(2x - 7)$.

(4)

Leave blank

Question 6 continued



Q6

(Total 10 marks)

Leave blank

7.

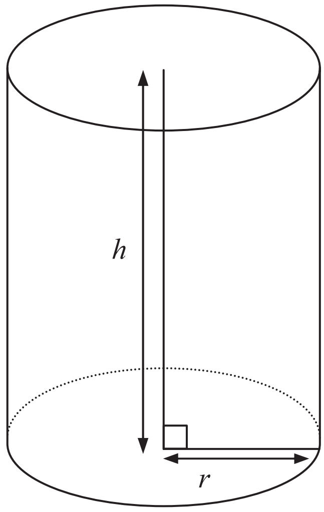


Figure 3

A water tank is in the shape of a right circular cylinder with no lid.
The base of the cylinder is a circle of radius r cm and the height is h cm.
The total external surface area of the tank is A cm^2.
The capacity of the tank is $50\,000\pi$ cm^3.

(a) Show that $A = \left(\frac{100\,000}{r} + r^2\right)\pi$.

(4)

(b) Find, to the nearest whole number, the minimum value of A.
Verify that the value you have found is a minimum.

(6)

Leave blank

Question 7 continued

(Total 10 marks)

Q7

Leave blank

8. The equation $x^2 + 2tx + t = 0$, where t is a non-zero constant, has roots α and β, where $\alpha > \beta$.

(a) Find, in terms of t,

(i) $\alpha^2 + \beta^2$, (ii) $\alpha^2\beta^2$.

(5)

Given that $10\alpha^2\beta^2 = 3(\alpha^2 + \beta^2)$,

(b) find the value of t.

(3)

Using your value of t,

(c) find the exact value of $\alpha - \beta$, giving your answer in the form $p\sqrt{q}$, where p and q are integers and $p \neq 1$.

(3)

Leave blank

Question 8 continued

Q8

(Total 11 marks)

Leave blank

9. Using $\cos(A + B) = \cos A \cos B - \sin A \sin B$,

(a) show that

(i) $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$,

(ii) $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$.

(4)

Given

$$f(\theta) = 1 + 10\sin^2\theta - 16\sin^4\theta$$

(b) Show that $f(\theta) = 3\cos 2\theta - 2\cos 4\theta$

(4)

(c) Solve the equation

$$1 + 10\sin^2\theta° - 16\sin^4\theta° + 2\cos 4\theta° = 0.25 \text{ for } 0 \leqslant \theta \leqslant 180,$$

giving your solutions to 1 decimal place.

(4)

Given that $\int_0^{\frac{\pi}{8}} f(\theta)\,d\theta = q + p\sqrt{2}$,

(d) find the value of p and the value of q.

(5)

Leave blank

Question 9 continued

Q9

(Total 17 marks)

Leave blank

10. The points A and B have coordinates (1, 6) and (6, –4) respectively.
The point K divides AB internally in the ratio 2:3

(a) Show that the coordinates of K are (3, 2).

(2)

The line l passes through K and is perpendicular to AB.

(b) Find an equation, with integer coefficients, for l.

(4)

The point E, with coordinates (7, e) lies on l.

(c) Find the value of e.

(1)

The line EK is produced to D so that $EK = KD$.

(d) Find the coordinates of D.

(2)

(e) Find the area of the kite $AEBD$.

(3)

The circle C passes through A, D and K.

(f) Find (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C,

(iii) the area of C, giving your answer in terms of π.

(3)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 15 marks)

TOTAL FOR PAPER: 100 MARKS

END